



School of Engineering  
Brown University

## EN40: Dynamics and Vibrations

**Final Examination**  
**Wednesday May 18 2011**

**NAME:** \_\_\_\_\_

### General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- *If you find you are unable to complete part of a question, proceed to the next part.*

### Please initial the statement below to show that you have read it

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

\_\_\_\_\_

**1-10: (20 PTS)** \_\_\_\_\_

**11: (15 PTS)** \_\_\_\_\_

**12: (5 PTS)** \_\_\_\_\_

**13: (10 PTS)** \_\_\_\_\_

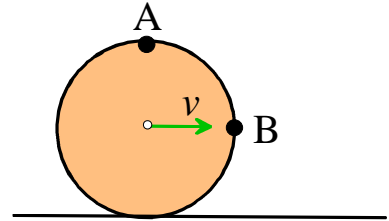
**14: (10 PTS)** \_\_\_\_\_

**TOTAL (60 PTS)** \_\_\_\_\_

FOR PROBLEMS 1-10 WRITE YOUR ANSWER IN THE SPACE PROVIDED.

ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED.  
ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. The figure on the right shows a disc with radius  $R$  rolling on the ground without slip.  $A$  and  $B$  are two points on the edge of the disc. The center of the disc moves with velocity  $v$ .



1.1 What is the speed of point  $A$ ?

- (a) Need more info
- (b)  $v_A = v$
- (c)  $v_A = \sqrt{2}v$
- (d)  $v_A = \sqrt{3}v$
- (e)  $v_A = 2v$

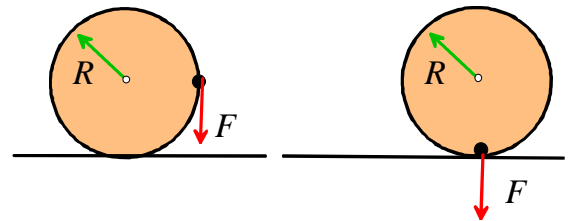
ANSWER \_\_\_\_\_ (1 POINT)

1.2 What is the speed of point  $B$ ?

- (a) Need more info
- (b)  $v_B = v$
- (c)  $v_B = \sqrt{2}v$
- (d)  $v_B = \sqrt{3}v$
- (e)  $v_B = 2v$

ANSWER \_\_\_\_\_ (1 POINT)

2. A vertical downward force  $F$  is applied at the edge of a disk of radius  $R$  as shown. The disk then rotates clockwise by 90 degrees, while rolling without slip. The force acts vertically throughout and acts on a fixed material point in the disk. What is the work done by  $F$ ?

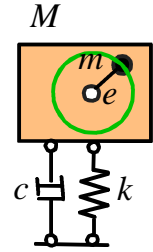


- (a) Need more info
- (b)  $FR$
- (c)  $\pi FR / 2$
- (d)  $-FR$
- (e)  $\pi FR$

ANSWER \_\_\_\_\_ (2 POINTS)



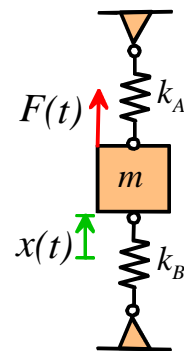
6. A motor with total mass  $M=50\text{kg}$  has rotating internal mass of  $m=1\text{kg}$  that rotates on a shaft with eccentricity  $e=1\text{mm}$  at angular rate  $\omega=100\text{rad/s}$ . The engine is mounted on vibration isolation pads with stiffness  $k=500\,000\text{ N/m}$  and a dashpot coefficient  $c=250\text{ Ns/m}$ . The system is found to have a severe vibration problem. Will the following changes reduce the vibration amplitude?



- |   |     |    |
|---|-----|----|
| (a) Increase the stiffness of the pads $k$            | YES | NO |
| (b) Decrease the stiffness of the pads $k$            | YES | NO |
| (c) Increase the speed of the motor                   | YES | NO |
| (d) Decrease the dashpot coefficient for the pads $c$ | YES | NO |

(2 POINTS)

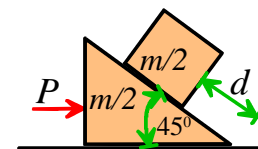
7. A mass-spring system shown in the figure is subjected to a harmonic force  $F(t) = F_0 \sin \omega t$ . Let  $x(t)$  denote the deflection of the mass from its static equilibrium position. What is the equation of the motion for this system?



- (a)  $\frac{d^2x}{dt^2} + \frac{k_A + k_B}{m} x = \frac{F_0}{m} \sin \omega t$
- (b)  $\frac{d^2x}{dt^2} + \frac{k_A - k_B}{m} x = \frac{F_0}{m} \sin \omega t$
- (c)  $\frac{d^2x}{dt^2} + \frac{k_B - k_A}{m} x = \frac{F_0}{m} \sin \omega t$
- (d)  $\frac{d^2x}{dt^2} + \frac{k_A k_B}{(k_A + k_B)m} x = \frac{F_0}{m} \sin \omega t$

ANSWER \_\_\_\_\_ (2 POINTS)

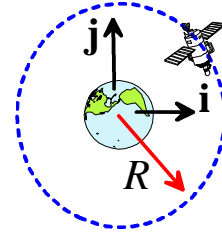
8. The figure shows a block with mass  $m/2$  on a wedge with identical mass. The entire assembly accelerates to the right. All contacts are frictionless. What force  $P$  is necessary to ensure that the block remains a fixed distance  $d$  from the base of the wedge?



- (a)  $P = mg$
- (b)  $P = mg/2$
- (c)  $P = 2mg$
- (d)  $P = mg / \sqrt{2}$
- (e) None of the above

ANSWER \_\_\_\_\_ (2 POINTS)

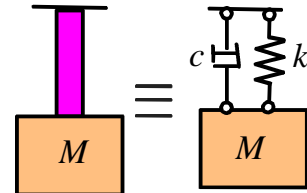
9. A satellite circles the earth in a circular low-earth orbit with radius  $R$ . In this orbit the total energy of the satellite (i.e. kinetic plus potential energy) is  $E$ . Appropriate rocket burns are then used to transfer the satellite into a circular geosynchronous orbit with radius  $5R$ . Assuming the PE very far from the earth is zero, the total energy (KE+PE) of the satellite in the new orbit is



- (a)  $E/5$
- (b)  $5E$
- (c)  $E$
- (d)  $4E$
- (e)  $4E/5$

ANSWER \_\_\_\_\_ (2 POINTS)

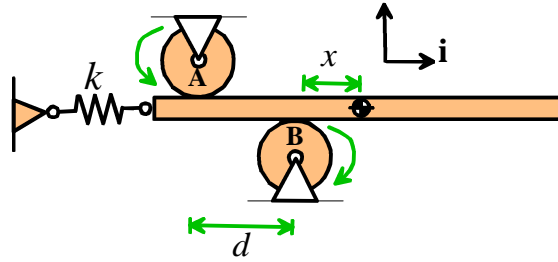
10. The mechanical behavior of some polymeric materials can be idealized as a spring-dashpot combination as shown in the figure. The following tests are conducted on a polymer specimen to determine its effective stiffness  $k$  and dashpot coefficient  $c$ . (i) A mass of 100kg is suspended from the bar, and its static deflection is measured to be 1mm. (ii) The mass is then struck to set it in motion, and its vibration response is measured. It is found that the amplitude of the sixth oscillation is reduced to about 30% of that of the first oscillation. The stiffness and damping coefficient are approximately



- (a)  $k=10^5 \text{ Nm}^{-1}$ ,  $c=770 \text{ Nsm}^{-1}$
- (b)  $k=10^6 \text{ Nm}^{-1}$ ,  $c=770 \text{ Nsm}^{-1}$
- (c)  $k=10^5 \text{ Nm}^{-1}$ ,  $c=240 \text{ Nsm}^{-1}$
- (d)  $k=10^6 \text{ Nm}^{-1}$ ,  $c=640 \text{ Nsm}^{-1}$
- (e) None of the above

ANSWER \_\_\_\_\_ (2 POINTS)

11. A bar of mass  $m$  is supported by two rollers, which spin rapidly in opposite directions as shown in the figure. At the instant shown, the center of mass of the bar is a distance  $x$  from roller  $B$ . Horizontal motion of the bar is resisted by a spring with stiffness  $k$ , which is unstretched when  $x=0$ . The coefficient of kinetic friction between the rollers and the bar is  $\mu$ .



11.1 Draw a free body diagram showing the forces acting on the bar

[2 POINTS]

11.2 Write down Newton's law of motion and the equation for rotational motion for the bar (the thickness of the bar can be neglected)

[3 POINTS]

11.3 Hence, show that  $x(t)$  satisfies the equation of motion

$$m \frac{d^2 x}{dt^2} + \left( k - \frac{2\mu mg}{d} \right) x = \mu mg$$

[3 POINTS]

11.4 Hence, find an expression for the natural frequency of vibration of the system.

**[2 POINTS]**

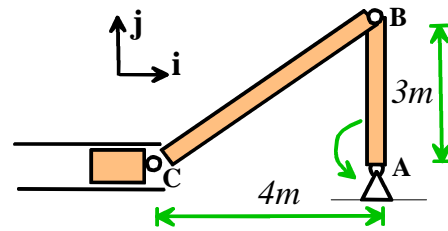
11.5 If the system is released from rest with  $x=0$ , what is the resulting amplitude of vibration?

**[3 POINTS]**

11.6 Describe briefly the motion of the system if  $k < 2\mu mg / d$  (no calculations are required). One point extra credit if your answer is in verse.

**[2 POINTS]**

12. The figure shows a piston-crank mechanism. The crank AB rotates with constant angular speed of 2 rad/sec. At the instant shown, calculate the following quantities, expressing your answer in the  $\{i, j, k\}$  basis shown.



12.1 The velocity of point B

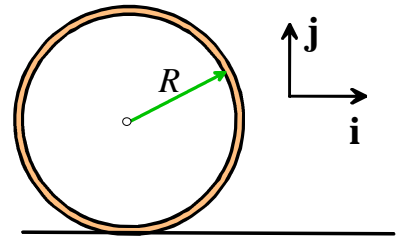
[2 POINTS]

12.2 The velocity of point C and the angular velocity of member BC

[3 POINTS]



13. Consider a ring of radius mass  $m$  and radius  $R$ , initially spinning at angular speed  $\omega_0$  in the clockwise direction. The thickness of the ring is negligible compared to its radius  $R$ . The spinning ring is placed on a horizontal surface with coefficient of friction  $\mu$ . The ring initially slips on the surface, then begins to roll without slip.



13.1 Draw the free body diagram for the ring just after it comes into contact with the surface.

[2 POINTS]

13.2 Show that, when the ring is slipping, the acceleration of the center of the ring is  $\mathbf{a} = \mu g \mathbf{i}$  and its angular acceleration is  $\boldsymbol{\alpha} = \mu g / R \mathbf{k}$

[2 POINTS]

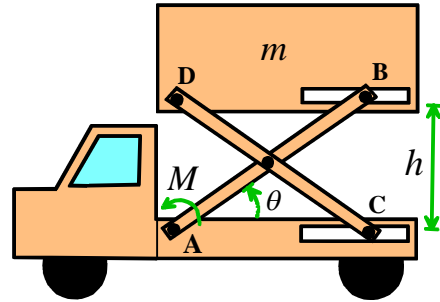
13.3 If  $t = 0$  when the ring comes into contact with the surface, at what time does rolling without slip commence?

**[3 POINTS]**

13.4 What are the velocity of the center of the ring and the angular velocity of the ring when rolling without slip commences?

**[3 POINTS]**

14. The figure shows the scissor-lift mechanism on an aircraft catering truck. Members AB and CD both have length  $L$ . Member AB rotates about A and moves through a slider at B; similarly, member BC rotates about D and moves along a slider at C. The payload has mass  $m$ , and is raised by a moment (or torque)  $M$  applied to member AB at point A.



14.1 Write down the height  $h$  of the mass  $m$  in terms of  $\theta$ . Hence, determine a formula for the vertical speed  $dh/dt$  of the payload in terms of  $d\theta/dt$  and any other relevant variables.

[2 POINTS]

14.2 Assume that the vertical speed of the mass is constant. Using energy methods, determine the moment  $M$  as a function of  $\theta$  and any other relevant variables.

[3 POINTS]

14.3 Now, assume that the moment  $M$  is constant. Use energy methods to find a formula for the vertical acceleration of the mass in terms of  $M$ ,  $\theta$ , and any other relevant variables.

**[5 POINTS]**