EN40: Dynamics and Vibrations

Midterm Examination
Tuesday March 5 2013

School of Engineering
Brown University

NAME: _____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

_____

1 (10 points) _____
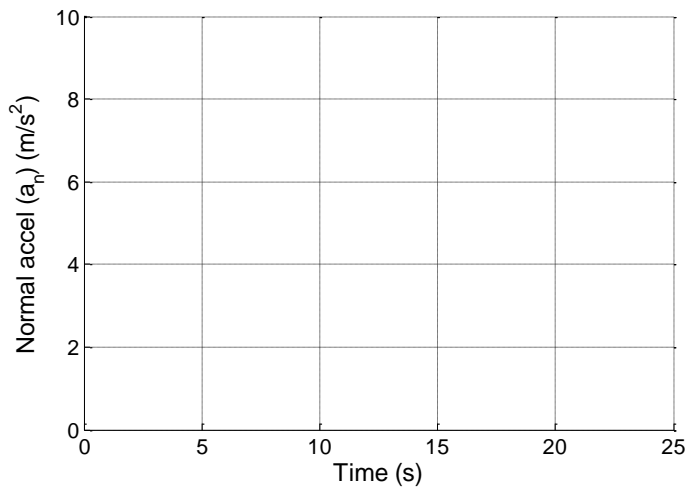
2. (8 points) _____

3. (8 points) _____
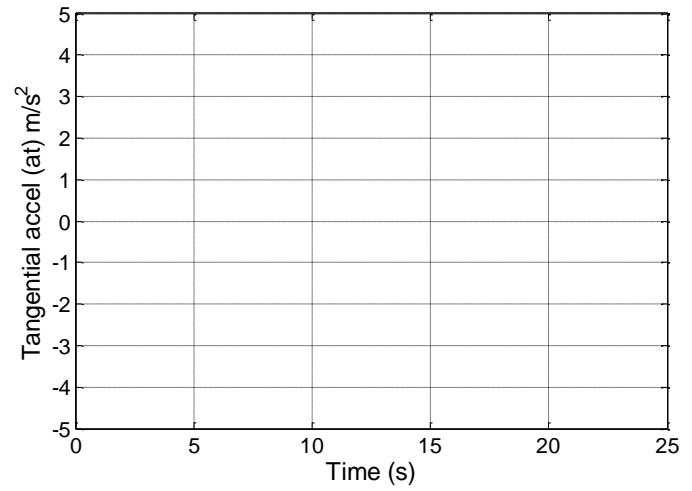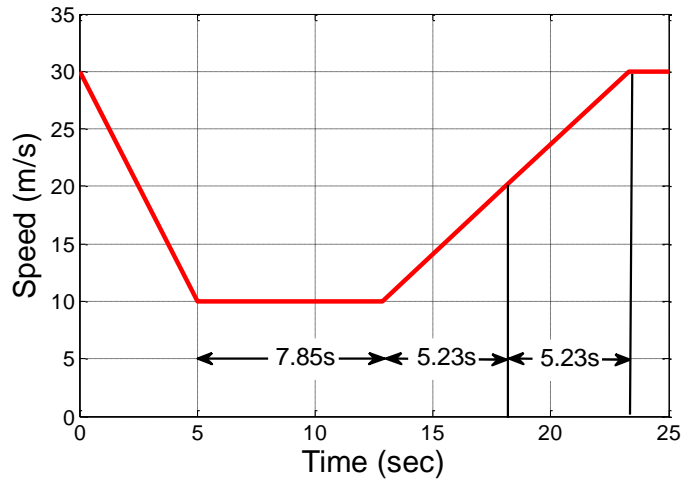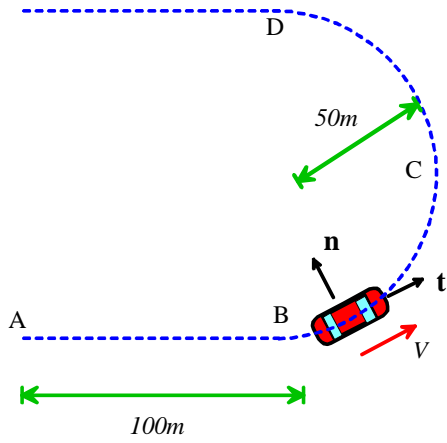
4. (8 points) _____

TOTAL (34 points) _____

1. The path and speed of a vehicle driving around a sharp bend is shown in the figure below (the vehicle is at point A at time $t=0$). Sketch graphs of the normal and tangential acceleration of the vehicle on the axes provided. Explain briefly how you calculated relevant quantities.

2. In a Rutherford scattering experiment, a positively charged alpha particle with mass m is fired with speed $V_0\mathbf{i}$ from a distant point (at infinity) towards a stationary, positively charged nucleus. The charged particle experiences a repulsive radial force

$$\mathbf{F} = \frac{K}{r^2}\mathbf{e}_r$$

where K is a constant. Gravity may be neglected. The trajectory of the particle is sketched in the figure below.
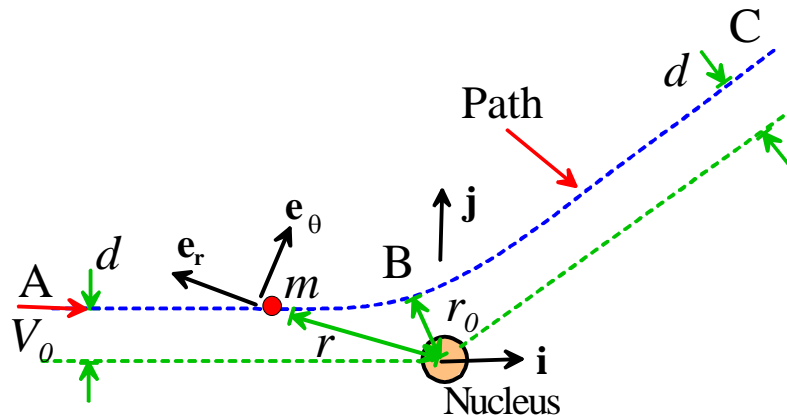
2.1 Find an expression for the potential energy of the force acting on the alpha particle, in terms of K and r.

[2 POINTS]

2.2 Use energy conservation to show that the speed of the particle v is related to its distance from the nucleus r by

$$v^2 = V_0^2 - \frac{2K}{mr}$$

[1 POINT]

3

2.3 Write down the angular momentum vector of the particle about the origin at time $t=0$. (Don't forget to specify the direction as well as the magnitude)
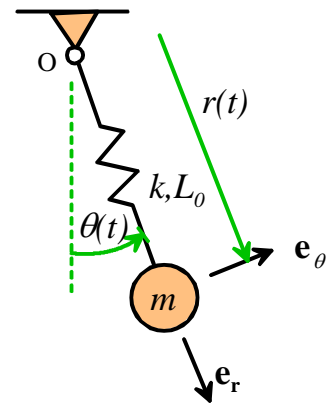
<div align="right">**[2 POINTS]**</div>

2.4 Hence, show that the minimum distance of the particle from the nucleus is

$$r_0 = \frac{K}{mV_0^2}\left[1 + \sqrt{1 + \left(\frac{V_0^2 md}{K}\right)^2}\right]$$
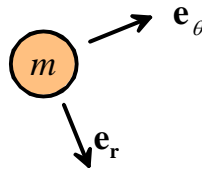
<div align="right">**[3 POINTS]**</div>

3. The figure shows a flexible pendulum, idealized as a point mass m at the end of a spring with stiffness k and un-stretched length L_0. The pivot at O is frictionless.

3.1 Write down the acceleration of the mass, expressed as components in the polar-coordinate basis $\{\mathbf{e}_r, \mathbf{e}_\theta\}$, in terms of geometric variables shown in the figure.

[2 POINTS]

3.2 Draw the forces acting on the mass on the figure provided below.

[2 POINTS]

3.3 Hence, show that r, θ satisfy the differential equations

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 - g\cos\theta + \frac{k}{m}(r - L_0) = 0 \qquad r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt} + g\sin\theta = 0$$
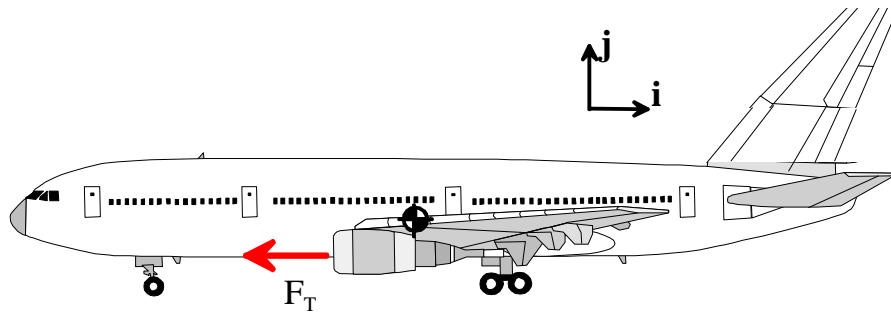
[2 POINTS]

5

3.4 Re-write the equations into a form that MATLAB could solve

[2 POINTS]

4. The figure shows an aircraft just starting its take-off roll. The two engines together provide a total thrust F_T that acts a height h below the center of mass, producing an acceleration $a_x = -(g/2)\mathbf{i}$. Since the aircraft is not yet moving lift and drag forces are zero, and all wheels roll freely.
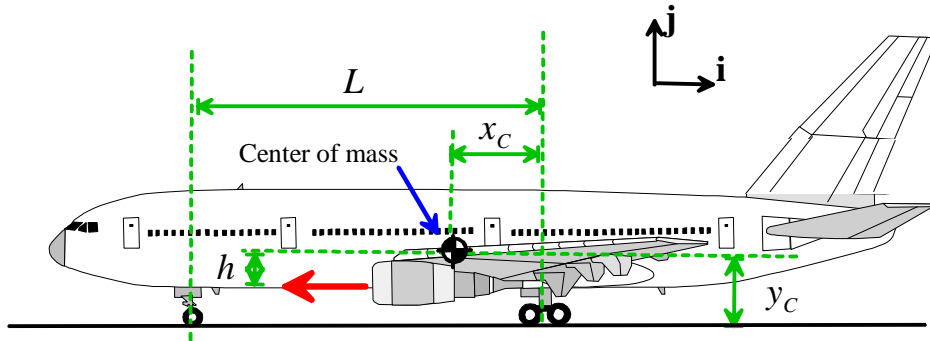
4.1 Draw the forces acting on the aircraft on the figure provided below (you can represent the forces acting on the rear wheels as a single statically equivalent force).

[2 POINTS]

4.2 Write down Newton's law of motion and the equation of rotational motion for the aircraft (assume straight line motion without rotation)

[2 POINTS]

4.3 Hence, find formulas for the reaction forces on the wheels in terms of $m, g,\ x_c L, h$ (for the rear wheels, calculate the single statically equivalent force).

4.4 Hence, show that the front wheel will lose contact with the ground if the distance h of the engine below the center of mass exceeds a critical value, and find a formula for this critical value of h.