School of Engineering
Brown University

# EN40: Dynamics and Vibrations

## Final Examination
## Wednesday May 18 2011

**NAME:** ________________________________________

## General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- *If you find you are unable to complete part of a question, proceed to the next part.*

## Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

_____________

**1-10: (20 PTS)** _____________

**11: (15 PTS)** _____________

**12: (5 PTS)** _____________

**13: (10 PTS)** _____________

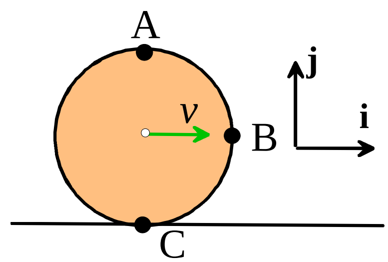
**14: (10 PTS)** _____________

**TOTAL (60 PTS)** ___________

**FOR PROBLEMS 1-10 WRITE YOUR ANSWER IN THE SPACE PROVIDED.**

**ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.**

**1.** The figure on the right shows a disc with radius $R$ rolling on the ground without slip. $A$ and $B$ are two points on the edge of the disc. The center of the disc moves with velocity $v$.




1.1 What is the speed of point $A$?

(a) Need more info

(b) $v_A = v$

(c) $v_A = \sqrt{2}v$

(d) $v_A = \sqrt{3}v$

(e) $v_A = 2v$

Recall $v = -\omega R$, and $\mathbf{v}_A = \omega\mathbf{k} \times \mathbf{r}_{A/C}$ so $\mathbf{v}_A = \omega\mathbf{k} \times \mathbf{r}_{A/C} = (v/R)\mathbf{k} \times 2R\mathbf{j} = 2v\mathbf{i}$

**ANSWER_______E_______ (1 POINT)**

1.2 What is the speed of point B?

(a) Need more info

(b) $v_B = v$

(c) $v_B = \sqrt{2}v$

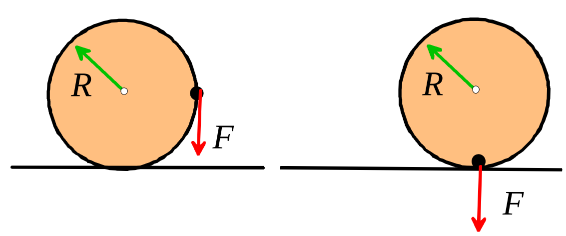
(d) $v_B = \sqrt{3}v$

(e) $v_B = 2v$

$\mathbf{v}_B = \omega\mathbf{k} \times \mathbf{r}_{B/C}$ so $\mathbf{v}_B = (v/R)\mathbf{k} \times (R\mathbf{i} + R\mathbf{j}) = -v\mathbf{j} + v\mathbf{i}$ and the speed is the magnitude of $\mathbf{v}_B$

**ANSWER_______**C**_______ (1 POINT)**

**2.** A vertical downward force $F$ is applied at the edge of a disk of radius $R$ as shown. The disk then rotates clockwise by 90 degrees, while rolling without slip. The force acts vertically throughout and acts on a fixed material point in the disk. What is the work done by $F$?




(a) Need more info

(b) $FR$

(c) $\pi FR/2$

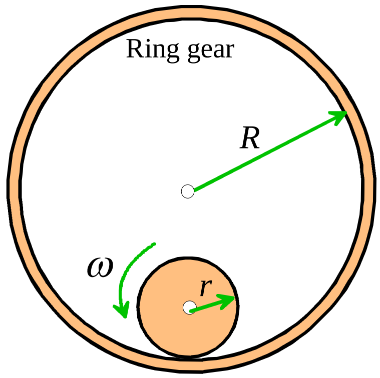
(d) $-FR$

(e) $\pi FR$

The work done is $\int_{\mathbf{r}0}^{\mathbf{r}1} \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}0}^{\mathbf{r}1} -F\mathbf{j} \cdot (dx\mathbf{i} + dy\mathbf{j}) = \int_{R}^{0} -F dy = FR$.

**ANSWER_______**B**_______ (2 POINTS)**

**3.** In the figure shown, the small gear rotates counterclockwise with angular speed $\omega$. The centers of both gears are fixed. The large ring gear has angular speed




(a) $\omega_r = \omega \frac{R}{r}$ clockwise

(b) $\omega_r = \omega \frac{R}{r}$ counterclockwise

(c) $\omega_r = \omega \frac{r}{R}$ clockwise

(d) $\omega_r = \omega \frac{r}{R}$ counterclockwise

(e) None of the above

The points on the two bodies where they touch must have the same velocities. So $\omega r = \omega_r R \Rightarrow \omega_r = \omega r / R$ and since this is positive the rotation direction is counterclockwise

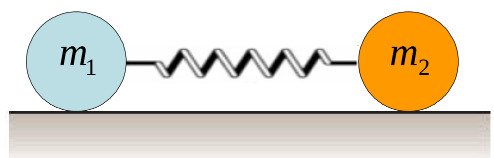
**ANSWER**_____D_________ **(2 POINTS)**

**4.** Mark each of the forces below as conservative (C) or non-conservative (NC)

| | |
|---|---|
| (a) Gravity | C |
| (b) The force acting between two magnets | C |
| (c) Buoyancy | C |
| (d) Air drag | NC |

**(2 POINTS)**

**5.** Two objects of masses $m_1$ and $m_2$ are placed on a table and are connected by a spring as shown in the figure below. Assume there is no friction between the table surface and the objects. If $m_1$ is held fixed, the natural frequency of vibration of $m_2$ is found to be $\omega_2$. When $m_2$ is held fixed, mass $m_1$ has natural frequency of vibration




(a) Need more info

(b) $\omega_2$

(c) $\frac{m_1}{m_2}\omega_2$

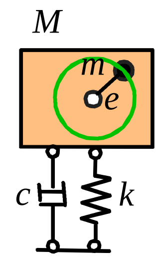
(d) $\sqrt{\frac{m_2}{m_1}}\omega_2$

(e) $\sqrt{\frac{m_1}{m_2}}\omega_2$

Both are just simple spring mass systems - $\omega_1 = \sqrt{k / m_1}$ $\quad \omega_2 = \sqrt{k / m_2} \Rightarrow \omega_1 = \omega_2 \sqrt{m_2 / m_1}$

**ANSWER**_______D_______ **(2 POINTS)**

**6.** A motor with total mass $M$=50kg has rotating internal mass of $m$=1kg that rotates on a shaft with eccentricity $e$=1mm at angular rate $\omega = 100$ rad/s. The engine is mounted on vibration isolation pads with stiffness $k$=500 000 N/m and a dashpot coefficient $c$=250 Ns/m. The system is found to have a severe vibration problem. Will the following changes reduce the vibration amplitude?

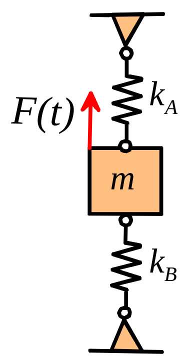



(a) Increase the stiffness of the pads $k$ YES
(b) Decrease the stiffness of the pads $k$ YES
(c) Increase the speed of the motor YES
(d) Decrease the dashpot coefficient for the pads $c$ NO

The resonant frequency is $\omega_n = \sqrt{500000/50} = 100$rad/s. The system is therefore at resonance. The damping factor $\zeta = c/2\sqrt{kM} = 0.025$. With this $\zeta$ the resonant peak is very sharp, so changing $k$, or changing the motor speed will take the system away from resonance and so decrease the vibration amplitude. Decreasing $c$ will reduce $\zeta$ and so make the vibration worse.

**(2 POINTS)**

**7.** A mass-spring system shown in the figure is subjected to a harmonic force $F(t) = F_0 \sin \omega t$. Let $x(t)$ denote the deflection of the mass from its static equilibrium position. What is the equation of the motion for this system?




(a) $\frac{d^2x}{dt^2} + \frac{k_A + k_B}{m} x = \frac{F_0}{m} \sin \omega t$

(b) $\frac{d^2x}{dt^2} + \frac{k_A - k_B}{m} x = \frac{F_0}{m} \sin \omega t$

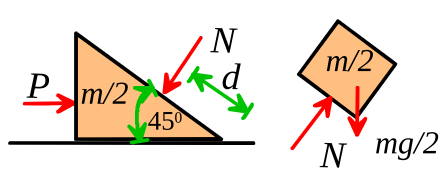
(c) $\frac{d^2x}{dt^2} + \frac{k_B - k_A}{m} x = \frac{F_0}{m} \sin \omega t$

(d) $\frac{d^2x}{dt^2} + \frac{k_A k_B}{(k_A + k_B)m} x = \frac{F_0}{m} \sin \omega t$

The springs are in parallel and so have effective stiffness $k_A + k_B$. This is a standard undamped forced system, so has the standard EOM with this effective stiffness

**ANSWER**_______A_______ **(2 POINTS)**

**8.** The figure shows a block with mass *m/2* on a wedge with identical mass. The entire assembly accelerates to the right. All contacts are frictionless. What force *P* is necessary to ensure that the block remains a fixed distance *d* from the base of the wedge?



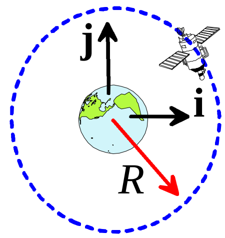

(a) $P = mg$
(b) $P = mg/2$
(c) $P = 2\,mg$
(d) $P = mg/\sqrt{2}$
(e) None of the above

A FBD for the small mass is shown. Newton for the small mass gives

$(N/\sqrt{2})\mathbf{i} + (N/\sqrt{2} - mg/2)\mathbf{j} = (m/2)a\mathbf{i} \Rightarrow N = mg/\sqrt{2} \quad a = g$. **F**=m**a** for the whole system gives $P$=$mg$

**ANSWER**________A______ **(2 POINTS)**

**9**. A satellite circles the earth in a circular low-earth orbit with radius $R$. In this orbit the total energy of the satellite (i.e. kinetic plus potential energy) is $E$. Appropriate rocket burns are then used to transfer the satellite into a circular geosynchronous orbit with radius $5R$. The total energy (KE+PE) of the satellite in the new orbit is




(a) *E/5*

(b) 5*E*

(c) *E*

(d) *4E*

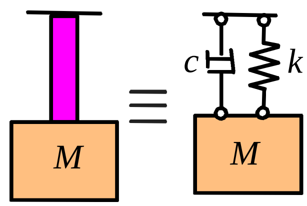
(e) *4E/5*

Circular motion with gravitational force $F_g = -GMm/r^2 \mathbf{e}_r$ so **F**=m**a** in the radial direction

gives $mv^2/r = GMm/r^2$. The total energy is $-GMm/r + mv^2/2 = -GMm/2r$. So $E = -GMm/2R$ changes to $-GMm/10R = E/5$ when $r$ is increased from $R$ to $5R$.

**ANSWER**_______A_______ **(2 POINTS)**

**10.** The mechanical behavior of some polymeric materials can be idealized as a spring-dashpot combination as shown in the figure. The following tests are conducted on a polymer specimen to determine its effective stiffness $k$ and dashpot coefficient $c$. (i) A mass of 100kg is suspended from the bar, and its static deflection is measured to be 1mm. (ii) The mass is then struck to set it in motion, and its vibration response is measured. It is found that the amplitude of the sixth oscillation is reduced to about 30% of that of the first oscillation. The stiffness and damping coefficient are approximately



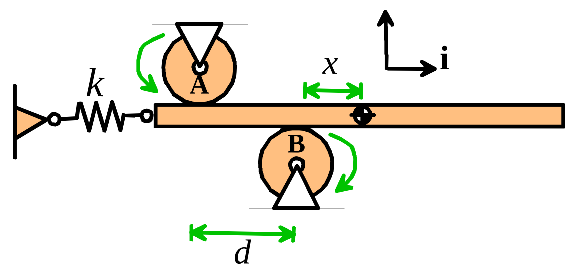

(a) $k=10^5\ Nm^{-1}$, $c=770\ Nsm^{-1}$
(b) $k=10^6\ Nm^{-1}$, $c=770\ Nsm^{-1}$
(c) $k=10^5\ Nm^{-1}$, $c=240\ Nsm^{-1}$
(d) $k=10^6\ Nm^{-1}$, $c=640\ Nsm^{-1}$
(e) None of the above

For the static test $F = mg = kx \Rightarrow k = mg/x = 100 \times 10/10^{-3} = 10^6\ N/m$. The log decrement is $\delta = (1/5)\log(1/0.3) = 0.2407$ and so $\zeta = \delta/(4\pi^2 + \delta^2) = 0.0383$.

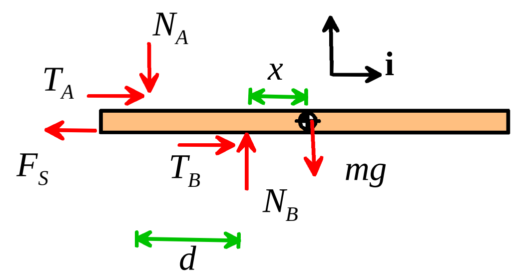
Finally $\zeta = c/2\sqrt{km} \Rightarrow c = 2\zeta\sqrt{km} = 766$

**ANSWER**______B________ **(2 POINTS)**

**11.** A bar of mass *m* is supported by two rollers, which spin rapidly in opposite directions as shown in the figure. At the instant shown, the center of mass of the bar is a distance *x* from roller *B*. Horizontal motion of the bar is resisted by a spring with stiffness *k*, which is unstretched when *x=0*. The coefficient of kinetic friction between the rollers and the bar is $\mu$.




11.1 Draw a free body diagram showing the forces acting on the bar




**[2 POINTS]**

11.2 Write down Newton's law of motion and the equation for rotational motion for the bar. (The thickness of the bar can be neglected)

Newton's law gives $(T_A + T_B - F_s)\mathbf{i} + (N_B - N_A - mg)\mathbf{j} = m\dfrac{d^2x}{dt^2}\mathbf{i}$

For rotational motion we have $N_A(d+x) - N_B x = 0$

**[3 POINTS]**

11.3 Hence, show that *x(t)* satisfies the equation of motion

$$m\frac{d^2x}{dt^2} + \left(k - \frac{2\mu mg}{d}\right)x = \mu mg$$

The two contacts slip, so $T_A = \mu N_A$, $T_B = \mu N_B$ and the spring force law gives $F_s = kx$. Also from the rotational equation and the **j** component of **F**=m**a** we have that

$$N_B - N_A = mg$$

$$-N_B + N_A\left(1 + \frac{d}{x}\right) = 0$$

Add these equations to see that $N_A = mgx/d$ then substitute back to see $N_B = mg(1 + x/d)$. Therefore $T_A + T_B - F_s = \mu mg(1 + 2x/d) - kx$ and the **i** component of the EOM gives $m\dfrac{d^2x}{dt^2} = \mu mg(1 + 2x/d) - kx$. This can be rearranged into the equation given.

**[3 POINTS]**

11.4 Hence, find an expression for the natural frequency of vibration of the system.

Rearrange into standard form

$$\frac{d^2x}{dt^2}+\left(\frac{k}{m}-\frac{2\mu g}{d}\right)\left(x-\frac{\mu g}{k/m-2\mu g/d}\right)=0$$

The natural frequency is therefore

$$\omega_n=\sqrt{\left(\frac{k}{m}-\frac{2\mu g}{d}\right)}$$

**[2 POINTS]**

11.5 If the system is released from rest with *x=0*, what is the resulting amplitude of vibration?

If we set $y=x-\dfrac{\mu g}{k/m-2\mu g/d}$ in the EOM and note that the initial conditions for *y* are $y=-\dfrac{\mu g}{k/m-2\mu g/d}$ and dy/dt=0 at *t=0* , we can read off the solution from the standard results.

$$y=-\frac{\mu g}{k/m-2\mu g/d}\cos\omega_n t\,.$$

The vibration amplitude is therefore $Y_0=\dfrac{\mu g}{k/m-2\mu g/d}$

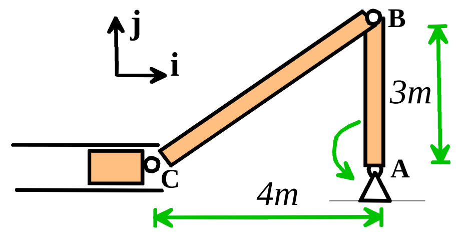
**[3 POINTS]**

11.6 Describe briefly the motion of the system if $k<2\mu mg/d$ (no calculations are required). One point extra credit if your answer is in verse.

> It appears to me
> If $k<2\mu mg/d$
> The eom is case 2
> On the handout we gave you
> From this we can see
> For large values of *t*, $x=A\exp(\lambda t)$
> With A arbitrary, and $\lambda=\sqrt{-k/m+2\mu g/d}$
> Instead of vibration
> This is just translation.
> So, the bar moves to the right
> And soon vanishes from sight…

**[2 POINTS]**

**12.** The figure shows a piston-crank mechanism. The crank AB rotates with constant angular speed of 2 rad/sec. At the instant shown, calculate the following quantities, expressing your answer in the **{i,j,k}** basis shown.




12.1 The velocity of point B

$$\mathbf{v}_B = \omega \mathbf{k} \times \mathbf{r}_{B/O} = 2\mathbf{k} \times 3\mathbf{j} = -6\mathbf{i} m/s$$

**[2 POINTS]**

12.2 The velocity of point C and the angular velocity of member BC

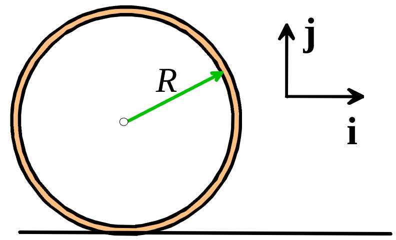
Note that C must move in the **i** direction. Therefore

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC}\mathbf{k} \times \mathbf{r}_{C/B} = -6\mathbf{i} + \omega_{BC}\mathbf{k} \times (-4\mathbf{i} - 3\mathbf{j}) = v_c \mathbf{i}$$

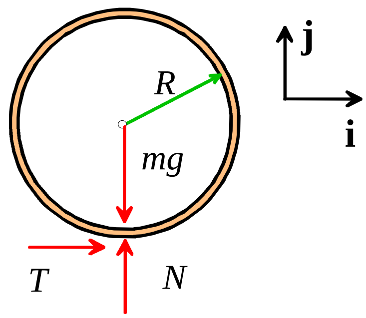
The **i** and **j** components of this equation give two equations for $\omega_{BC}, v_c$. Clearly $\omega_{BC} = 0$ $v_c = -6\mathbf{i}$.

**[3 POINTS]**

**13.** Consider a ring of radius mass *m* and radius *R*, initially spinning at angular speed $\omega_0$ in the clockwise direction. The thickness of the ring is negligible compared to its radius *R*. The spinning ring is placed on a horizontal surface with coefficient of friction $\mu$. The ring initially slips on the surface, then begins to roll without slip.




13.1 Draw the free body diagram for the ring just after it comes into contact with the surface.




**[2 POINTS]**

13.2 Show that, when the ring is slipping, the acceleration of the center of the ring is $\mathbf{a} = \mu g \mathbf{i}$ and its angular acceleration is $\boldsymbol{\alpha} = \mu g / R \ \mathbf{k}$

Newtons law gives $T\mathbf{i} + (N - mg)\mathbf{j} = ma_G \mathbf{i}$

The rotational equation of motion gives $TR = I_G \alpha \mathbf{k}$

Recall also that since the contact slips $T = \mu N$ and for a ring $I_G = mR^2$

Solving these gives $N=mg$, $T = \mu mg$, $a_G = \mu g \quad \alpha = \mu g / R$

**[2 POINTS]**

13.3 If t = 0 when the ring comes into contact with the surface, at what time does rolling without slip commence?

Integrating the angular acceleration gives $\omega = -\omega_0 + \alpha t = -\omega_0 + \mu g t / R$
Integrating the linear acceleration gives $v = \mu g t$ .

For rolling without slip $v = -\omega R$ and therefore $\mu g t = \omega_0 R - \mu g t \Rightarrow t = \omega_0 R / (2\mu g)$

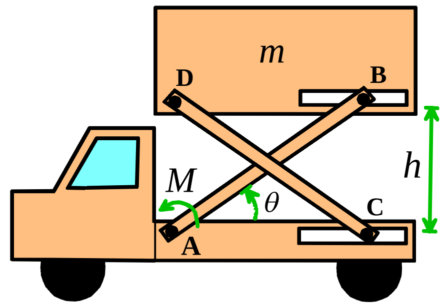
**[3 POINTS]**

13.4 What are the velocity of the center of the ring and the angular velocity of the ring when rolling without slip commences?

Substituting for the times in the expressions for the velocity and angular velocity gives
$\omega = -\omega_0 / 2 \qquad v = -\omega_0 R / 2$ so the angular velocity of the ring is always halved, regardless of the coefficient of friction…

**[3 POINTS]**

**14.** The figure shows the scissor-lift mechanism on an aircraft catering truck. Members AB and CD both have length $L$. Member AB rotates about A and moves through a slider at B; similarly, member BC rotates about D and moves along a slider at C. The payload has mass $m$, and is raised by a moment (or torque) $M$ applied to member AB at point A.




14.1 Write down the height $h$ of the mass $m$ in terms of $\theta$. Hence, determine a formula for the vertical speed $dh/dt$ of the payload in terms of $d\theta / dt$ and any other relevant variables.

Simple geometry gives $h = L\sin\theta \Rightarrow \frac{dh}{dt} = L\cos\theta \frac{d\theta}{dt}$

**[2 POINTS]**

14.2 Assume that the vertical speed of the mass is constant. Using energy methods, determine the moment $M$ as a function of $\theta$ and any other relevant variables.

The truck is a conservative system so the rate of work done by the moment $M$ has to be equal to its rate of change of potential energy (since the speed is constant)

Therefore $M\frac{d\theta}{dt} = mg\frac{dh}{dt} = mgL\cos\theta\frac{d\theta}{dt} \Rightarrow M = mgL\cos\theta$

**[3 POINTS]**

14.3 Now, assume that the moment *M* is constant. Use energy methods to find a formula for the vertical acceleration of the mass.

In this case the rate of work done by M has to be the rate of change of PE and KE. Therefore

$$M\frac{d\theta}{dt} = \frac{d}{dt}\left(mgh + \frac{1}{2}m\left(\frac{dh}{dt}\right)^2\right) = mg\frac{dh}{dt} + m\frac{dh}{dt}\frac{d^2h}{dt^2}$$

$$M\frac{1}{mL\cos\theta} - g = \frac{d^2h}{dt^2}$$

**[5 POINTS]**