School of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Saturday May 10 2014: 2pm-5pm

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1-15 [40 points] ______________

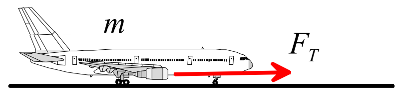
16 [10 POINTS] ______________

17 [10 POINTS] ______________

TOTAL [60 POINTS] ________________

FOR PROBLEMS 1-8 WRITE YOUR ANSWER IN THE SPACE PROVIDED. ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1.1 The aircraft shown in the figure has mass of 10000kg and is at rest at time *t=0*. It is then subjected to an engine thrust that varies with the aircraft's speed according to the formula



$$F = F_0\left(1 - \frac{v}{v_0}\right)$$

where $v_0 = 50m/s$ and $F_0 = 200kN$. After 10sec its speed is

(a) $200m/s$
(b) $50(1-e^{-200})m/s$
(c) $50(1-e^{-4})m/s$
(d) $50(1-e^{200})m/s$
(e) None of the above

$$F = ma \Rightarrow m\frac{dv}{dt} = F_0\left(1 - \frac{v}{v_0}\right)$$

$$\Rightarrow \int_0^v \frac{dv}{1 - v/v_0} = \int_0^t \frac{F_0}{m}dt \Rightarrow -v_0 \log(1 - v/v_0) = \frac{F_0}{m}t$$

$$\Rightarrow v = v_0\left(1 - \exp(-F_0 t/mv_0)\right) = 50\left(1 - \exp(-200\times 10^3 \times 10/10000\times 50)\right)$$

ANSWER_______C_______ **(2 POINTS)**

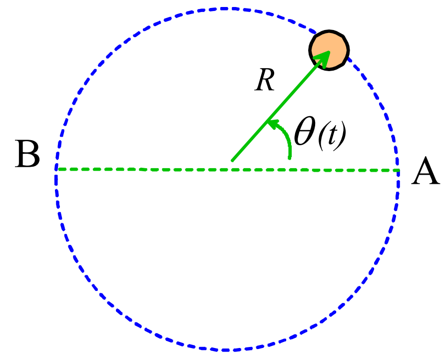
1.2 During the 2 second period the engine exerts an impulse

(a) $2\times 10^6 Ns$
(b) $500000(1-e^{-4})Ns$
(c) $500000(1-e^{-200})Ns$
(d) $500000(1-e^{200})Ns$
(e) None of the above

Linear impulse-momentum gives $I = p_1 - p_0 = mv_1 - 0 = 500000(1-e^{-4})Ns$

ANSWER_______B_______ **(2 POINTS)**

2. The figure shows a particle traveling around a circular path. The angle θ varies with time as $\theta = \pi t^2/4$. When the particle reaches B its tangential and normal acceleration are



(a) $a_t = \frac{\pi}{2}R \quad a_n = \pi^2 R$

(b) $a_t = \pi R \quad a_n = 4\pi^2 R$

(c) $a_t = 0 \quad a_n = \pi^2 R$

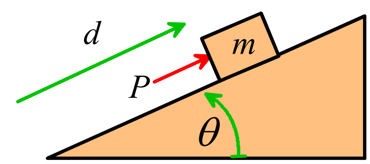
(d) $a_t = \pi R/2 \quad a_n = \pi R/2$

The formulas are $\mathbf{a} = \alpha R\mathbf{t} + R\omega^2\mathbf{n} \qquad \omega = \frac{d\theta}{dt} = \frac{\pi}{2}t \qquad \alpha = \frac{d\omega}{dt} = \frac{\pi}{2}$

At B $\theta = \pi \Rightarrow t = 2 \Rightarrow \mathbf{a} = \frac{\pi}{2}R\mathbf{t} + R\left(\frac{\pi}{2}2\right)^2\mathbf{n} \Rightarrow a_t = \frac{\pi}{2}R \quad a_n = \pi^2 R$

ANSWER______A________ **(2 POINTS)**

3. A constant force P pushes a mass m a distance d up a frictionless slope (the speed of the mass is not constant). The work done by the force is



(a) Pd

(b) $Pd\cos\theta$

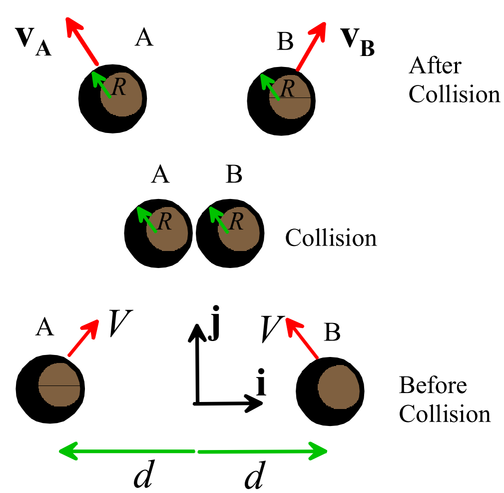
(c) $Pd\sin\theta$

(d) $mgd\sin\theta$

Work done is $\int \mathbf{F}\cdot d\mathbf{r} = Pd$

ANSWER_______A_______ **(2 POINTS)**

4. The two spheres shown in the figure have the same mass. At $t=0$ they have position and velocity vectors $\mathbf{r}_A = -d\mathbf{i}\ \ \mathbf{v}_A = V(\mathbf{i}+\mathbf{j})$, $\mathbf{r}_B = d\mathbf{i}\ \ \mathbf{v}_B = V(-\mathbf{i}+\mathbf{j})$. The collision is frictionless with restitution coefficient $e=0$. After the collision the spheres have velocities



(a) $\mathbf{v}_A = V\mathbf{j} \quad \mathbf{v}_B = V\mathbf{j}$

(b) $\mathbf{v}_A = V(-\mathbf{i}+\mathbf{j}) \quad \mathbf{v}_B = V(\mathbf{i}+\mathbf{j})$

(c) $\mathbf{v}_A = V(\mathbf{i}+\mathbf{j}) \quad \mathbf{v}_B = V(-\mathbf{i}+\mathbf{j})$

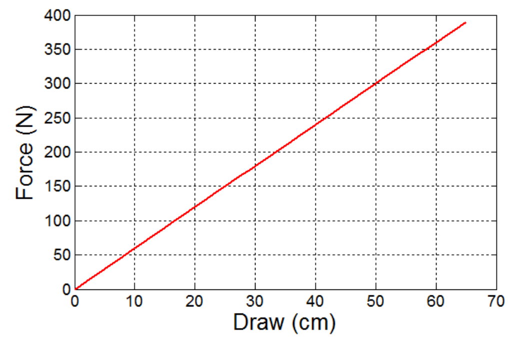
(d) $\mathbf{v}_A = \mathbf{0} \quad \mathbf{v}_B = \mathbf{0}$

In a 3D collision the tangential velocities (parallel to **j**) are unchanged, the normal collisions obey the 1D formulas. If $e=0$ the relative normal velocity (parallel to **i**) is zero after collision. Only (a) satisfies these,.

ANSWER_______A_______ **(2 POINTS)**

5. The figure shows a force-draw curve for a longbow. The bow is drawn to a distance of 50cm and fires an arrow with mass 50 grams at a speed of 30 m/s. Its dynamic efficiency (the ratio of the arrow kinetic energy to the work done in drawing the bow) is

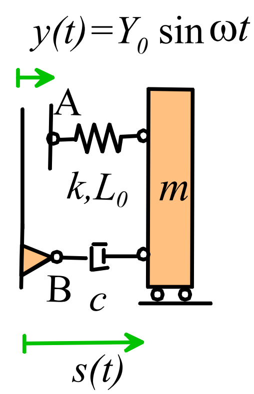
(a) 60%
(b) 45%
(c) 30%
(d) 20%



The dynamic efficiency is the ratio of the KE of the arrow to the energy stored in the bow. $\dfrac{\frac{1}{2}\times 0.05\times(30)^2}{\frac{1}{2}\times 300\times 0.5}=\dfrac{0.1\times 900}{300}=0.3$

ANSWER______C________ **(2 POINTS)**

6. The end of the spring at A moves with a prescribed time dependent displacement $y(t)$. With the definitions $\omega_n=\sqrt{k/m},\ \zeta=c/2\sqrt{km}\quad K=1$ the equation of motion for $s(t)$ is



(a) $\dfrac{1}{\omega_n^2}\dfrac{d^2s}{dt^2}+\dfrac{2\zeta}{\omega_n}\dfrac{ds}{dt}+s=L_0+Ky(t)$

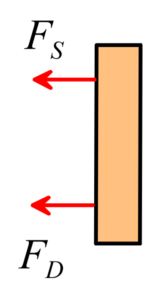
(b) $\dfrac{1}{\omega_n^2}\dfrac{d^2s}{dt^2}+\dfrac{2\zeta}{\omega_n}\dfrac{ds}{dt}+s=L_0+K\left\{y(t)+\dfrac{2\zeta}{\omega_n}\dfrac{dy}{dt}\right\}$

(c) $\dfrac{1}{\omega_n^2}\dfrac{d^2s}{dt^2}+\dfrac{2\zeta}{\omega_n}\dfrac{ds}{dt}+s=L_0+\dfrac{K}{\omega_n^2}\dfrac{d^2y}{dt^2}$

(d) $\dfrac{1}{\omega_n^2}\dfrac{d^2s}{dt^2}+\dfrac{2\zeta}{\omega_n}\dfrac{ds}{dt}+s=L_0$

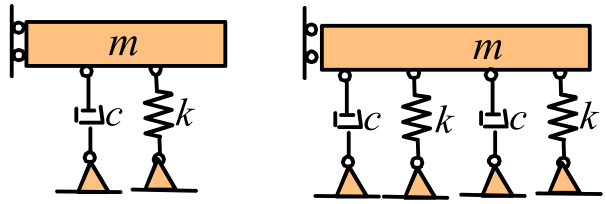
FBD is shown. With $F_S=k(s-y-L_0)$, $F_D=c\dfrac{ds}{dt}$ Newton's law gives

$-k(s-y-L_0)-c\dfrac{ds}{dt}=m\dfrac{d^2s}{dt^2}$. This can be rearranged into case (a)



ANSWER_______A_______ **(2 POINTS)**

7. In the figures shown, system A is critically damped. System B is



(a) Overdamped
(b) Underdamped
(c) Critically Damped
(d) Trivially Dry.

For the first system $\zeta = c/2\sqrt{km} = 1$ so for the second $\zeta = 2c/2\sqrt{2km} > 1$ so overdamped

ANSWER______A________ **(2 POINTS)**

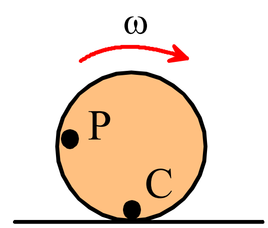
8. In the molecule shown in the figure, the atoms can be idealized as particles and the bonds between atoms as springs. The molecule has

(a) 12 degrees of freedom and 12 vibration modes
(b) 18 degrees of freedom and 18 vibration modes
(c) 18 degrees of freedom and 15 vibration modes
(d) 18 degrees of freedom and 12 vibration modes

6 particles, move in 3D, so 18 DOF. 6 rigid body modes so 18-6=12 vibration modes.

ANSWER________D______ **(2 POINTS)**

9.1 The disk shown in the figure rolls without slip, with a clockwise angular velocity. From the list below, pick the vector that best describes the velocity of point C



(a) Zero
(b) $\longrightarrow$
(c) $\longleftarrow$
(d) $\uparrow$
(e) $\downarrow$

For rolling without slip the contact point has zero velocity

ANSWER_________A_____ **(2 POINTS)**

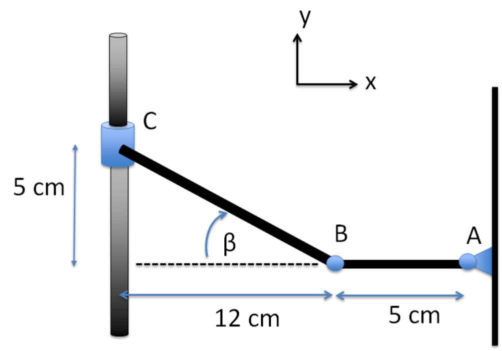
9.2 For the disk described in the preceding problem, pick the vector that best describes the velocity of point P

(a) $\longrightarrow$ (b) $\longleftarrow$ (c) $\nearrow$ (d) $\nwarrow$

The rigid body formula $\mathbf{v}_P = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{P/C}$ shows that the velocity of P must be perpendicular to PC.

ANSWER_______C_______ **(2 POINTS)**

10.1 In the figure shown, link AB has clockwise angular velocity 7 rad/s. The collar at C slides vertically up a fixed rod. The angular velocity of link BC is



(a) 35/12 rad/s (clockwise)
(b) 35/12 rad/s (counterclockwise)
(c) 7 rad/s (clockwise)
(d) Zero

The rigid body formulas for AB and BC give

$$\mathbf{v}_B = -7\mathbf{k} \times (-5\mathbf{i}) = 35\mathbf{j}$$
$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC}\mathbf{k} \times (-12\mathbf{i} + 5\mathbf{j}) = -5\omega_{BC}\mathbf{i} + (35 - 12\omega_{BC})\mathbf{j}$$

Since C moves only in the **j** direction $\omega_{BC} = 0$.

ANSWER_______D_______ **(2 POINTS)**

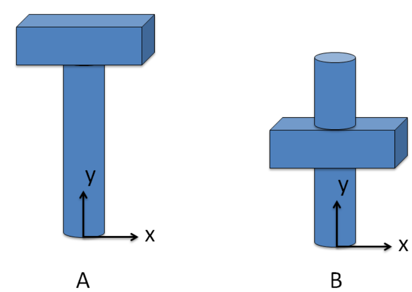
10.2 In the mechanism described in the preceding problem link AB has angular acceleration 5 rad/s^2 in the counterclockwise direction. The acceleration of **point B** is

(a) **0**
(b) $245\mathbf{i} - 25\mathbf{j}$ cm/s
(c) $5\mathbf{i} - 25\mathbf{j}$ cm/s
(d) $-5\mathbf{i} - 25\mathbf{j}$ cm/s

The rigid body formulas for AB gives $\mathbf{a}_B = 5\mathbf{k} \times (-5\mathbf{i}) - 7\mathbf{k} \times (-7\mathbf{k}) \times (-5\mathbf{i}) = 245\mathbf{i} - 25\mathbf{j}$

ANSWER______B________ **(2 POINTS)**

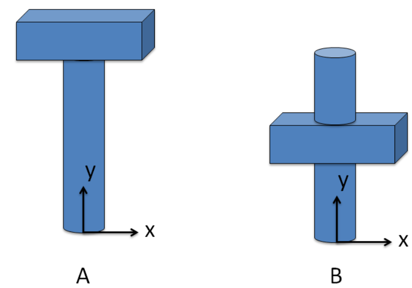
11. In the figure shown, both the cylinder and the rectangular prism have the same dimensions and are welded together to form a single rigid body. Both objects have the same, uniform mass density. The mass moments of inertia of the two objects about the *x* axis satisfy



(a) $I_x^A > I_x^B$
(b) $I_x^A < I_x^B$
(c) $I_x^A = I_x^B$
(d) Further information is needed to answer.

ANSWER_______A_______ **(2 POINTS)**

12. In the figure shown, both the cylinder and the rectangular prism have the same dimensions and are welded together to form a single rigid body. Both objects have the same, uniform mass density. The mass moments of inertia of the two objects about the *y* axis satisfy



(a) $I_y^A > I_y^B$

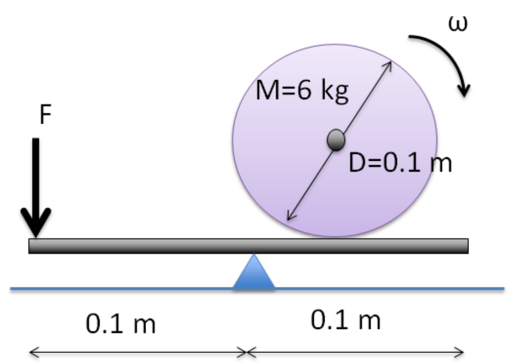
(b) $I_y^A < I_y^B$

(c) $I_y^A = I_y^B$

(d) Further information is needed to answer.

ANSWER______C________ **(2 POINTS)**

13. The thin disk brake drum is rotating at an angular speed of 10 rad/s at t=0. A force is applied to the thin rigid bar to stop the drum from rotating. How much work must be done on the drum in order to stop its rotation completely?



(a) 3 J

(b) 3/4 J

(c) 3/8 J

(d) None of the above

The work done is equal to the change in KE, i.e. $\frac{1}{2} I_G \omega^2 = \frac{1}{2}\frac{m}{2}\frac{D^2}{4}\omega^2 = \frac{6}{16}(0.1)^2 10^2 = \frac{3}{8} J$

ANSWER_______C_______ **(2 POINTS)**

14. Identify whether the statements below are true or false

(a) Angular impulse has the same units as energy — TRUE **[FALSE]**

(b) Angular impulse has the same units as angular momentum — **[TRUE]** FALSE

(c) Angular impulse is the time integral of an applied moment — **[TRUE]** FALSE

(d) Angular impulse is equal to the change in linear momentum — TRUE **[FALSE]**

(2 POINTS)

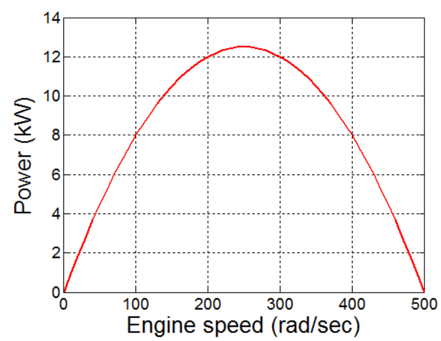
15.1 A car has a wheel radius of 0.5m and gear ratio (motor angular speed/axle angular speed) of 10. If the engine runs at 200 rad/s the car speed is

(a) 1 m/s
(b) 5 m/s
(c) 10 m/s
(d) 20 m/s

Axle angular speed is 20 rad/s and $v = R\omega \Rightarrow v = 10 m/s$

ANSWER_______C_______ **(2 POINTS)**

15.2 The vehicle described in the preceding problem is powered by an internal combustion engine with power-curve shown in the figure (with power in kW and engine speed in rad/s). The vehicle has mass 1000kg and drives on flat ground with the engine running at 200 rad/s. If transmission losses and air resistance can be neglected, the instantaneous acceleration of the vehicle is



(a) 10 m/s^2
(b) 15 m/s^2
(c) 20 m/s^2
(d) 25 m/s^2
(e) none of the above

The usual energy argument gives

$$P = \frac{d}{dt}(KE + PE) = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = P$$
$$\Rightarrow a = \frac{P}{mv} = \frac{12000}{1000 \times 10} = 1.2 m/s^2$$

ANSWER_______E_______ **(2 POINTS)**

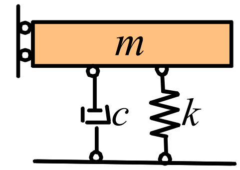
15.3 If the gear ratio is unchanged, the vehicle described in the preceding problems will continue to accelerate (assuming level ground and zero friction losses) until its speed reaches

(a) Infinity
(b) 50 m/s
(c) 25 m/s
(d) 2.5 m/s

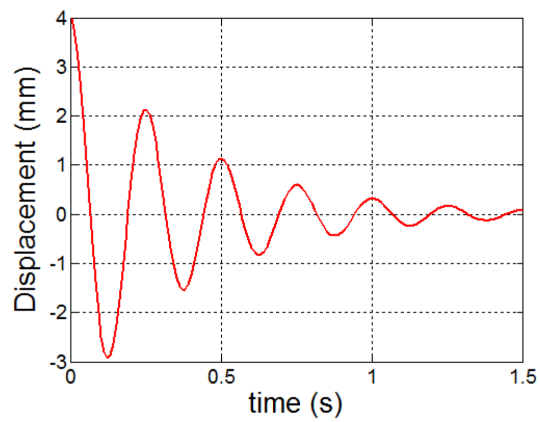
Traveling at constant speed on level ground requires zero power. The car will accelerate until the motor angular speed reaches 500 rad/s. Thus $v = r\omega = (500/10) \times 0.5 = 25 m/s$

ANSWER_______C_______ **(2 POINTS)**

16. A vibration isolation platform can be idealized as a spring-mass-damper system as shown in the figure. In a free vibration test on the table, the base is held fixed and the platform is disturbed slightly from its equilibrium position. The subsequent displacement of the table is plotted in the figure below as a function of time.



16.1 Use the graph provided to estimate the period of oscillation and the log decrement.



There are two cycles in 0.5 sec so the period is 0.25 sec

Using the first and 3rd peaks the log decrement is log(4)/2 (use the natural log) is 0.693

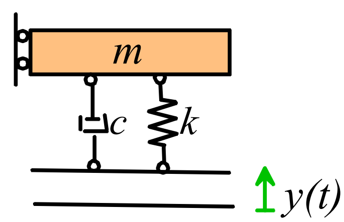
(2 POINTS)

16.2 Hence, calculate the natural frequency and damping factor that characterize the vibration isolation table.

The standard formulas give $\zeta = \dfrac{\delta}{\sqrt{4\pi^2 + \delta^2}} \approx 0.1 \qquad \omega_n = \dfrac{\sqrt{4\pi^2 + \delta^2}}{T} \approx 25 \text{ rad/s}$

(2 POINTS)

16.3 The base of the platform is subjected to a harmonic displacement $y(t) = Y_0 \sin \omega t$ with amplitude 5mm and frequency $(25/\pi)$ Hz. Calculate the amplitude of vibration of the platform.



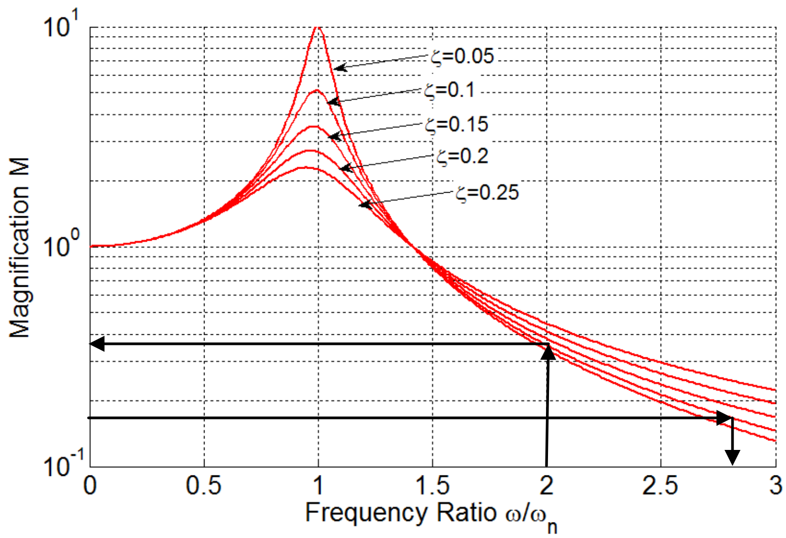
The angular frequency of the base is 50 rad/s.

The displacement amplitude follows from the standard formula

$$Y_0 \frac{\sqrt{1 + \left(\dfrac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \dfrac{\omega^2}{\omega_n^2}\right)^2 + \left(\dfrac{2\zeta\omega}{\omega_n}\right)^2}} = 5 \frac{\sqrt{1 + \left(\dfrac{2\times 50}{10\times 25}\right)^2}}{\sqrt{\left(1 - \dfrac{50^2}{25^2}\right)^2 + \left(\dfrac{2\times 50}{10\times 25}\right)^2}} = 1.8mm$$

(2 POINTS)

16.4 It is necessary to modify the vibration isolation system to further reduce the vibration amplitude by a factor of two. Using the graph provided, recommend changes to the values of *k,m*, and/or *c* necessary to achieve this (e.g. recommend that *k* should be increased by some factor, *m* should be reduced by some factor, etc).



The simplest approach to this is to change the value of ω_n for the system.

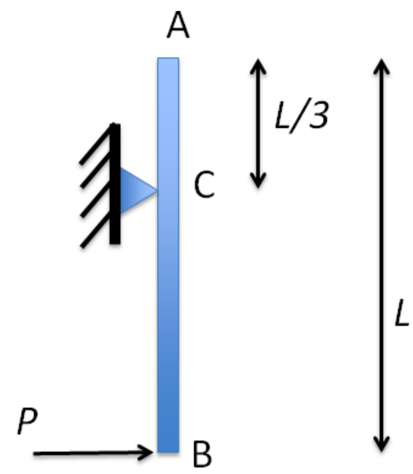
From the graph, in the original design the magnification is 0.35 at the original frequency $\omega / \omega_n = 2$. We need to reduce the magnification to 0.175, which requires $\omega / \omega_n > 2.75$.

Therefore ω_n must be divided by 2.75/2=1.4.

Since $\omega_n = \sqrt{\frac{k}{m}}$ and $\zeta = \frac{c}{2\sqrt{km}}$ we could, e.g. reduce *k* by a factor of 2. This would reduce the damping – this would help reduce the steady state vibration amplitude. If after this modification it was found that the transient vibrations decay too slowly the value of *c* could be increased by 1.4, or instead reduce *k* by 1.4 and increase *m* by 1.4. Other solutions are possible.

(4 POINTS)

17. The figure shows a bar with length L and mass m that is suspended from a pivot at point C. At the instant shown the bar is at rest.



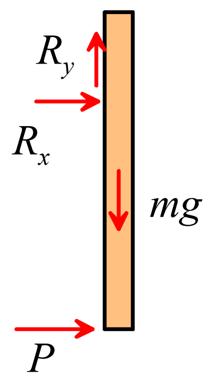
17.1 The bar has mass moment of inertia $I_G = mL^2/12$ about its center of mass. Find its mass moment of inertia about C

The parallel axis theorem gives

$$I_C = \frac{mL^2}{12} + m\left(\frac{L}{2} - \frac{L}{3}\right)^2 = \frac{1}{9}mL^2$$

(2 POINTS)

17.2 Draw a free body diagram showing all the forces acting on the rod



(2 POINTS)

17.3 Hence, find a formula for the angular acceleration of the rod at the instant shown in the figure.

Equation for rotational motion about C gives

$$P\left(\frac{L}{2} + \frac{L}{6}\right) = \frac{1}{9}mL^2\alpha \Rightarrow \alpha = \frac{6P}{mL}$$

(2 POINTS)

17.4 Find a formula for the acceleration of the center of mass of the rod at the instant shown

The rigid body formula gives $\mathbf{a}_G = \boldsymbol{\alpha} \times \mathbf{r}_{G/C} \Rightarrow \mathbf{a}_G = \frac{6P}{mL}\frac{L}{6}\mathbf{i} = \frac{P}{m}\mathbf{i}$

(2 POINTS)

17.5 Find formulas for the reaction forces at *C* at the instant shown.

$$\mathbf{F} = m\mathbf{a} \Rightarrow \left(P + R_x\right)\mathbf{i} + \left(R_y - mg\right)\mathbf{j} = m\frac{P}{m}\mathbf{i}$$
$$\Rightarrow R_x = 0 \qquad R_y = mg$$

(2 POINTS)