



Division of Engineering  
Brown University

## EN40: Dynamics and Vibrations

**Final Examination**  
**Thursday May 20 2010**

**NAME:** \_\_\_\_\_

### General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- *If you find you are unable to complete part of a question, proceed to the next part.*

### Please initial the statement below to show that you have read it

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

\_\_\_\_\_

**1-8: (40 PTS)** \_\_\_\_\_

**9: (20 PTS)** \_\_\_\_\_

**10: (10 PTS)** \_\_\_\_\_

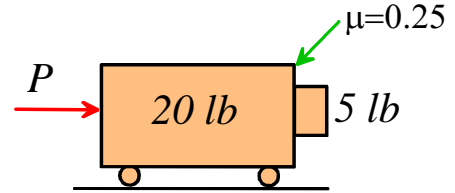
**11: (20 PTS)** \_\_\_\_\_

**TOTAL (90 PTS)** \_\_\_\_\_

**FOR PROBLEMS 1-8 WRITE YOUR ANSWER IN THE SPACE PROVIDED.**

**ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.**

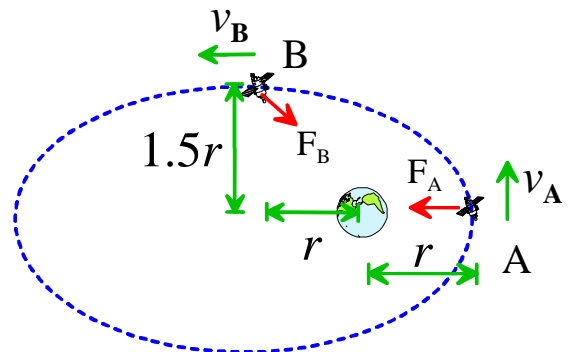
1. What is the minimum acceleration of the cart and block shown below so that the 5lb block does not slip off the front of the 20lb cart? (g denotes the gravitational acceleration, and  $\mu$  is the coefficient of friction.)



- (a) g      (b) 2g      (c) 3g      (d) 4g      (e) 10g

ANSWER \_\_\_\_\_ (3 POINTS)

2. A satellite is orbiting the earth. The velocity and force vectors for the satellite at two points A and B on the orbit are shown. What is the speed at point B in terms of the speed  $v_A$  at point A?



- (a) Need more info  
 (b)  $v_B = \frac{1}{3}v_A$   
 (c)  $v_B = \frac{1}{2}v_A$   
 (d)  $v_B = \frac{2}{3}v_A$   
 (e)  $v_B = v_A$

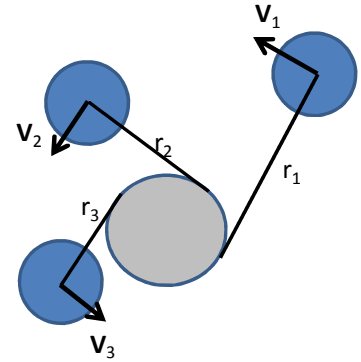
ANSWER \_\_\_\_\_ (3 POINTS)

3. An EN40 student needs to weigh her bowling ball before taking it home for the summer, because she does not want to pay over-weight charges for her luggage. She does not have a scale. So, she decides to use her EN40 Trifilar Pendulum to determine the mass. Her trifilar pendulum has a mass  $m=0.5\text{kg}$  and moment of inertia  $I=50 \text{ kg cm}^2$ . With the bowling ball, she obtains a combined moment of inertia of  $I=250 \text{ kg cm}^2$ . She knows that the moment of inertia for a sphere of mass  $M$  and radius  $r$  is  $I = \frac{2}{5}Mr^2$ . The bowling ball has radius 10cm. What is the mass of the bowling ball?

- (a) 0.5kg      (b) 2.5kg      (c) 5kg      (d) 7kg      (e) 10kg

ANSWER \_\_\_\_\_ (3 POINTS)

4. A ball is tied to an inextensible string that wraps around a thick pole. The pole is fixed. A top view is shown in the figure at several different times. The velocity of the ball is always at right angles to the direction of the string. *Neglect gravity.*



(i) (2 pts) Is linear momentum of the ball conserved during this motion? Why or why not?

(ii) (2 pts) Is energy of the ball conserved during this motion? Why or why not?

(iii) (2 pts) Is angular momentum of the ball conserved during this motion? Why or why not?

5. A motor has a torque curve given by

$$T = T_s \left( 1 - \frac{\omega}{\omega_{nl}} \right)^2$$

where  $T$  is the torque exerted by the motor,  $\omega$  is its angular speed of the motor, and  $T_s, \omega_{nl}$  are constants.

5.1 What is the maximum power that can be extracted from this motor?

- (a)  $\frac{4}{27} T_s \omega_{nl}$       (b)  $\frac{1}{3} T_s \omega_{nl}$       (c)  $\frac{4}{9} T_s \omega_{nl}$       (d)  $\frac{16}{27} T_s \omega_{nl}$       (e)  $\frac{2}{3} T_s \omega_{nl}$

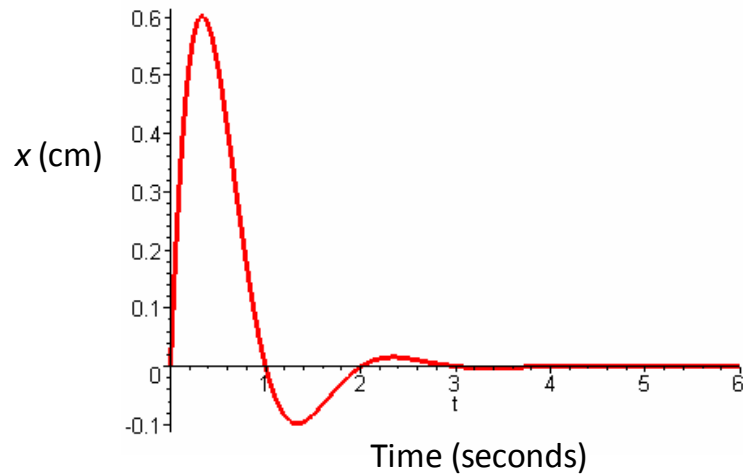
ANSWER \_\_\_\_\_ (3 POINTS)

5.2 The motor is used to power a vehicle of mass  $m$  and wheel radius  $R$  moving up a  $30^\circ$  incline. If friction in the transmission can be neglected, what is the gear ratio of the transmission (ie  $\omega_a / \omega$ , where  $\omega_a$  is the axle speed) needed to achieve the max speed?

- (a)  $\frac{4}{81} \frac{T_s}{mgR}$       (b)  $\frac{4}{27} \frac{T_s}{mgR}$       (c)  $\frac{16}{27} \frac{T_s}{mgR}$       (d)  $\frac{8}{9} \frac{T_s}{mgR}$       (e) need more info

ANSWER \_\_\_\_\_ (3 POINTS)

6. The displacement versus-time curve for the free vibration of a machine part is shown.



6.1 What is the best estimate of the damped natural period of the system?

- a) 0.4 sec      b) 1.0 sec      c) 1.4 sec      d) 2.0 sec      e) 2.4 sec

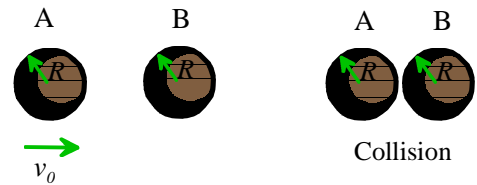
ANSWER \_\_\_\_\_ (3 POINTS)

6.2 What is the best estimate of the viscous damping factor  $\zeta$  for the system?

- a) 0.03      b) 0.5      c) 0.85      d) 1.2      d) need more info

ANSWER \_\_\_\_\_ (3 POINTS)

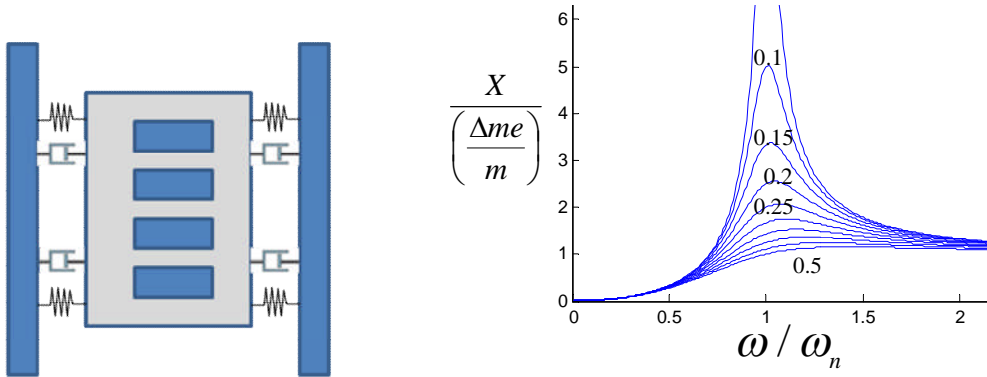
7. The figure shows a collision between two identical spheres. The restitution coefficient for the collision  $e=0$ . Before the collision, A moves with speed  $v_0$  and B is stationary. After the collision



- (a) Sphere A is stationary and sphere B moves to the right  
 (b) Sphere B is stationary and sphere A moves to the left  
 (c) Both spheres move to the right at the same speed  
 (d) Both spheres A and B move to the right with different speeds  
 (e) Sphere B moves to the right and sphere A moves to the left.  
 (f) Both spheres are stationary

ANSWER \_\_\_\_\_ (3 POINTS)

8. The Subaru Legacy has an unusual horizontally-aligned four-cylinder engine. The engine is connected to lateral motor mounts by 4 springs and 4 dashpots, as shown in the figure. When idling, slightly differences in the firing of the individual cylinders lead to an effective *rotor forcing* of the engine, with an effective mass imbalance of  $e\Delta m = 0.4kgm$  at a frequency corresponding to  $600/\pi$  RPM. The total mass of the engine is  $M=200kg$ . Brand new, each spring has stiffness  $k=12800N/m$  and each damper has damping coefficient  $c=400 N\cdot s/m$ .



8.1 What are the natural frequency and damping coefficient  $\zeta$  for the engine?

- (a)  $4 \frac{rad}{s}$ , 0.125    (b)  $16 \frac{rad}{s}$ , 0.125    (c)  $4 \frac{rad}{s}$ , 0.25    (d)  $16 \frac{rad}{s}$ , 0.25

ANSWER \_\_\_\_\_ (3 POINTS)

8.2 What is the approximate steady-state amplitude of the lateral vibrations of the engine?

- (a) 1mm    (b) 2mm    (c) 4mm    (d) 8mm    (e) need more info

ANSWER \_\_\_\_\_ (3 POINTS)

8.3 The main problem is not the engine vibration, but the forces caused on the attachment points to the body of the car. These attachment fixtures fatigue under load. What is the approximate amplitude of the force exerted on one damper attachment point for the new car?

- (a) 2 N    (b) 4 N    (c) 8 N    (d) 16 N    (e) 32 N

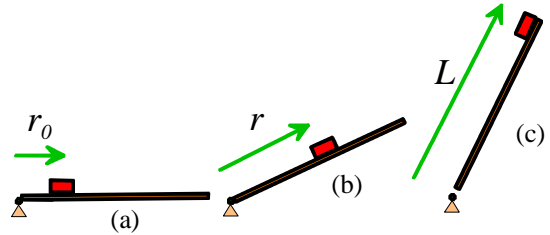
ANSWER \_\_\_\_\_ (4 POINTS)

8.4 As the car ages, the spring stiffness gets smaller. How does this change the vibration amplitude?

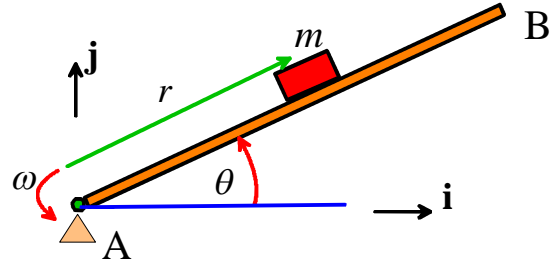
- a) Increases    b) Decreases    c) Stays the same    d) Need more info

ANSWER \_\_\_\_\_ (3 POINTS)

9. (20 pts) The figure shows a simple idealization of a centrifugal pump. The vane AB rotates with constant angular velocity  $d\theta/dt = \omega$  about A. A small particle of fluid with mass  $m$  slides along the vane (figs a,b), and is eventually ejected from the pump (fig c). The goal of this problem is to derive the equation of motion for the distance  $r(t)$  shown in the figure. **NEGLECT FRICTION AND GRAVITY.**



9.1 (2 pts) Write down the position vector  $\mathbf{r}$  of the particle in terms of  $r$  and  $\theta$ , using the  $\{\mathbf{i}, \mathbf{j}\}$  coordinate system shown.



9.2 (4 pts) Hence, determine an expression for the acceleration vector of the particle in terms of  $r$ ,  $\theta$ , and their time derivatives. Note that  $d\theta/dt = \omega$  is constant.

9.3 (2 pts) Draw the forces acting on the particle on the figure provided. **NEGLECT GRAVITY AND FRICTION**



9.4 (2 pts) Hence, write down  $\mathbf{F} = m\mathbf{a}$  for the fluid particle.

9.5 (4 pts) Show that the radial position  $r$  satisfies the differential equation

$$\frac{d^2 r}{dt^2} - r\omega^2 = 0$$

9.6 (4 pts) Show that the expression

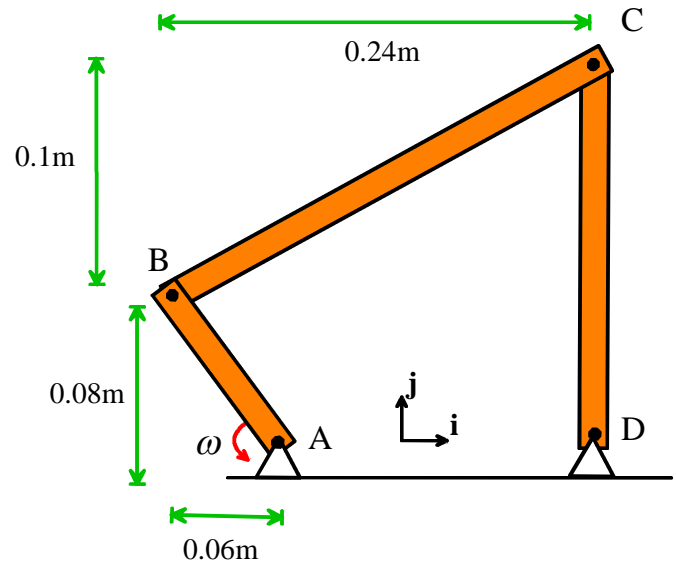
$$r(t) = \frac{1}{2}r_0(e^{\omega t} + e^{-\omega t})$$

satisfies the equation derived in part (v), and satisfies initial conditions  $r = r_0$ ,  $dr/dt = 0$  at time  $t=0$ .

9.7 (2 points) Hence, find a formula, in terms of  $r_0$ , for the vane length  $L$  such that the particle is ejected after one complete revolution of the vane.

10. The figure shows a crank-rocker mechanism. The link AB rotates at a constant angular velocity of  $\omega = 10 \mathbf{k}$  rad/s. The link CD is vertical.

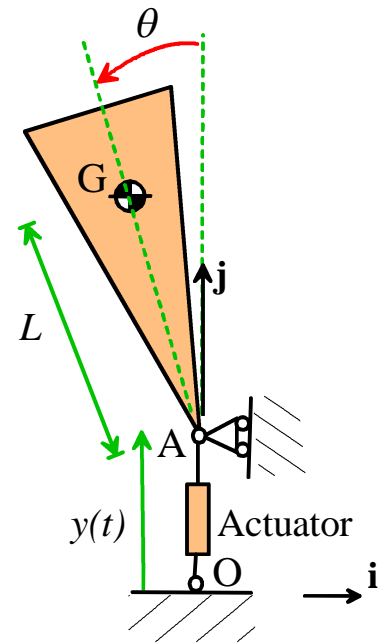
10.1 (3 pts) Calculate the velocity vector of point B at the instant shown in the figure, expressing your answer as components in the  $\{\mathbf{i}, \mathbf{j}\}$  coordinate system shown.



10.2 (7 pts) Determine the angular velocities of members BC and CD, and the velocity vector of point C.



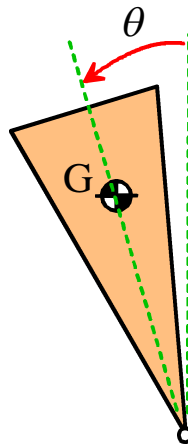
11. (20 pts) The figure shows an inverted pendulum supported by a frictionless pivot at A. The pendulum is a rigid body with mass  $m$ , and moment of inertia  $I_G = \frac{1}{10}mL^2$ . Its center of mass is a distance  $L$  from the pivot. An actuator causes the pivot to move vertically with a displacement  $y(t)$ . The goal of this problem is to derive a differential equation of motion relating the angle  $\theta$  to  $y(t)$ .



11.1 (2 pts) Write down the position vector  $\mathbf{r}$  of the center of mass in  $\{\mathbf{i}, \mathbf{j}\}$  components in terms of  $L$ ,  $\theta$ , and  $y$ .

11.2 (3 pts) Hence, calculate the acceleration vector of the center of mass in terms of  $\theta$ ,  $y$ , and their time derivatives.

11.3 (2 pts) Draw a free body diagram for the rigid body pendulum on the figure shown below.



11.4 (4 pts) Write down Newton's law of motion and the equation of rotational motion for the pendulum.

11.5 (6 pts) Combine these equations appropriately to obtain a *single* differential equation of motion for  $\theta$ , in terms of  $m$ ,  $L$ ,  $g$ , and  $y(t)$  and its time derivatives.

11.6 (3 pts) Rearrange the equation into a form that could be solved by MATLAB