School of Engineering
Brown University

EN40: Dynamics and Vibrations

Midterm Examination
Tuesday March 8 2011

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1 (7 points) _____________

2 (14 points) _____________

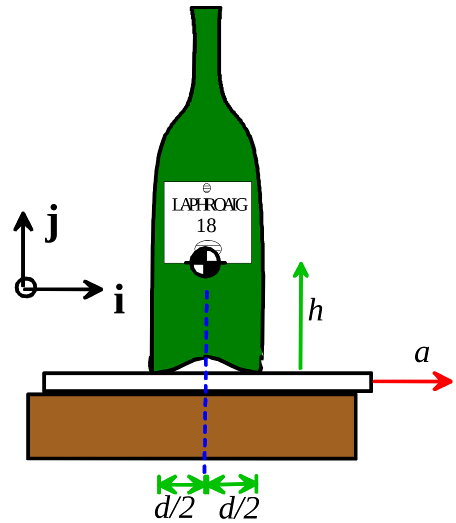
3. (9 points) _____________

4. (5 points) _____________

5. (5 points) _____________

TOTAL (40 points)___________

1. The figure shows the 'tablecloth' trick demonstrated in class. The bottle has diameter d at the base, and its center of mass is a height h above the table. The coefficient of friction between cloth and bottle is μ. The cloth is pulled horizontally with an acceleration $a > \mu g$ so the cloth slips under the bottle.



1.1 Draw the forces acting on the bottle on the figure below.

[2 POINTS]

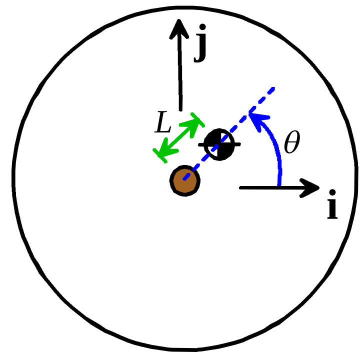
1.2 Assuming that the bottle does not tip, calculate its horizontal acceleration.

[2 POINTS]

1.3 Show that the bottle will tip over if *h/d* exceeds a critical value, and give an expression for the maximum allowable value of *h/d* for the trick to work.

[3 POINTS]

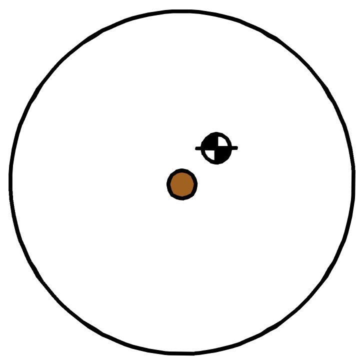
2. An unbalanced rotor that is spun at constant speed by a motor attached to its hub can be idealized as a particle with mass *m* located at the center of mass of the rotor, which is a distance *L* from the hub as shown in the figure.



2.1 Write down the position vector of the particle (i.e. center of mass) in terms of L and and the angle θ. Hence, derive expressions for its acceleration in terms of θ, $\omega = d\theta / dt$ and L. Use the basis shown, and assume that ω is constant.

[3 POINTS]

2.2 Draw the forces and moments acting on the rotor on the figure provided. Gravity should be included.

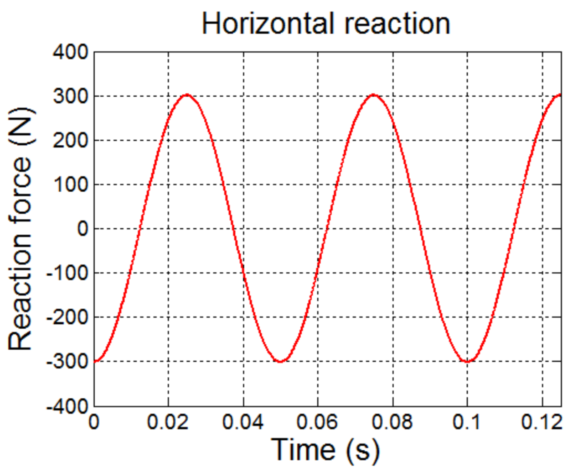


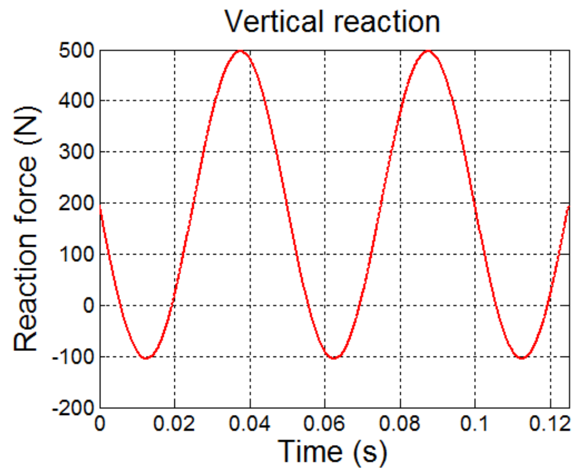
[3 POINTS]

2.3 Hence, calculate expressions for the horizontal and vertical reaction forces acting at the rotor hub as functions of time.

[3 POINTS]

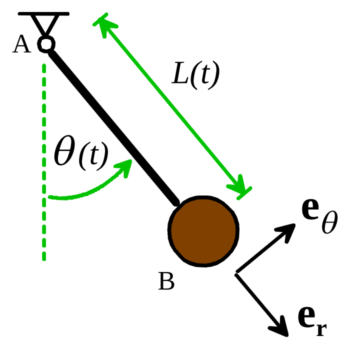
2.4 The graphs show reaction forces measured experimentally. Determine the mass of the rotor, and the distance of the center of mass from the axis of rotation. Use SI units.





[5 POINTS]

3. A gymnast swinging on a high horizontal bar can be idealized as a pendulum shown in the figure, with a point mass at B and pin joint at A. The length of member AB varies with time according to the equation $L(t) = L_0 + \Delta L \sin \Omega t$, were, L_0 and ΔL are constants, and Ω is the (constant) frequency at which the athlete 'pumps' to start swinging.



3.1 Find the acceleration vector of the mass at B, in terms of the angle θ and its time derivatives, L_0, ΔL and Ω. Express your answer using the polar coordinate basis vectors shown in the figure.

[2 POINTS]

3.2 Draw a free body diagram showing the force acting on the mass at B.

[3 POINTS]

3.3 Using Newton's law, show that the equation of motion for the angle θ is

$$\left(L_0 + \Delta L \sin \Omega t\right)\frac{d^2\theta}{dt^2} + 2\Delta L \Omega \cos \Omega t \frac{d\theta}{dt} + g \sin \theta = 0$$

[2 POINTS]

3.5 Rearrange the equation of motion into a form that MATLAB could solve.

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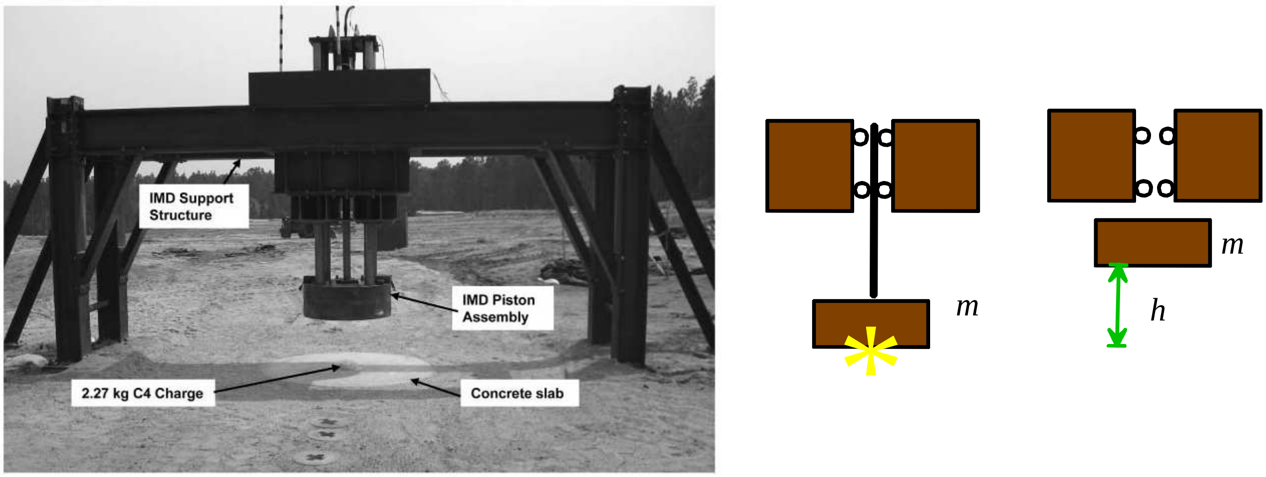
[2 POINTS]

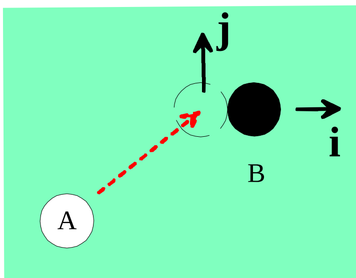
4. The figure shows an apparatus to measure the impulse exerted by a sub-surface explosive device. It consists of a piston with mass *m* supported by a frame. The system is initially at rest. The explosion then propels the piston vertically, and its maximum height *h* is measured. Derive an expression that relates the piston mass *m* and the height *h* to the impulse *I* exerted on the piston by the explosion. Friction can be neglected.



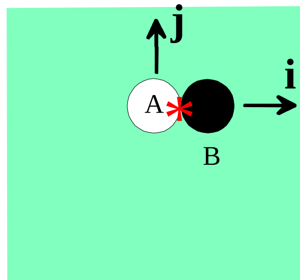
(Figure from Ehrgott, et al *Experimental Techniques,* doi: 10.1111/j.1747-1567.2009.00604.x)

[5 POINTS]

5. The figure shows a collision between two spheres with identical mass. Before the collision, sphere *A* moves at 45^0 to the **i** direction while sphere *B* is at rest. The coefficient of restitution between the spheres is *e=0*. One of figures (a)-(e) shows correctly the position of the spheres a short time after the collision (the dashed circles show the positions of the spheres at the instant the collision occurs, for reference). By answering the true/false questions below, identify the correct figure.

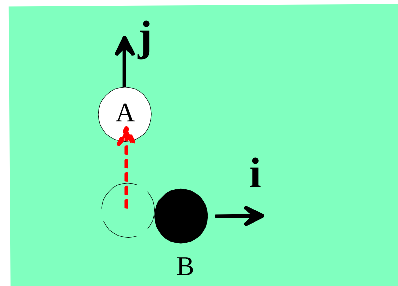


Before collision

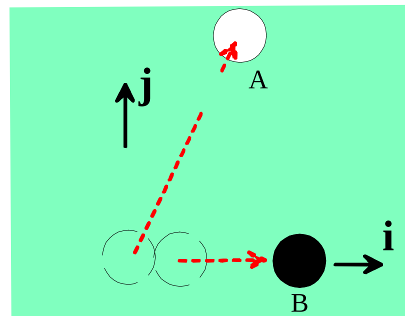


Collision

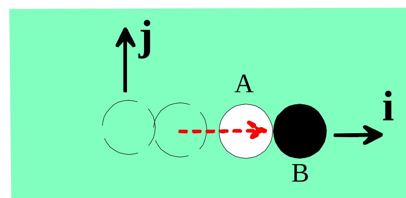
(a)



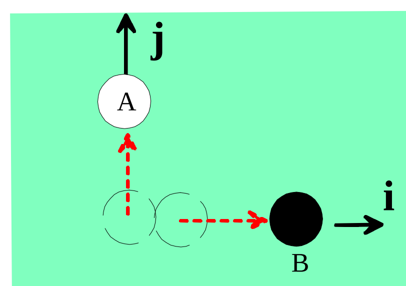
(b)



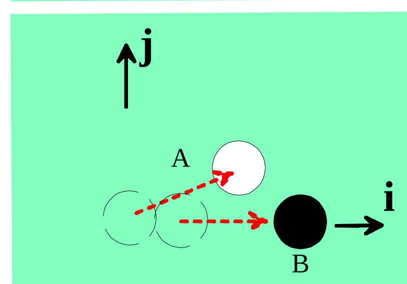
(c)



(d)



(e)



(a) Momentum is conserved T F
The restitution coefficient formula is satisfied T F

(b) Momentum is conserved T F
The restitution coefficient formula is satisfied T F

(c) Momentum is conserved T F
The restitution coefficient formula is satisfied T F

(d) Momentum is conserved T F
The restitution coefficient formula is satisfied T F

(e) Momentum is conserved T F
The restitution coefficient formula is satisfied T F

[5 POINTS]