Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Thursday May 20 2010

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- *If you find you are unable to complete part of a question, proceed to the next part.*

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1-8: (40 PTS) _____________

9: (20 PTS) _____________

10: (10 PTS) _____________

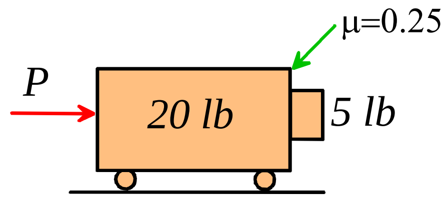
11: (20 PTS) _____________

TOTAL (90 PTS) ___________

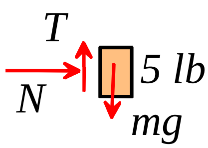
FOR PROBLEMS 1-8 WRITE YOUR ANSWER IN THE SPACE PROVIDED.

ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. What is the minimum acceleration of the cart and block shown below so that the 5lb block does not slip off the front of the 20lb cart? (g denotes the gravitational acceleration, and μ is the coefficient of friction.)



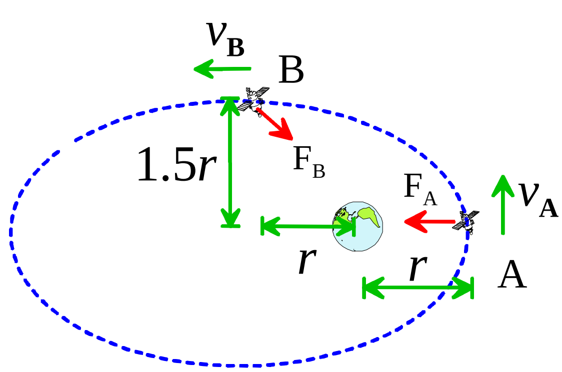
(a) g (b) 2g (c) 3g (d) 4g (e) 10g



A FBD is shown on the left. Newton's law gives $T=mg$, $N=ma$. For no slip, $T < \mu N \Rightarrow mg < \mu ma \Rightarrow a > g / \mu$

ANSWER_______D_______ (3 POINTS)

2. A satellite is orbiting the earth. The velocity and force vectors for the satellite at two points A and B on the orbit are shown. What is the speed at point B in terms of the speed v_A at point A?



(a) Need more info

(b) $v_B = \frac{1}{3} v_A$

(c) $v_B = \frac{1}{2} v_A$

(d) $v_B = \frac{2}{3} v_A$

(e) $v_B = v_A$

Angular momentum is conserved, $\mathbf{r} \times m\mathbf{v} = const$ so $r v_A = 1.5 r v_B \Rightarrow v_B = 2 v_A / 3$

ANSWER________D______ (3 POINTS)

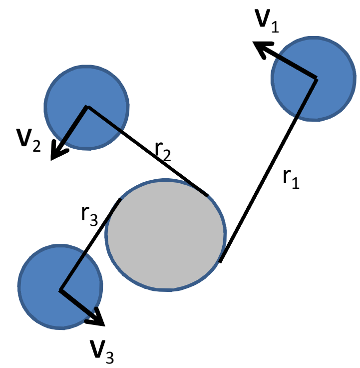
3. An EN40 student needs to weigh her bowling ball before taking it home for the summer, because she does not want to pay over-weight charges for her luggage. She does not have a scale. So, she decides to use her EN40 Trifilar Pendulum to determine the mass. Her trifilar pendulum has a mass m=0.5kg and moment of inertia I=50 kg cm^2. *With the bowling ball*, she obtains a combined moment of inertia of I=250 kg cm^2. She knows that the moment of inertia for a sphere of mass M and radius r is $I = \frac{2}{5} M r^2$. The bowling ball has radius 10cm. What is the mass of the bowling ball?

(a) 0.5kg (b) 2.5kg (c) 5kg (d) 7kg (e) 10kg

Inertia of ball = Itotal-Iplatform=200 kgcm^2 so M=5.200/2/10^2=5kg

ANSWER______C________ (3 POINTS)

4. A ball is tied to an inextensible string that wraps around a thick pole. The pole is fixed. A top view is shown in the figure at several different times. The velocity of the ball is always at right angles to the direction of the string. ***Neglect gravity***.



(i) **(2 pts)** Is linear momentum of the ball conserved during this motion? Why or why not?

No, because the string exerts a force on the ball, causing its momentum to change.

(ii) **(2 pts)** Is energy of the ball conserved during this motion? Why or why not?

Yes. The ball by itself is a conservative system, and the force exerted by the string is perpendicular to the ball's velocity, so the string does no work on the ball (it is a constraint force). No external work done on a system means energy is conserved.

(iii) **(2 pts)** Is angular momentum of the ball conserved during this motion? Why or why not?

No. Angular momentum of a particle about a point is conserved if the forces acting on the particle exert no moment about that point. There is no fixed point in the system for which the net moment exerted by the tension in the string is zero for all time.

5. A motor has a torque curve given by

$$T = T_s \left(1 - \frac{\omega}{\omega_{nl}}\right)^2$$

where T is the torque exerted by the motor, ω is its angular speed of the motor, and T_s, ω_{nl} are constants.

5.1 What is the maximum power that can be extracted from this motor?

(a) $\frac{4}{27} T_s \omega_{nl}$ (b) $\frac{1}{3} T_s \omega_{nl}$ (c) $\frac{4}{9} T_s \omega_{nl}$ (d) $\frac{16}{27} T_s \omega_{nl}$ (e) $\frac{2}{3} T_s \omega_{nl}$

Power is $T\omega = T_s \omega (1 - \omega / \omega_{nl})^2$ and at max power

$dP / d\omega = T_s (1 - \omega / \omega_{nl})^2 - 2T_s (1 - \omega / \omega_{nl}) \omega / \omega_{nl} = T_s (1 - \omega / \omega_{nl})(1 - 3\omega / \omega_{nl}) = 0 \Rightarrow \omega = \omega_{nl} / 3$

Substituting back for the power gives $P = 4T_s \omega_{nl} / 27$

ANSWER_______A_______ (3 POINTS)

5.2 The motor is used to power a vehicle of mass m and wheel radius R moving up a 30^o incline. What is the gear ratio (ie ω_a / ω, where ω_a is the axle speed) of the system connecting the motor to the wheels that is needed to achieve the max speed?

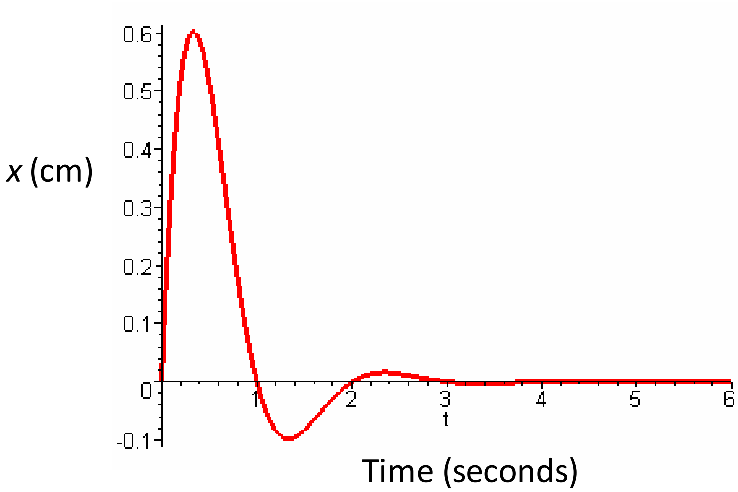
(a) $\frac{4}{81} \frac{T_s}{mgR}$ (b) $\frac{4}{27} \frac{T_s}{mgR}$ (c) $\frac{16}{27} \frac{T_s}{mgR}$ (d) $\frac{8}{9} \frac{T_s}{mgR}$ (e) need more info

Car speed is $P_{\max} = mgv \sin 30^0 = mgv / 2 \Rightarrow v = 8T_s \omega_{nl} / 27mg$. Axle speed is *v/R*. Hence

$\omega_a / \omega = (8T_s \omega_{nl} / 27mgR) / (\omega_{nl} / 3) = 24T_s / 27mgR$

ANSWER____D__________ (3 POINTS)

6. The displacement versus-time curve for the free vibration of a machine part is shown.



6.1 What is the best estimate of the damped natural period of the system?

a) 0.4 sec b) 1.0 sec c) 1.4 sec d) 2.0 sec e) 2.4 sec

The period is the time between two successive positive zero crossings, i.e. 2 sec

ANSWER_______D_______ (3 POINTS)

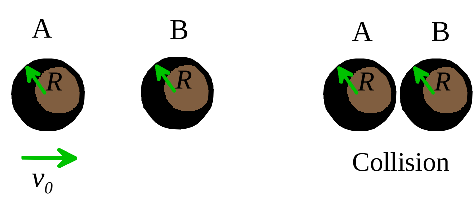
6.2 What is the best estimate of the viscous damping factor ς for the system?

a) 0.03 b) 0.5 c) 0.85 d) 1.2 d) need more info

The log decrement is $\delta = \log(x_1 / x_2) = \log(0.6 / 0.02) = 3.4$ The damping coefficient follows as

$\zeta = \delta / \sqrt{\delta^2 + 4\pi^2} \approx 0.48$

ANSWER_______B_______ (3 POINTS)



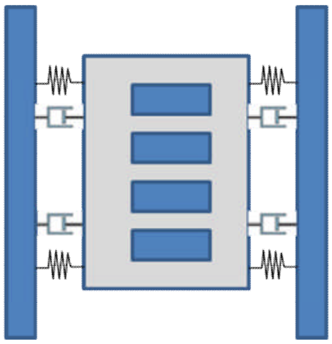
7. The figure shows a collision between two identical spheres. The restitution coefficient for the collision *e=0.* Before the collision, A moves with speed v_0 and B is stationary. After the collision

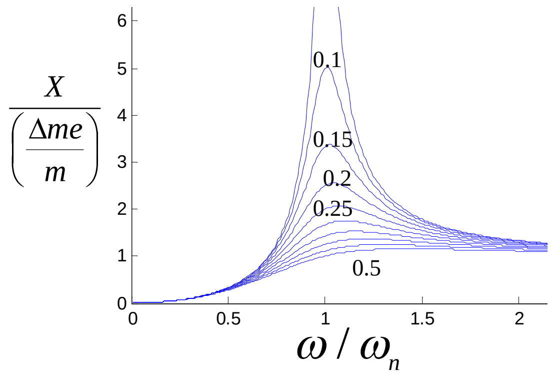
(a) Sphere A is stationary and sphere B moves to the right
(b) Sphere B is stationary and sphere A moves to the left
(c) Both spheres move to the right at the same speed
(d) Both spheres A and B move to the right with different speeds
(e) Sphere B moves to the right and sphere A moves to the left.
(f) Both spheres are stationary

If $e=0$ the relative velocity after collision is zero, so the answer has to be C.

ANSWER______C________(3 POINTS)

8. The Subaru Legacy has an unusual horizontally-aligned four-cylinder engine. The engine is connected to lateral motor mounts by 4 springs and 4 dashpots, as shown in the figure. When idling, slightly differences in the firing of the individual cylinders lead to an effective *rotor forcing* of the engine, with an effective mass imbalance of $e\Delta m = 0.4kgm$ at a frequency corresponding to $600/\pi$ RPM. The total mass of the engine is M=200kg. Brand new, each spring has stiffness k=12800N/m and each damper has damping coefficient c=400 N-s/m.





8.1 What are the natural frequency and damping coefficient ς for the engine?

(a) $4\frac{rad}{s}$, 0.125 (b) $16\frac{rad}{s}$, 0.125 (c) $4\frac{rad}{s}$, 0.25 (d) $16\frac{rad}{s}$, 0.25

From formulas $\omega_n = \sqrt{k/m} = \sqrt{4 \times 12800/200} = 16 rad/s$. The damping coefficient is $\zeta = c/2\sqrt{km} = 4 \times 400/2\sqrt{4 \times 12800 \times 200} = 1/4$

ANSWER______D________ (3 POINTS)

8.2 What is the typical steady-state amplitude of the lateral vibrations of the engine?

(a) $1mm$ (b) $2mm$ (c) $4mm$ (d) $8mm$ (e) need more info

The forcing frequency is $\omega = 2\pi \times 600/\pi/60 = 20 rad/s \Rightarrow \omega/\omega_n = 20/16$. Doing the calculation with the formula,

$X/(\Delta me/m) = (\omega/\omega_n)^2 / \sqrt{(1-\omega^2/\omega_n^2)^2 + 4\zeta^2\omega^2/\omega_n^2} = 1.86 \Rightarrow X = 1.86\Delta me/m = 0.0037m$. Or more quickly reading off the graph $X/(\Delta me/m)$ is about 2…

ANSWER_______C_______ (3 POINTS)

8.3 The main problem is not the engine vibration, but the forces caused on the attachment points to the body of the car. These attachment fixtures fatigue under load. What is the approximate amplitude of the force exerted on one damper attachment point for the new car?

(a) 2 N (b) 4 N (c) 8 N (d) 16 N (e) 32 N

The force is $cdx/dt = cX\omega\cos(\omega t + \phi) \approx 29.6\cos(\omega t + \phi)N$

ANSWER_______E_______ (4 POINTS)

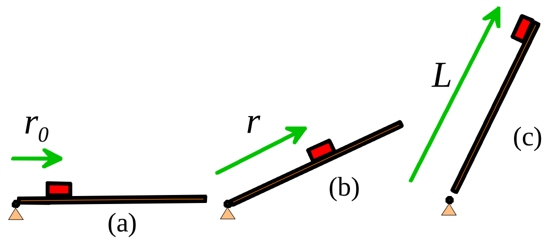
8.4 As the car ages, the spring stiffness gets smaller. How does this change the vibration amplitude?

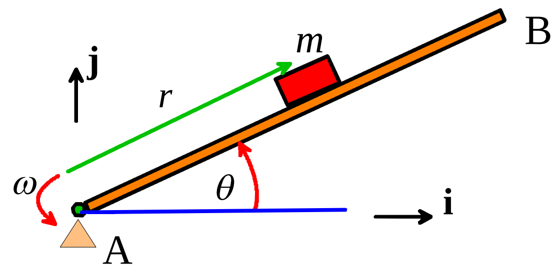
a) Increases b) Decreases c) Stays the same d) Need more info

This decreases ω_n so the operating point shifts to the right on the curve, and the amplitude decreases.

ANSWER_______B_______ (3 POINTS)

9. (20 pts) The figure shows a simple idealization of a centrifugal pump. The vane AB rotates with constant angular velocity $d\theta/dt = \omega$ about A. A small particle of fluid with mass m slides along the vane (figs a,b), and is eventually ejected from the pump (fig c). The goal of this problem is to derive the equation of motion for the distance *r(t)* shown in the figure. **NEGLECT FRICTION AND GRAVITY.**





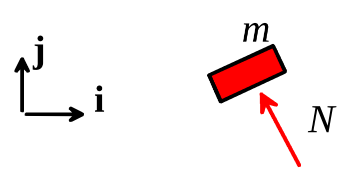
9.1 (2 pts) Write down the position vector **r** of the particle in terms of r and θ, using the **{i,j}** coordinate system shown.

$$\mathbf{r} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}$$

9.2 (4 pts) Hence, determine an expression for the acceleration vector of the particle in terms of r, θ, and their time derivatives. Note that $d\theta/dt = \omega$ is constant.

$$\mathbf{v} = \frac{dr}{dt}(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) + r\omega(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j})$$
$$\mathbf{a} = \frac{d^2r}{dt^2}(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) - r\omega^2(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) + 2\frac{dr}{dt}\omega(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j})$$

9.3 (2 pts) Draw the forces acting on the particle on the figure provided. **NEGLECT GRAVITY AND FRICTION**



9.4 (2 pts) Hence, write down **F**=m**a** for the fluid particle.

$$\mathbf{F} = m\mathbf{a} \Rightarrow N\left(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}\right) = m\left[\frac{d^2r}{dt^2}(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) - r\omega^2(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) + 2\frac{dr}{dt}\omega(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j})\right]$$

9.5 (4 pts) Show that the radial position *r* satisfies the differential equation

$$\frac{d^2r}{dt^2} - r\omega^2 = 0$$

Take dot product of both sides of the equation of motion with $(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$ gives

$$0 = m\left[\frac{d^2r}{dt^2} - r\omega^2\right]$$

9.6 (4 pts) Show that the expression

$$r(t) = \tfrac{1}{2}r_o(e^{\omega t} + e^{-\omega t})$$

satisfies the equation derived in part (v), and satisfies initial conditions $r = r_0$, $dr/dt = 0$ at time *t=0.*

Substituting the solution into the ODE gives

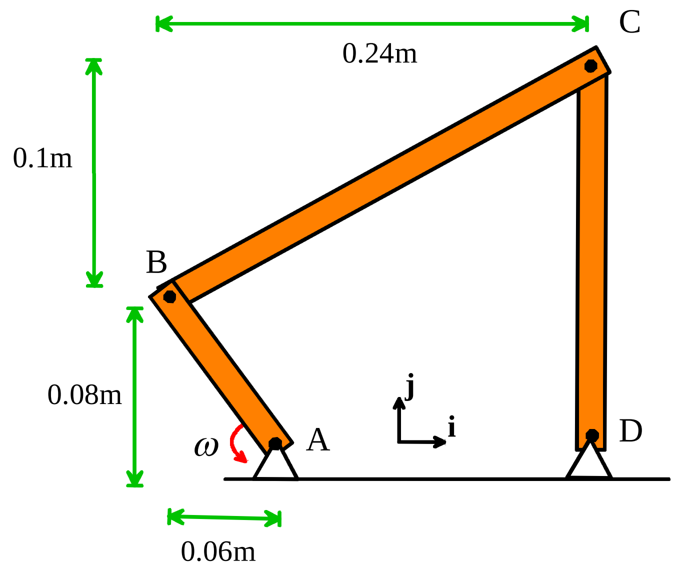
$$r(t) = \omega^2 \tfrac{1}{2}r_o(e^{\omega t} + e^{-\omega t}) - \tfrac{1}{2}r_o(e^{\omega t} + e^{-\omega t})\omega^2 = 0$$

so the equation is satisfied. Substituting $t = 0$ into the solution clearly satisifies the initial conditions stated.

9.7 (2 points) Hence, find a formula, in terms of r_0, for the vane length *L* such that the particle is ejected after one complete revolution of the vane.

At one complete revolution, $\omega t = 2\pi$, so that $L = \tfrac{1}{2}r_o(e^{2\pi} + e^{-2\pi}) \Rightarrow L = 267r_0$

10. The figure shows a crank-rocker mechanism. The link AB rotates at a constant angular velocity of $\omega = 10\,\mathbf{k}$ rad/s. The link CD is vertical.



10.1 (3 pts) Calculate the velocity vector of point B at the instant shown in the figure, expressing your answer as components in the $\{\mathbf{i}, \mathbf{j}\}$ coordinate system shown.

Rigid body equation

$$\mathbf{v}_B = \mathbf{v}_A + 10\mathbf{k} \times (-0.06\mathbf{i} + 0.08\mathbf{j})$$
$$= -0.6\mathbf{j} - 0.8\mathbf{i} \qquad m/s$$

10.2 (7 pts) Determine the angular velocities of members BC and CD, and the velocity of vector of point C.

Apply the rigid body equation to BC

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC}\mathbf{k} \times (0.24\mathbf{i} + 0.1\mathbf{j})$$
$$= \mathbf{v}_B - 0.1\omega_{BC}\mathbf{i} + 0.24\omega_{BC}\mathbf{j}$$

and CD

$$\mathbf{v}_C = \omega_{CD}\mathbf{k} \times (0.18\mathbf{j})$$
$$= -0.18\omega_{CD}\mathbf{i}$$

Eliminate $\mathbf{v}_C$ and use 10.1:

$$-0.8\mathbf{i} - 0.6\mathbf{j} - 0.1\omega_{BC}\mathbf{i} + 0.24\omega_{BC}\mathbf{j} = -0.18\omega_{CD}\mathbf{i}$$

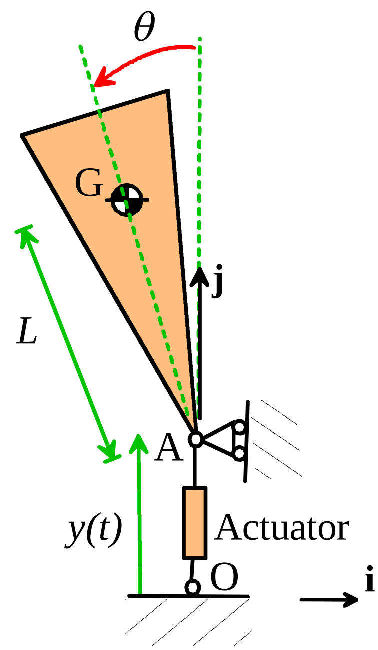
Using the **i** and **j** components of this equation, we see that

$$-0.8 - 0.1\omega_{BC} + = -0.18\omega_{CD}$$
$$-0.6 + 0.24\omega_{BC} = 0$$

Hence $\omega_{BC} = 2.5$, $\omega_{CD} = 5.833$, both in rad/s

The velocity of C follows as **-**1.05**i** m/s

11. **(20 pts)** The figure shows an inverted pendulum supported by a frictionless pivot at A. The pendulum is a rigid body with mass *m*, and moment of inertia $I_G = \frac{1}{10} mL^2$. Its center of mass is a distance *L* from the pivot. An actuator causes the pivot to move vertically with a displacement $y(t)$. The goal of this problem is to derive a differential equation of motion relating the angle θ to $y(t)$.



11.1 **(2 pts)** Write down the position vector **r** of the center of mass in {**i,j**} components in terms of L, θ, and y.

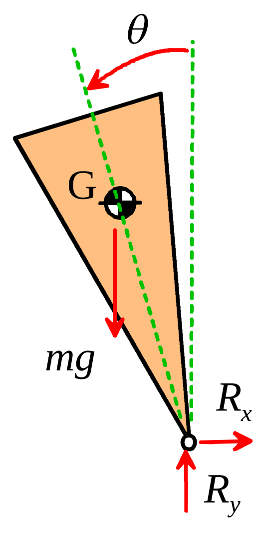
$$\mathbf{r} = -L\sin\theta\mathbf{i} + (y + L\cos\theta)\mathbf{j}$$

11.2 **(3 pts)** Hence, calculate the acceleration vector of the center of mass in terms of θ, y, and their time derivatives.

$$\mathbf{v} = -L\frac{d\theta}{dt}(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) + \frac{dy}{dt}\mathbf{j}$$

$$\mathbf{a} = -L\frac{d^2\theta}{dt^2}(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) + L\left(\frac{d\theta}{dt}\right)^2 (\sin\theta\mathbf{i} - \cos\theta\mathbf{j}) + \frac{d^2 y}{dt^2}\mathbf{j}$$

11.3 **(2 pts)** Draw a free body diagram for the rigid body pendulum on the figure shown below.



11.4 **(4 pts)** Write down Newton's law of motion and the equation of rotational motion for the pendulum.

$$R_x\mathbf{i}+(R_y-mg)\mathbf{j}=m\left\{-L\frac{d^2\theta}{dt^2}(\cos\theta\mathbf{i}+\sin\theta\mathbf{j})+L\left(\frac{d\theta}{dt}\right)^2(\sin\theta\mathbf{i}-\cos\theta\mathbf{j})+\frac{d^2y}{dt^2}\mathbf{j}\right\}$$

$$R_xL\cos\theta+R_yL\sin\theta=I_G\frac{d^2\theta}{dt^2}$$

11.5 **(6 pts)** Combine these equations appropriately to obtain a *single* differential equation of motion for θ , in terms of *m, L, g,* and $y(t)$ and its time derivatives.

$$R_x=mL\left(-\frac{d^2\theta}{dt^2}\cos\theta+\left(\frac{d\theta}{dt}\right)^2\sin\theta\right)$$

$$(R_y-mg)=m\left(\frac{d^2y}{dt^2}-L\frac{d^2\theta}{dt^2}\sin\theta-L\left(\frac{d\theta}{dt}\right)^2\cos\theta\right)$$

$$mL^2\left(-\frac{d^2\theta}{dt^2}\cos\theta+\left(\frac{d\theta}{dt}\right)^2\sin\theta\right)\cos\theta+Lm\left(\frac{d^2y}{dt^2}-L\frac{d^2\theta}{dt^2}\sin\theta-L\left(\frac{d\theta}{dt}\right)^2\cos\theta+g\right)\sin\theta=I_G\frac{d^2\theta}{dt^2}$$

$$\Rightarrow 1.1L\frac{d^2\theta}{dt^2}+\left(\frac{d^2y}{dt^2}-g\right)\sin\theta=0$$

11.6 (**3 pts)** Rearrange the equation into a form that could be solved by MATLAB

We introduce $\omega=d\theta/dt$ as an additional variable. Then

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \omega\end{bmatrix}=\begin{bmatrix}\omega\\ \left(g-d^2y/dt^2\right)\sin\theta/1.1L\end{bmatrix}$$

(the d^2y/dt^2 is on the right hand side of the equation because it is a known function of time).