School of Engineering
Brown University

EN40: Dynamics and Vibrations

Midterm Examination
Tuesday March 6 2012

NAME: **Wm. Shakespeare**

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

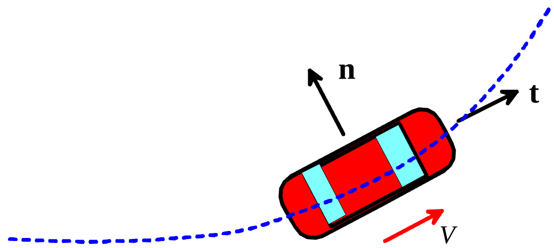
`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

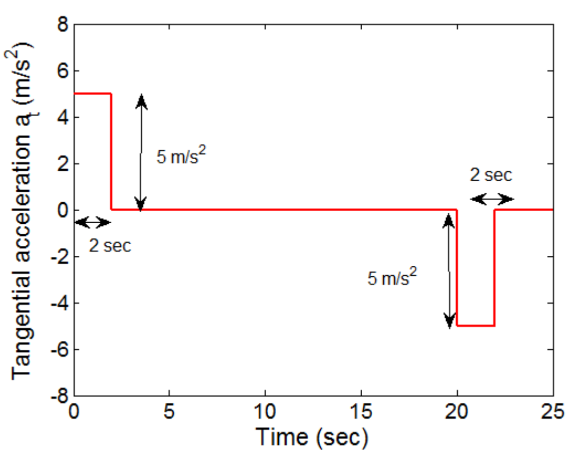
WS

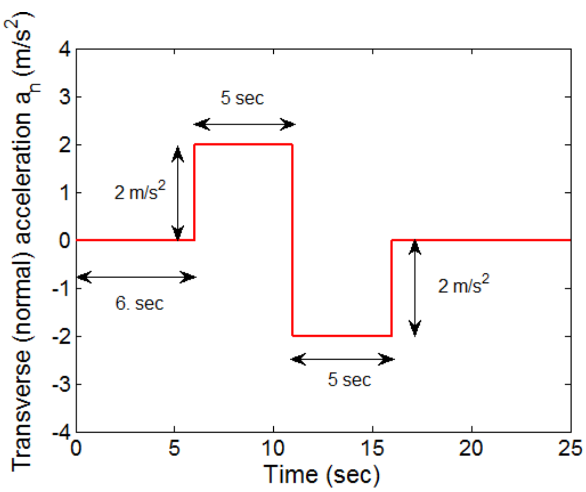
2 (10 points)	**10**
3. (11 points)	**11**
4. (6 points)	**6**
5. (8 points)	**8**

TOTAL (35 points) **35**

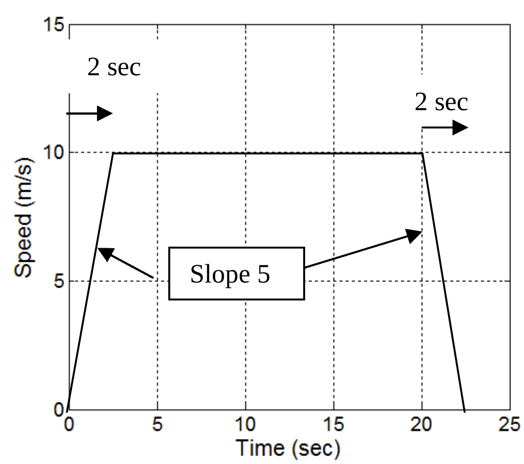
2. A vehicle is instrumented with accelerometers that measure acceleration components a_t, a_n in directions parallel and perpendicular to the car's direction of motion, respectively. (A positive value for a_n means the car accelerates to the left). The graphs below show the variation of a_t, a_n measured in an experiment.







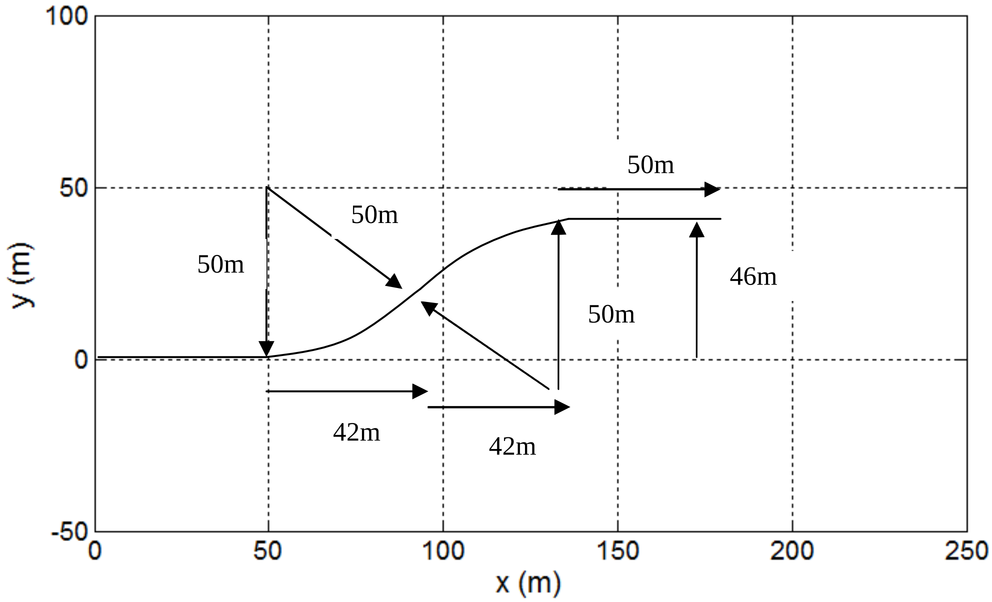
2.1 Assuming the car is at rest at time *t=0*, sketch a graph showing the car's speed as a function of time. Explain how you determined values for relevant quantities.



The speed is just the integral of the tangential acceleration – which can be determined from the area under the acceleration curve.

[2 POINTS]

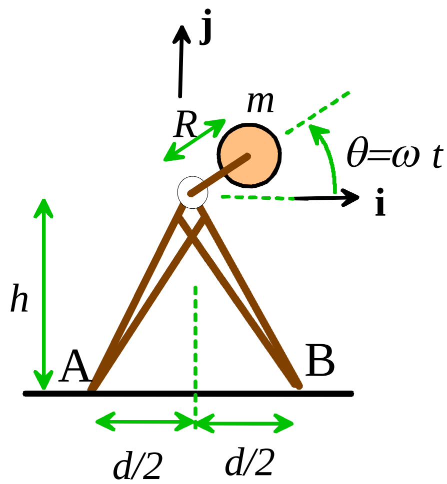
2.2 Assume that at time $t=0$ the car is at the origin, and facing in the positive x direction. Sketch the subsequent path of the vehicle in the space provided below. Provide as much quantitative information regarding the geometry of the path as possible. Explain how you arrived at your conclusions.



- During the first 2 sec, the car moves $10 \times 2/2 = 10m$
- During the subsequent 4 sec, the car moves $10 \times 4. = 40m$
- At $t=6s$ the car has constant speed of 10m/s and constant transverse acceleration. It must therefore move around a circular path. The constant speed circular motion formula shows that the radius of curvature of the path is $R = V^2 / a = 100/2 = 50m$. It must turn to the left, since the acceleration is positive.
- The car travels around this circular path for 5 sec – this means that it travels an arc-length 50m around the circular path. Simple geometry shows that during this segment of the path the car travels in the x direction by 50sin(1)=42m, while it travels 50(1-cos(1))=23m vertically.
- Then the path repeats in sequence but in reverse, starting with a left turn.
- The path must have a continuous slope everywhere, to ensure that the acceleration is finite (a slope discontinuity has zero radius of curvature, and if the car is moving at finite speed this gives infinite acceleration)

[8 POINTS]

3. The figure shows a mass m on the end of a rotor with length R that spins at constant angular rate ω. It is mounted on a frame whose mass can be neglected.



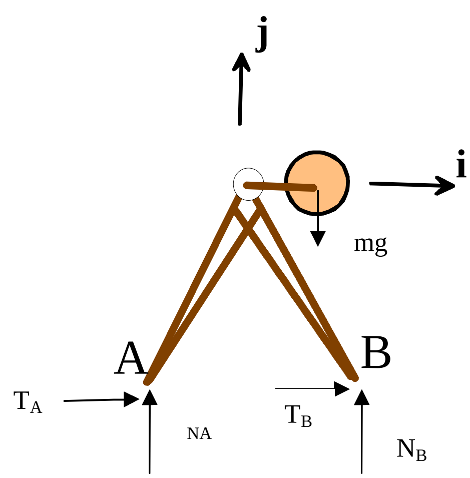
3.1 Write down the position vector of the mass m in terms of the angle θ, using the coordinate system shown. Hence, determine a formula for the acceleration vector of the mass.

The position vector is $\mathbf{r} = R\cos\theta\mathbf{i} + R\sin\theta\mathbf{j}$. Differentiating twice leads to the usual result

$$\mathbf{a} = -\omega^2 R(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

[2 POINTS]

3.2 Draw the external forces acting on the mass and its supporting frame (at the instant that $\theta = 0$) on the figure provided. Gravity should be included (but do not show internal forces).



[3 POINTS]

3.3 Write down Newton's laws **F**=m**a** and $\mathbf{M}_c = \mathbf{0}$ for the system at the instant shown in 3.2, and hence determine expressions for the vertical reaction forces acting at the supports A and B, in terms of *m,g,R,* ω *, h, d* .

$$\left(T_A + T_B\right)\mathbf{i} + (N_A + N_B - mg)\mathbf{j} = -mR\omega^2\mathbf{i}$$
$$h(T_A + T_B) - N_A(d/2 + R) + N_B(d/2 - R) = 0$$

The first equation shows that $T_A + T_B = -m\omega^2 R \qquad N_A + N_B = mg$. Substituting back into the second equation shows that

$$N_A = \left\{ mg(d/2 - R) - hmR\omega^2 \right\} / d$$
$$N_B = \left\{ mg(d/2 + R) + hmR\omega^2 \right\} / d$$

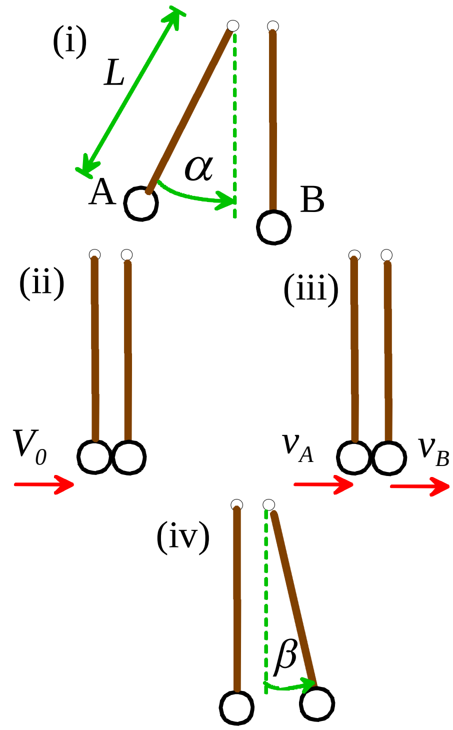
[4 POINTS]

3.4 Find a formula for the maximum possible speed that the rotor can spin without tipping over the frame, in terms of g, h, d, R .

The frame will tip if either reaction goes to zero. The condition $N_A = 0$ shows that

$$\sqrt{\frac{g(d-2R)}{2hR}} = \omega$$

[2 POINTS]



4. The figure shows an experiment conducted by planetary geologists to determine the restitution coefficient between large masses of rock. Two large granite spheres with identical mass *m* are suspended from cranes to form large pendula. One pendulum is released from rest at an angle α to the vertical. It then collides with the second sphere, causing it to swing through an angle β before coming to rest. The goal of this problem is to find a formula relating α and β to the restitution coefficient.

4.1 Using energy methods, find a formula for the speed V_0 of sphere A just before impact, in terms of *g* and α.

Conservative system, so total energy is constant. This shows that

$$mV_0^2/2 = mgL(1-\cos\alpha)$$
$$\Rightarrow V_0 = \sqrt{2gL(1-\cos\alpha)}$$

[2 POINTS]

4.2 By considering the collisions, find a formula for the speed v_B of sphere B just after impact, in terms of V_0 and *e*.

Momentum is conserved during the impact, which yields $mV_0 = mv_A + mv_B$.
The restitution coefficient formula gives $eV_0 = v_B - v_A$
Dividing the first equation through by *m* and adding to the second gives $v_B = (1+e)V_0/2$

[2 POINTS]

4.3 Find a formula for the angle β in terms of v_B. Hence, determine the required relationship between *e*, α, β.

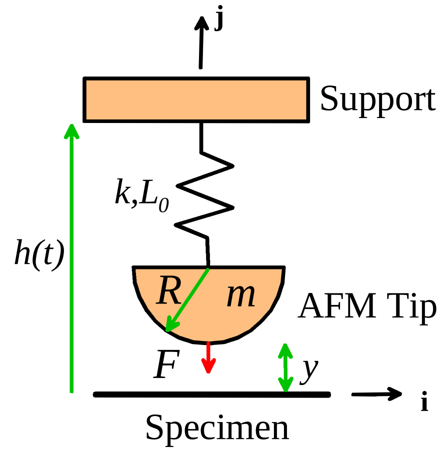
This is 4.1 in reverse - $v_B = \sqrt{2gL(1-\cos\beta)}$. From 4.2 we see that $e = 2v_B/V_0 - 1 = 2\sqrt{\dfrac{1-\cos\beta}{1-\cos\alpha}} - 1$

[2 POINTS]

5. The tip of an atomic force microscope that operates in 'Tapping mode' can be idealized as hemisphere with radius R and mass m, which is suspended from a spring with stiffness k and un-stretched length L_0. The specimen surface exerts an attractive force on the tip with magnitude

$$F = \frac{A}{6y^2}$$

where y is the separation between the sample surface and tip, and A is a constant. The support vibrates with angular frequency ω, so that its height above the sample is $h(t) = H_0 + \Delta H \sin \omega t$. The goal of this problem is to derive an equation of motion for $y(t)$. Horizontal motion may be neglected.

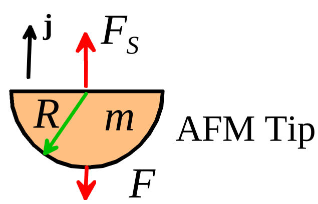


5.1 Write down the acceleration of the AFM tip in terms of y.

$$\mathbf{a} = \frac{d^2 y}{dt^2}\mathbf{j}$$

[2 POINTS]

5.2 Draw the forces acting on the microscope tip on the figure provided below. Gravity may be neglected.



[2 POINTS]

5.3 Hence, show that y satisfies the differential equation

$$m\frac{d^2 y}{dt^2} + ky + \frac{A}{6y^2} = k(H_0 + \Delta H \sin \omega t - R - L_0)$$

The spring force law gives $F_S = k(h - y - L_0)$. Newton's law gives

$$m\frac{d^2 y}{dt^2} = F_S - F = k(H_0 + \Delta H \sin \omega t - L_0 - y) - \frac{A}{6y^2}$$

Rearranging this result gives the required answer.

[2 POINTS]

5.4 Re-write the differential equation in part 5.3 into a form that MATLAB could solve.

We introduce $dy/dt = v_y$ as an additional variable, in the usual way. The EOM then becomes

$$\frac{d}{dt}\begin{bmatrix} y \\ v_y \end{bmatrix} = \begin{bmatrix} v_y \\ k(H_0 + \Delta H \sin \omega t - R - L_0) - ky - A/(6y^2) \end{bmatrix}$$

[2 POINTS]