School of Engineering
Brown University

EN40: Dynamics and Vibrations

Midterm Examination
Tuesday March 6 2012

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1 (10 points) _____________

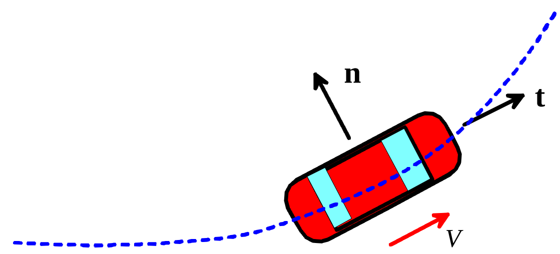
2. (11 points) _____________

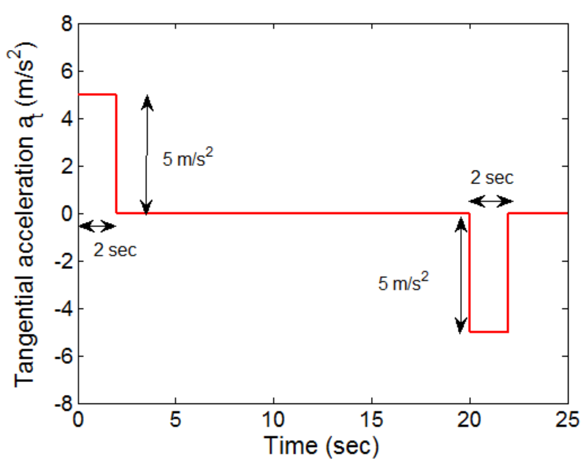
3. (6 points) _____________

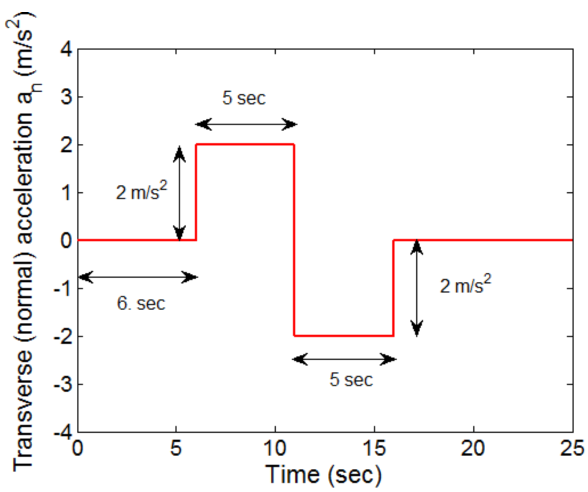
4. (8 points) _____________

TOTAL (35 points) ___________

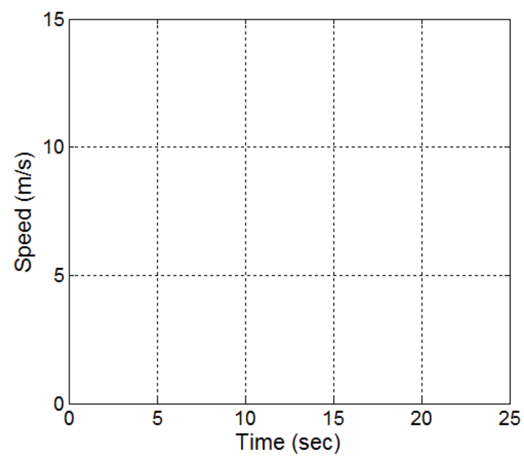
1. A vehicle is instrumented with accelerometers that measure acceleration components a_t, a_n in directions parallel and perpendicular to the car's direction of motion, respectively. (A positive value for a_n means the car accelerates to the left). The graphs below show the variation of a_t, a_n measured in an experiment.





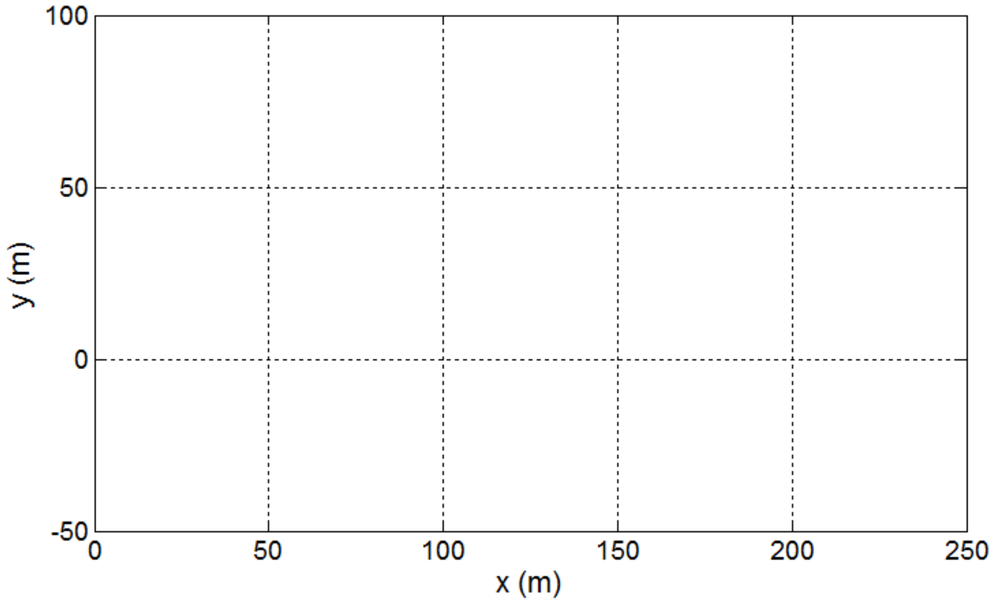


1.1 Assuming the car is at rest at time $t=0$, sketch a graph showing the car's speed as a function of time. Explain briefly how you determined values for relevant quantities.



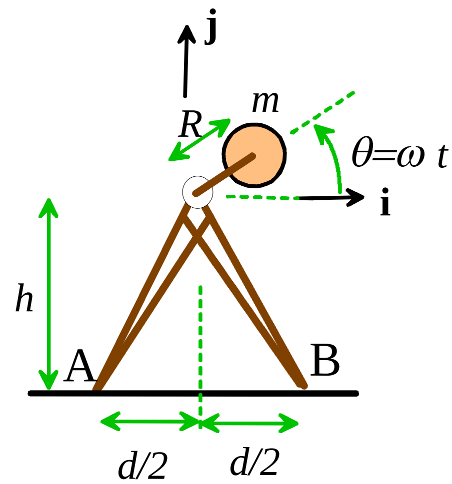
[2 POINTS]

1.2 Assume that at time *t=0* the car is at rest at the origin, and facing in the positive *x* direction. Sketch the subsequent path of the vehicle in the space provided below. Provide as much quantitative information regarding the geometry of the path as you can (e.g. if part of the path is a circle, state the radius of the circle). Explain briefly how you arrived at your conclusions.



[8 POINTS]

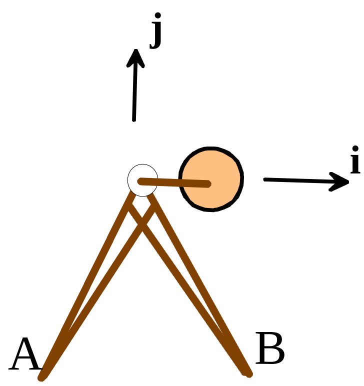
2. The figure shows a mass m on the end of a rotor with length R that spins at constant angular rate ω. It is mounted on a frame whose mass can be neglected. The frame rests on the ground, and friction prevents slip at A and B.



2.1 Write down the position vector of the mass m in terms of the angle θ, using the coordinate system shown. Hence, determine a formula for the acceleration vector of the mass.

[2 POINTS]

2.2 Draw the external forces acting on the mass and its supporting frame (at the instant that $\theta = 0$) on the figure provided. Gravity should be included (but do not show internal forces).



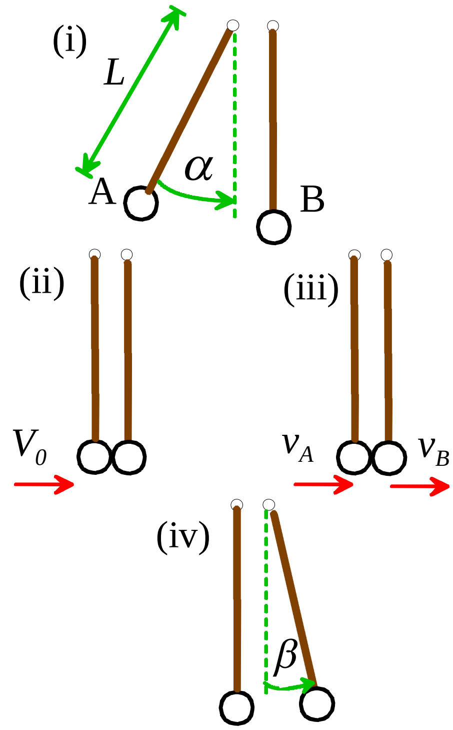
[3 POINTS]

2.3 Write down Newton's laws **F**=m**a** and $\mathbf{M}_c = \mathbf{0}$ for the system at the instant shown in 2.2 (i.e. continue to assume $\theta = 0$), and hence determine expressions for the vertical reaction forces acting at the supports A and B, in terms of m, g, R, ω, h, d.

[4 POINTS]

2.4 Find a formula for the maximum possible speed that the rotor can spin without tipping over the frame in the configuration shown (i.e. with $\theta = 0$), in terms of g, h, d, R.

[2 POINTS]



3. The figure shows an experiment conducted by planetary geologists to determine the restitution coefficient between large masses of rock. Two large granite spheres with identical mass *m* are suspended from cranes to form large pendula. One pendulum is released from rest at an angle α to the vertical. It then collides with the second sphere, causing it to swing through an angle β before coming to rest. The goal of this problem is to find a formula relating α and β to the restitution coefficient.

3.1 Using energy methods, find a formula for the speed V_0 of sphere A just before impact, in terms of g and α.

[2 POINTS]

3.2 By considering the collisions, find a formula for the speed v_B of sphere B just after impact, in terms of V_0 and e.

[2 POINTS]

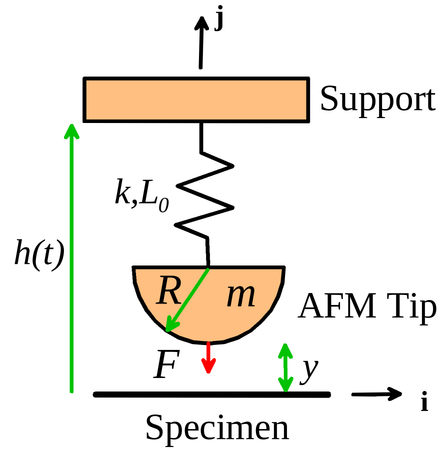
3.3 Find a formula for the angle β in terms of v_B. Hence, determine a formula for e, in terms of α, β.

[2 POINTS]

4. The tip of an atomic force microscope that operates in 'Tapping mode' can be idealized as hemisphere with radius R and mass m, which is suspended from a spring with stiffness k and un-stretched length L_0. The specimen surface exerts an attractive force on the tip with magnitude

$$F = \frac{A}{6y^2}$$

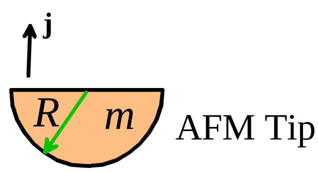
where y is the separation between the sample surface and tip, and A is a constant. The support vibrates with angular frequency ω, so that its height above the sample is $h(t) = H_0 + \Delta H \sin \omega t$. The goal of this problem is to derive an equation of motion for $y(t)$. Horizontal motion may be neglected.



4.1 Write down the acceleration of the AFM tip in terms of y (and its time derivatives)

[2 POINTS]

4.2 Draw the forces acting on the microscope tip on the figure provided below. Gravity may be neglected.



[2 POINTS]

4.3 Hence, show that *y* satisfies the differential equation

$$m\frac{d^2y}{dt^2} + ky + \frac{A}{6y^2} = k(H_0 + \Delta H \sin \omega t - R - L_0)$$

[**2 POINTS]**

4.4 Re-write the differential equation in part 5.3 into a form that MATLAB could solve.

[2 POINTS]