School of Engineering
Brown University

# EN40: Dynamics and Vibrations

## Midterm Examination
## Thursday April 2 2015

**NAME:** _____

### General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

### Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

_____

**1 (5 points)** _____

**2. (10 points)** _____

**3. (5 points)** _____

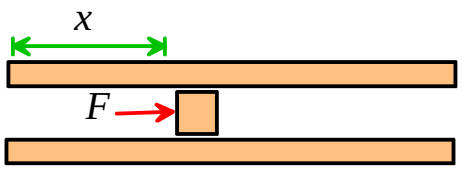
**4. (10 points)** _____

**TOTAL (30 points)**_____

**1.** The projectile in a gas gun is subjected to a propulsive force

$$F = F_0\left(1-\frac{v}{3c}\right)^5$$

where $F_0, c$ are constants and $v$ is the projectile's speed. The projectile has mass $m$. It starts at rest at time $t=0$ at position $x=0$.



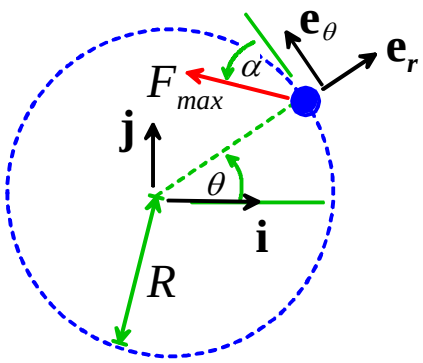

1.1 Use Newton's law to determine the acceleration of the projectile and hence determine an expression for its speed as a function of time and other parameters. Air resistance and friction may be neglected.

**[3 POINTS]**

1.2 Find a formula for the distance traveled by the projectile as a function of time.

**[2 POINTS]**

**2.** A 'prey' particle with mass *m* is subjected to a propulsive force with magnitude $F_{\max}$ that is applied at a constant angle $\alpha$ to its direction of motion. It also experiences a drag force (not shown in the figure) with magnitude $cV$ (where *V* is the magnitude of the velocity and *c* is a constant) that acts opposite to its direction of motion. The random force is zero. As a result, it travels at constant speed *V* around a circular path with radius *R*.




2.1 Write down expressions for the propulsive force vector and drag force vector acting on the particle, in terms of $F_{\max}$, $\alpha$, $c$ and *V*, expressing your answer in polar coordinates $\{\mathbf{e}_r, \mathbf{e}_\theta\}$.

**[2 POINTS]**

2.2 Write down an expression for the acceleration vector in terms of *V* and *R*, and hence use Newton's law to show that

$$V = \frac{F_{\max}}{c}\cos\alpha \qquad R = \frac{mF_{\max}}{c^2}\frac{\cos^2\alpha}{\sin\alpha}$$

**[3 POINTS]**

2.3 Find a formula for the time required for the particle to complete a full circle.

**[1 POINT]**

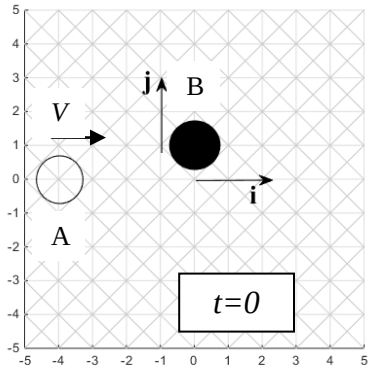
2.4 In the MATLAB code controlling the prey, the propulsive force is calculated from the formula $\mathbf{F} = F_{\max}\cos\omega t\mathbf{i} + F_{\max}\sin\omega t\mathbf{j}$, where $\omega$ is a constant. Use the solutions to 2.2 and 2.3 to find a formula relating the radius of the path $R$ to $\omega$, $F_{\max}$, $m$ and $c$. The formula $\cos\theta = 1/\sqrt{1+\tan^2\theta}$ might be helpful.

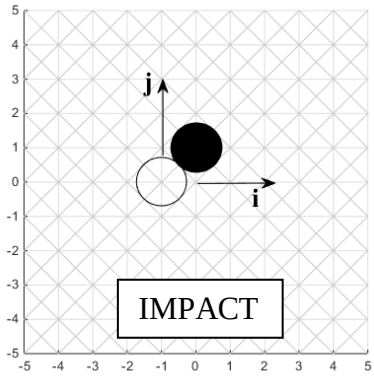
**[2 POINTS]**

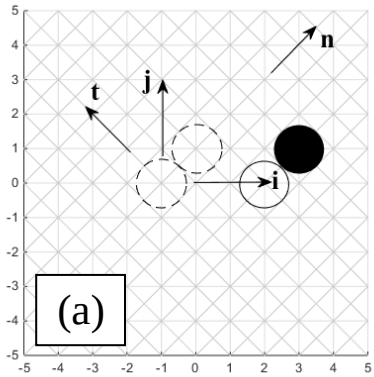
2.5 The prey starts with a total energy supply $E$. Find a formula for the maximum time that the particle can continue traveling around the circular path without exhausting the energy supply, in terms of $E, F_{\max}, m, \omega, c$.

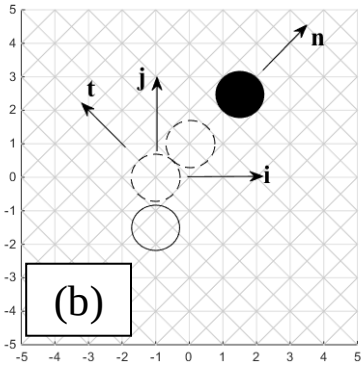
**[2 POINTS]**

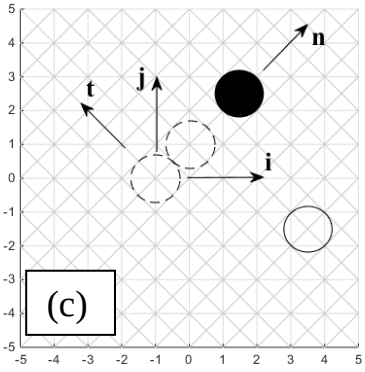
**3.** Two spheres with identical mass and restitution coefficient *e=0* have initial positions shown in the figure below. Before impact sphere B is stationary and sphere *A* has velocity *V***i.** The collision is frictionless. By answering the true/false questions below, identify which of the figures (a-d) shows the correct position of the spheres after collision.

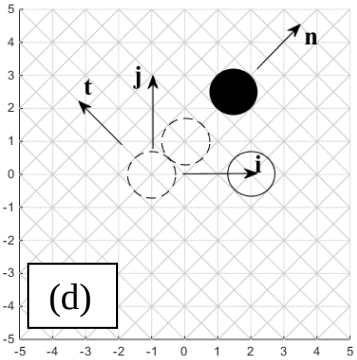



CIRCLE ONE RESPONSE TO EACH STATEMENT BELOW

(a) Total Momentum is conserved in the **j** direction T F
Momentum of B is conserved in the **t** direction T F
The restitution formula is satisfied in the **n** direction T F

(b) Total Momentum is conserved in the **j** direction T F
Momentum of B is conserved in the **t** direction T F
The restitution formula is satisfied in the **n** direction T F

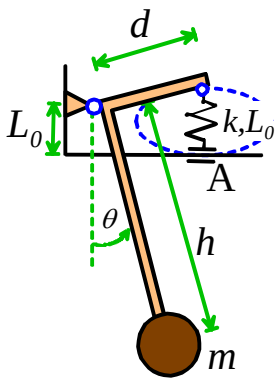
(c) Total Momentum is conserved in the **j** direction T F
Momentum of B is conserved in the **t** direction T F
The restitution formula is satisfied in the **n** direction T F

(d) Total Momentum is conserved in the **j** direction T F
Momentum of B is conserved in the **t** direction T F
The restitution formula is satisfied in the **n** direction T F

Correct figure: a b c d

**[5 POINTS]**

**4.** The figure shows a schematic diagram of a pendulum apparatus designed to measure the stiffness of an intervertebral disk. The disk is idealized as a spring with stiffness $k$ and unstretched length $L_0$ (the slider at A allows the spring to remain vertical at all times). The goal of this problem is to find the relationship between $k$ and the measured period of oscillation of the pendulum.




4.1 Write down a formula for the speed of the mass $m$ in terms of $h$ and $d\theta/dt$

**[1 POINT]**

4.2 Hence, write down the total potential and kinetic energy of the system, in terms of $k,d,h,m,g$ and $\theta$ and its time derivatives.

**[2 POINTS]**

4.3 Hence, show that $\theta$ satisfies the equation of motion

$$mh^2 \frac{d^2\theta}{dt^2} + kd^2 \sin\theta\cos\theta + mgh\sin\theta = 0$$

**[2 POINTS]**

4.4 Linearize the equation of motion for small $\theta$ and hence find a formula for the natural frequency of vibration of the pendulum, in terms of $k, d, h, m, g$ .

**[3 POINTS]**

4.5 Rearrange the equation in 4.3 into a form that MATLAB could solve using the ode45 function.

**[2 POINTS]**