EN40: Dynamics and Vibrations

Midterm Examination
Tuesday March 8 2011

School of Engineering
Brown University

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General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

S-N

1 (7 points)	7
2 (14 points)	14
3. (9 points)	9
4. (5 points)	5
5. (5 points)	5
TOTAL (40 points)	40

1. The figure shows the 'tablecloth' trick demonstrated in class.   The bottle has diameter d at the base, and its center of mass is a height h above the table. The coefficient of friction between cloth and bottle is μ. The cloth is pulled horizontally with an acceleration $a > \mu g$ so the cloth slips under the bottle.

1.1 Draw the forces acting on the bottle on the figure below.
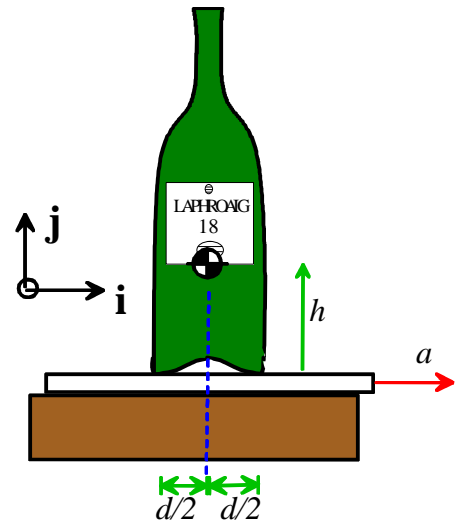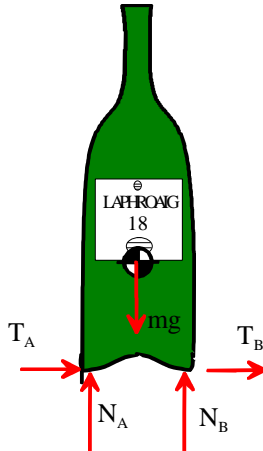
[2 POINTS]

1.2 Assuming that the bottle does not tip, calculate its horizontal acceleration.

Newton's law gives $(T_A + T_B)\mathbf{i} + (N_A + N_B - mg)\mathbf{j} = ma_{Bottle}\mathbf{i}$

The cloth slips, so $T_A = \mu N_A \quad T_B = \mu N_B$

Hence $N_A + N_B = mg \qquad (T_A + T_B) = ma_{Bottle} \quad = \mu(N_A + N_B) = \mu mg$

The acceleration is therefore $a_{Bottle} = \mu g$

[2 POINTS]

1.3 Show that the bottle will tip over if h/d exceeds a critical value, and give an expression for the maximum allowable value of h/d for the trick to work.

Moments about the COM gives
$$(T_A + T_B)h + (N_B - N_A)d/2 = 0$$
$$\Rightarrow 2\mu mgh/d + (N_B - N_A) = 0$$

Recall that $N_A + N_B = mg$ and solve for N_A, N_B: $N_B = mg/2 - \mu mgh/d \qquad N_A = mg/2 + \mu mgh/d$

Notice that $N_B = 0$ if $h/d > 1/2\mu$ which means the bottle will tip. So make sure that $h/d < 1/2\mu$

[3 POINTS]

2. An unbalanced rotor that is spun at constant speed by a motor attached to its hub can be idealized as a particle with mass m located at the center of mass of the rotor, which is a distance L from the hub as shown in the figure.
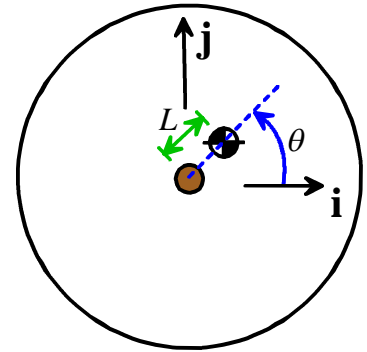
2.1 Write down the position vector of the particle (i.e. center of mass) in terms of L and and the angle θ. Hence, derive expressions for its acceleration in terms of θ, $\omega = d\theta / dt$ and L. Use the basis shown, and assume that ω is constant.
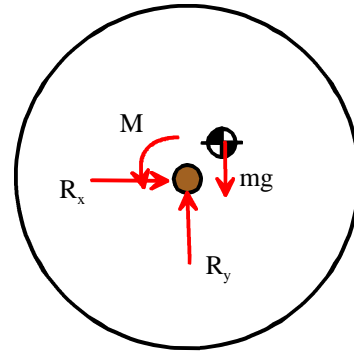
$$\mathbf{r} = L\cos\theta\mathbf{i} + L\sin\theta\mathbf{j}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = L\omega(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) \quad \textbf{[3 POINTS]}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -L\omega^2(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

2.2 Draw the forces and moments acting on the rotor on the figure provided. Gravity should be included.
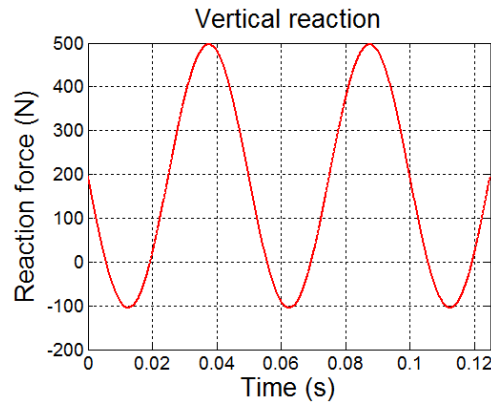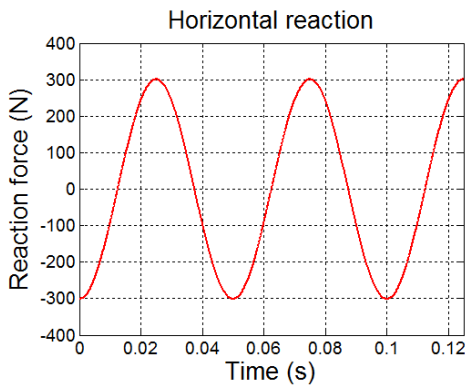
[3 POINTS]

2.3 Hence, calculate expressions for the horizontal and vertical reaction forces acting at the rotor hub as functions of time.

Newton's law gives
$$R_x\mathbf{i} + (R_y - mg)\mathbf{j} = -mL\omega^2(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$
[3 POINTS]
$$\Rightarrow R_x = -Lm\omega^2\cos\omega t \qquad R_y = mg - Lm\omega^2\sin\omega t$$

2.4 The graphs show reaction forces measured experimentally. Determine the mass of the rotor, and the distance of the center of mass from the axis of rotation. Use SI units.

The mean value of Ry is 200N and (from 2.3) must equal mg. Therefore $m = 200 / 9.81 = 20.39 kg$

Note that Rx and Ry are both harmonic (as predicted in 2.3), with amplitude 300N. The period is 0.05sec. Therefore $\omega = 2\pi / T = 40\pi \ rad / s$.

From 2.3 we see that $Lm\omega^2 = 300N \qquad \Rightarrow L = 300 / (m\omega^2) = 0.93 \times 10^{-3} m \quad (0.93mm)$ **[5 POINTS]**

3. A gymnast swinging on a high horizontal bar can be idealized as a pendulum shown in the figure, with a point mass at B and pin joint at A. The length of member AB varies with time according to the equation $L(t) = L_0 + \Delta L \sin \Omega t$, were, L_0 and ΔL are constants, and Ω is the (constant) frequency at which the athlete 'pumps' to start swinging.
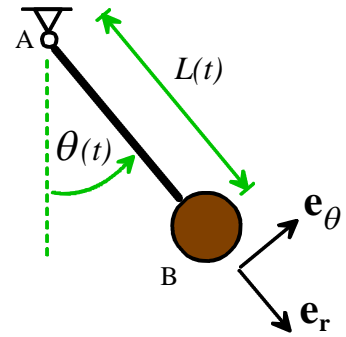
3.1 Find the acceleration vector of the mass at B, in terms of the angle θ and its time derivatives, L_0, ΔL and Ω. Express your answer using the polar coordinate basis vectors shown in the figure.

From class notes the acceleration in polar-coords is

$$\mathbf{a} = \left(\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right) \mathbf{e}_r + \left(2 \frac{dr}{dt} \frac{d\theta}{dt} + r \frac{d^2\theta}{dt^2} \right) \mathbf{e}_\theta$$
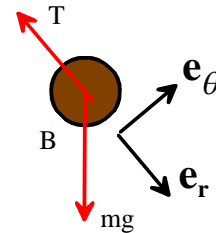
For this problem $r = L_0 + \Delta L \sin \Omega t \quad dr/dt = \Delta L \Omega \cos \Omega t \quad d^2 r / dt^2 = -\Delta L \Omega^2 \sin \Omega t$
Therefore

$$\mathbf{a} = \left(-\Delta L \Omega^2 \sin \Omega t - (L_0 + \Delta L \sin \Omega t) \left(\frac{d\theta}{dt} \right)^2 \right) \mathbf{e}_r + \left(2\Delta L \Omega \cos \Omega t \frac{d\theta}{dt} + (L_0 + \Delta L \sin \Omega t) \frac{d^2\theta}{dt^2} \right) \mathbf{e}_\theta$$

[2 POINTS]

3.2 Draw a free body diagram showing the force acting on the mass at B.

[3 POINTS]

3.3 Using Newton's law, show that the equation of motion for the angle θ is

$$(L_0 + \Delta L \sin \Omega t) \frac{d^2\theta}{dt^2} + 2\Delta L \Omega \cos \Omega t \frac{d\theta}{dt} + g \sin \theta = 0$$

Newton's law is
$(mg \cos \theta - T) \mathbf{e}_r - mg \sin \theta \mathbf{e}_\theta =$

$$m \left(-\Delta L \Omega^2 \sin \Omega t - (L_0 + \Delta L \sin \Omega t) \left(\frac{d\theta}{dt} \right)^2 \right) \mathbf{e}_r + m \left(2\Delta L \Omega \cos \Omega t \frac{d\theta}{dt} + (L_0 + \Delta L \sin \Omega t) \frac{d^2\theta}{dt^2} \right) \mathbf{e}_\theta$$

The $\mathbf{e}_\theta$ component of this equation gives the answer stated.

[2 POINTS]

3.5 Rearrange the equation of motion into a form that MATLAB could solve.
We turn the 2nd order ODE into two first order ODEs in the usual way

$$\backslash \frac{d}{dt} \begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} \omega \\ \left(-2\Delta L \Omega \cos \Omega t \frac{d\theta}{dt} - g \sin \theta \right) / (L_0 + \Delta L \sin \Omega t) \end{bmatrix}$$

[2 POINTS]

4. The figure shows an apparatus to measure the impulse exerted by a sub-surface explosive device. It consists of a piston with mass m supported by a frame. The system is initially at rest. The explosion then propels the piston vertically, and its maximum height h is measured. Derive an expression that relates the piston mass m and the height h to the impulse I exerted on the piston by the explosion. Friction can be neglected.
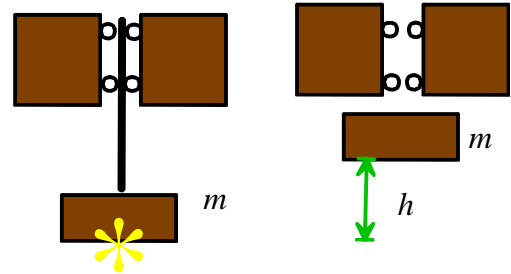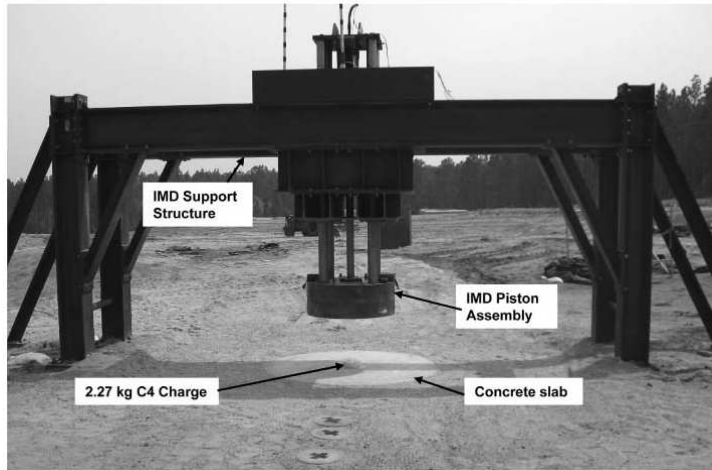

(Figure from Ehrgott, et al *Experimental Techniques,* doi: 10.1111/j.1747-1567.2009.00604.x)

Just after the explosion, the mass is traveling vertically with speed V. During subsequent flight, energy is conserved, so $mV^2 / 2 = mgh \Rightarrow V = \sqrt{2gh}$.

The impulse-momentum equation then gives $\mathbf{I} = \mathbf{p}_1 - \mathbf{p}_0 \Rightarrow I\mathbf{j} = mV\mathbf{j} \Rightarrow I = m\sqrt{2gh}$.

[5 POINTS]

5. The figure shows a collision between two spheres with identical mass. Before the collision, sphere *A* moves at 45^0 to the **i** direction while sphere *B* is at rest. The coefficient of restitution between the spheres is *e=0*. One of figures (a)-(e) shows correctly the position of the spheres a short time after the collision (the dashed circles show the positions of the spheres at the instant the collision occurs, for reference). By answering the true/false questions below, identify the correct figure.

Before collision   Collision

 (a)
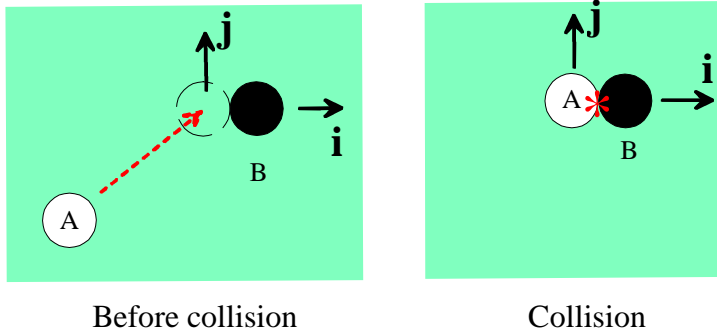
For momentum to be conserved, $\mathbf{v}_{A0} = \mathbf{v}_{A1} + \mathbf{v}_{B1}$

With e=0, **n= i** the restitution formula is $\mathbf{v}_{B1} - \mathbf{v}_{A1} = -\mathbf{v}_{A0} + [\mathbf{v}_{A0} \cdot \mathbf{i}]\mathbf{i}$

Dot this with **i** $(\mathbf{v}_{B1} - \mathbf{v}_{A1}) \cdot \mathbf{i} = 0$ i.e. the spheres must have the same **i** component of velocity after impact. Dot it with **j** and note that B moves parallel to **i** in all the figures to see that $\mathbf{v}_{A1} \cdot \mathbf{j} = \mathbf{v}_{A0} \cdot \mathbf{j}$. This means that A must have the same **j** velocity before and after impact to satisfy the restitution formula. Recall the animation of this problem shown in class…

 (b)

(a) Momentum is conserved   T   (F) (no **i** momentum)
      The restitution coefficient formula is satisfied (T)(F)
      (both answers were accepted - the **i** component of
      restitution is satisfied, but the **j** component can't be
      determined. F is a better answer if we assume all figs are
      for the same time.

 (c)

(b) Momentum is conserved (T)   F
      The restitution coefficient formula is satisfied (T)   F

(c) Momentum is conserved   T   (F) (no **j** momentum)
      The restitution coefficient formula is satisfied   T   (F)
      (A has no **j** velocity)

 (d)

(d) Momentum is conserved (T)   F
      The restitution coefficient formula is satisfied   T (F) **i**
      velocities of A and B not equal)

 (e)

(e) Momentum is conserved   T   (F) (**i** momentum exceeds
      j momentum)
      The restitution coefficient formula is satisfied   T (F)(**j**
      velocity of A is smaller than before impact)

[5 POINTS]