School of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Tuesday May 15, 2011

NAME: ______________________________

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- *If you find you are unable to complete part of a question, proceed to the next part.*

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1-10: (20 PTS) ______________

11: (10 PTS) ______________

12: (10 PTS) ______________

13: (10 PTS) ______________

14: (10 PTS) ______________

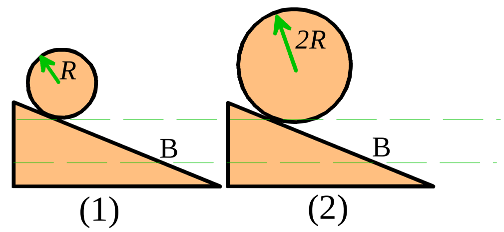
TOTAL (60 PTS) ____________

FOR PROBLEMS 1-10 WRITE YOUR ANSWER IN THE SPACE PROVIDED.

ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. Two cylinders with equal mass density start at rest, and roll without slipping down an incline. Cylinder 1 has radius R and cylinder 2 has radius $2R$. Which cylinder will have a higher velocity when it arrives at point B?

(a) Cylinder 1
(b) Cylinder 2
(c) Both the same
(d) Neither will ever reach B.



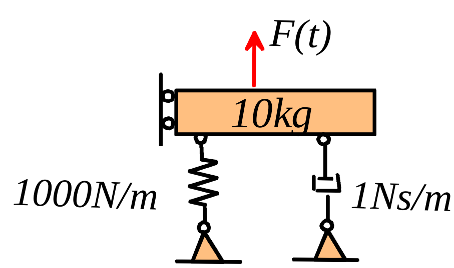
ANSWER______________ (2 POINTS)

2. The unit kg m^2/s is used for:

(a) Rotational Kinetic Energy
(b) Power
(c) Angular Momentum
(d) All of the above
(e) None of the above

ANSWER______________ (2 POINTS)

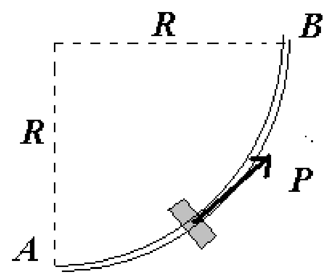
3. A vibration isolation table can be idealized as shown in the figure. It is subjected to a harmonic force *F(t)* with amplitude 0.1N and angular frequency 10 rad/s. The amplitude of vertical vibration is



(a) 0.1 mm
(b) 0.2 mm
(c) 1 mm
(d) 10 mm
(e) 20 mm
(f) None of the above

ANSWER______________ (2 POINTS)

4. A bead of mass m slides on frictionless ring of radius R in a vertical plane. The block is subjected to a vertical gravitational force mg as well as a force P=2mg that is always oriented along the direction of sliding. The block starts from rest at point A. What is the velocity of the block when it reaches point B?



(a) $v=\sqrt{2Rg}$
(b) $v=\sqrt{2(\pi-1)Rg}$
(c) $v=\sqrt{4Rg}$
(d) $v=\sqrt{2\left(2\sqrt{2}-1\right)Rg}$
(e) None of the above

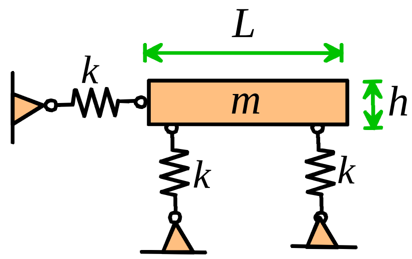
ANSWER______________ (2 POINTS)

5. A bird lands near the tip of a branch, and is observed to oscillate up and down about once a second. The bird on the branch can be idealized as a lightly damped spring-mass system. When the vibration stops, the (static) deflection of the tip of the branch is approximately equal to

(a) 0 m
(b) 0.05 m
(c) 0.25 m
(d) 0.50 m
(e) 0.75 m
(f) 1 m

ANSWER______________ (2 POINTS)

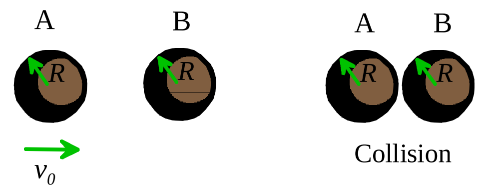
6. A vehicle mounted on its suspension system is idealized as a rigid body supported by three springs as shown in the figure. How many vibration frequencies does the system have, assuming that motion is confined to the plane of the figure?



(a) 2
(b) 3
(c) 4
(d) 6
(e) None of the above

ANSWER______________ (2 POINTS)

7. The figure shows a collision between two identical spheres. The restitution coefficient for the collision $e=0$. Before the collision, A moves with speed v_0 and B is stationary. During the collision



(a) Momentum and energy are both conserved
(b) Momentum is conserved, and the energy increases
(c) Momentum is conserved, and the energy decreases
(d) Energy is conserved and momentum increases
(e) Energy is conserved and momentum decreases

ANSWER______________ **(2 POINTS)**

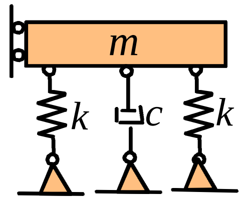
8. A 'Critically Damped' vibrating system
(a) Vibrates forever if it is disturbed from equilibrium
(b) Vibrates if disturbed from equilibrium but the vibrations decay quickly
(c) Returns to equilibrium following a disturbance without vibration
(d) Never returns to its equilibrium configuration if disturbed
(e) Feels wet and insulted.

ANSWER______________ **(2 POINTS)**

9. Beats are heard when two sounds have
(a) nearly the same amplitude
(b) nearly the same frequencies
(c) twice the amplitude
(d) exactly twice the wavelength

ANSWER______________ **(2 POINTS)**

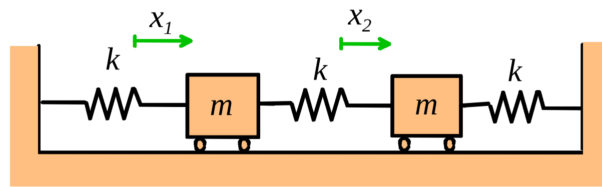
10. The viscous damping factor for the system shown in the figure is



(a) $\zeta = c/\sqrt{2km}$
(b) $\zeta = c/2\sqrt{2km}$
(c) $\zeta = 2c/\sqrt{2km}$
(d) $\zeta = c/2\sqrt{km}$

ANSWER______________ **(2 POINTS)**

11. The figure shows two identical masses that are connected to springs with stiffness k. The masses vibrate with displacements $x_1(t), x_2(t)$ from their equilibrium positions. When $x_1 = x_2 = 0$ there is no force in the springs.



11.1 Draw a free body diagram for each mass on the figure provided below

(3 POINTS)

11.2 Write down the changes in length of each of the three springs in terms of x_1, x_2. Hence, use Newton's laws to show that $x_1(t), x_2(t)$ satisfy the equations of motion

$$m\frac{d^2x_1}{dt^2} + 2kx_1 - kx_2 = 0$$

$$m\frac{d^2x_2}{dt^2} - kx_1 + 2kx_2 = 0$$

(4 POINTS)

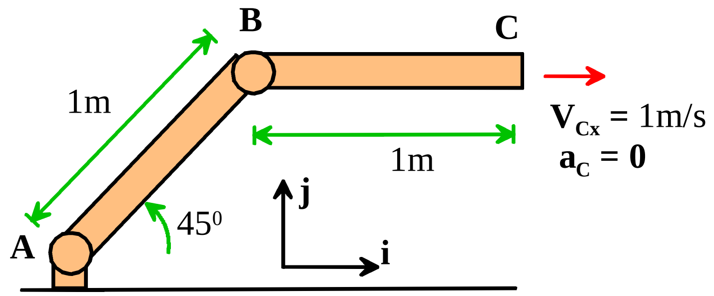
11.3 Add and subtract the equations of motion to show that the normal modes $q_1 = x_1 + x_2 \qquad q_2 = x_1 - x_2$ satisfy equations of the form

$$\frac{d^2 q_1}{dt^2} + \omega_1^2 q_1 = 0 \qquad \frac{d^2 q_2}{dt^2} + \omega_2^2 q_2 = 0$$

Hence, determine formulas for the two natural frequencies ω_1, ω_2.

(3 POINTS)

12. The figure shows a robot arm. Point C on the arm is required to move horizontally with constant speed 1m/s. This is accomplished by rotating links AB and BC with appropriate angular speeds ω_{AB}, ω_{BC} and angular accelerations α_{AB}, α_{BC}. The goal of this problem is to calculate values for ω_{AB}, ω_{BC}, α_{AB}, α_{BC} at the instant shown.



12.1 Determine formulas for the velocity vectors $\mathbf{v}_B, \mathbf{v}_C$ of points B and C, in terms of ω_{AB}, ω_{BC}. (You do not need to solve for ω_{AB}, ω_{BC} until 12.3).

(3 POINTS)

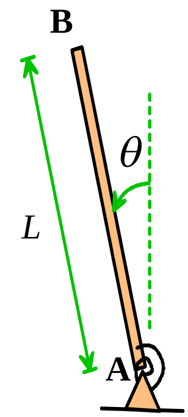
12.2 Determine formulas for the acceleration vectors $\mathbf{a}_B, \mathbf{a}_C$ of points B and C in terms of $\alpha_{AB}, \alpha_{BC}, \omega_{AB}, \omega_{BC}$. (You do not need to solve for α_{AB}, α_{BC} until 12.3)

(3 POINTS)

12.3 Hence, calculate the required values of $\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}$

(4 POINTS)

13. The figure shows a bar with mass m and length L that is pivoted about point A. The bar is stabilized by a torsional spring with stiffness κ, which exerts a restoring moment with magnitude $\kappa\theta$ at A. The goal of this problem is to determine the natural frequency of small amplitude vibrations of the bar.



13.1 State, or derive, the mass moment of inertia of the bar about point A, in terms of m and L.

(2 POINTS)

13.2 Write down the total potential energy of the system, in terms of m, g, L, κ, θ.

(2 POINTS)

13.3 Write down the total kinetic energy of the system, in terms of $m, L, \frac{d\theta}{dt}$.

(2 POINTS)

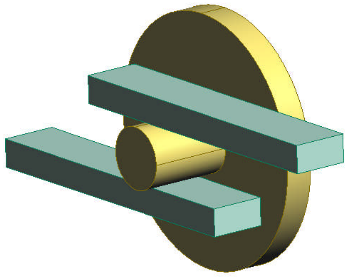
13.4 Hence, use energy conservation to show that θ satisfies

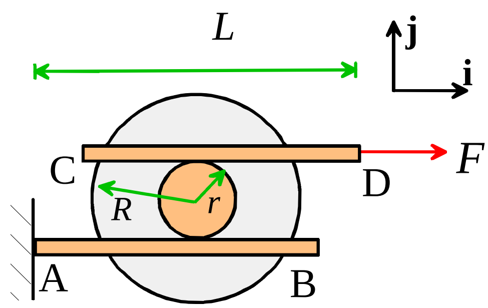
$$\frac{mL^2}{3}\frac{d^2\theta}{dt^2} + \kappa\theta - mg\frac{L}{2}\sin\theta = 0$$

(**2 POINTS)**

13.5 Finally, determine a formula for the natural frequency of vibration.

(2 POINTS)



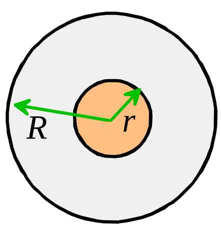


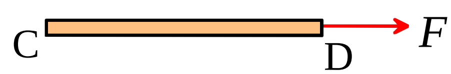
14. An ‘inerter’ is a suspension element that exerts a force F that is related to its length L by

$$F = \mu \frac{d^2 L}{dt^2}$$

where μ is a constant (NOT friction coefficient!). The figure shows a proposed design for such a device. It consists of a disk with mass m, radius R and mass moment of inertia $mR^2/2$ which is rigidly connected to an axle with radius r. The axle rolls without slip between platens AB and CD (which have negligible mass). The objective of this problem is to derive an equation for the coefficient μ

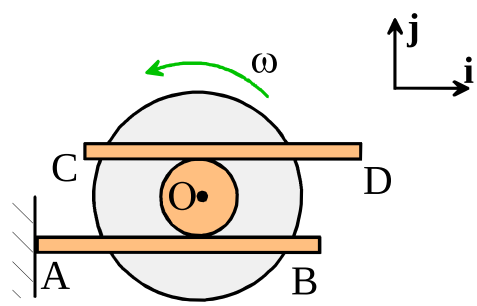
14.1 Draw a free body diagram showing the forces acting on the disk/axle and platen CD on the figures provided below. Gravity may be neglected.





(3 POINTS)

14.2 Suppose AB is stationary and the disk and axle rotate with angular velocity $\omega \mathbf{k}$. Find the velocity vectors $\mathbf{v}_O$ and $\mathbf{v}_D$ of the center O of the axle and point D in terms of r and ω



(2 POINTS)

14.3 Write down the equations of linear (**F**=m**a**) and rotational ($\mathbf{M} = I\boldsymbol{\alpha}$) motion for the disk.

(**2 POINTS)**

14.4 Hence, show that

$$\mu = \frac{m}{4}(1 + \frac{R^2}{2r^2})$$

(**3 POINTS)**