School of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Friday May 8 2015: 2pm-5pm

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1-20 [40 points] ______________

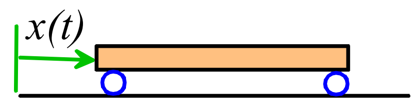
21 [12 POINTS] ______________

22 [10 POINTS] ______________

TOTAL [62 POINTS] ______________

FOR PROBLEMS 1-20 WRITE YOUR ANSWER IN THE SPACE PROVIDED. ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. The platform shown in the figure vibrates horizontally with a displacement $x(t) = X_0(1-\cos\omega t)$. Its horizontal acceleration is



(a) $a(t) = -X_0\omega^2(1-\cos\omega t)$

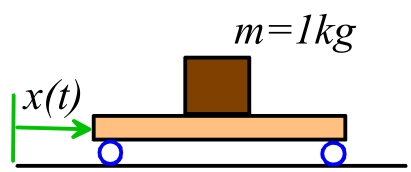
(b) $a(t) = -X_0\omega^2 \sin\omega t$

(c) $a(t) = X_0\omega^2 \cos\omega t$

(d) $a(t) = X_0\omega \sin\omega t$

ANSWER_______________ **(2 POINTS)**

2. A 1 kg mass rests on a horizontal surface with friction coefficient 0.5. At time *t=0* the surface begins to vibrate horizontally with a displacement $x(t) = X_0(1-\cos\omega t)$. The frequency of vibration is $\omega = 10$ rad/sec. For each value of X_0 listed below, state whether or not slip will occur at the contact between the mass and the surface.



(a) $X_0 = 1cm$ SLIP NO SLIP

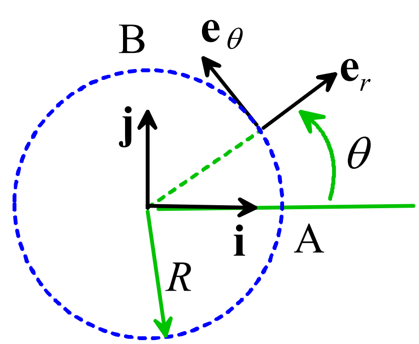
(b) $X_0 = 2cm$ SLIP NO SLIP

(c) $X_0 = 4cm$ SLIP NO SLIP

(d) $X_0 = 8cm$ SLIP NO SLIP

(2 POINTS)

3. A particle starts at rest at point A and travels with constant tangential acceleration around a circular path with radius *R=4m*. After 2 seconds it has a speed of 4 m/s. The acceleration at time *t*=2 sec is



(a) $\mathbf{a} = 2\mathbf{e}_r + 4\mathbf{e}_\theta \quad m/s^2$

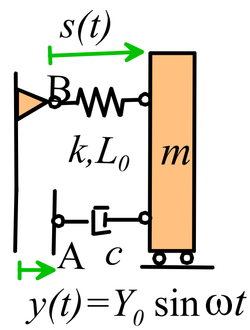
(b) $\mathbf{a} = -4\mathbf{e}_r + 2\mathbf{e}_\theta \quad m/s^2$

(c) $\mathbf{a} = -2\mathbf{e}_r + 4\mathbf{e}_\theta \quad m/s^2$

(d) $\mathbf{a} = 4\mathbf{e}_r + 2\mathbf{e}_\theta \quad m/s^2$

ANSWER_______________ **(2 POINTS)**

4. The end of the dashpot at A moves with a prescribed time dependent displacement $y(t)$. With the definitions $\omega_n = \sqrt{k/m},\ \zeta = c/2\sqrt{km}\quad K = 1$ the equation of motion for $s(t)$ is



(a) $\frac{1}{\omega_n^2}\frac{d^2 s}{dt^2} + \frac{2\zeta}{\omega_n}\frac{ds}{dt} + s = L_0 + Ky(t)$

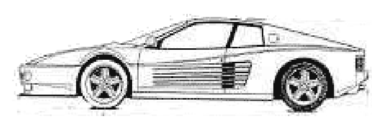
(b) $\frac{1}{\omega_n^2}\frac{d^2 s}{dt^2} + \frac{2\zeta}{\omega_n}\frac{ds}{dt} + s = L_0 + K\frac{2\zeta}{\omega_n}\frac{dy}{dt}$

(c) $\frac{1}{\omega_n^2}\frac{d^2 s}{dt^2} + \frac{2\zeta}{\omega_n}\frac{ds}{dt} + s = L_0 + K\left\{y(t) + \frac{2\zeta}{\omega_n}\frac{dy}{dt}\right\}$

(d) $\frac{1}{\omega_n^2}\frac{d^2 s}{dt^2} + \frac{2\zeta}{\omega_n}\frac{ds}{dt} + s = L_0 + \frac{K}{\omega_n^2}\frac{d^2 y}{dt^2}$

ANSWER_______________ **(2 POINTS)**

5. An electric vehicle with mass 2000 kg is powered by a battery that is capable of developing a maximum power of 200kW. Neglecting air resistance, the shortest possible time for the vehicle to accelerate from rest to 20m/s while traveling on level ground is approximately



(a) 0.5 sec

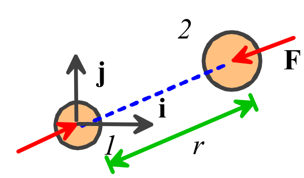
(b) 1 sec

(c) 2 sec

(d) 4 sec

ANSWER_______________ **(2 POINTS)**

6. The bond in a diatomic molecule has a potential energy that is approximated using the function $V(r) = -E_0(r/d)\exp(-r/d)$, where r is the distance between atoms and E_0 and d are constants. When the atoms are separated by a distance $r=2d$, the force of attraction between them is



(a) $F = E_0 \exp(-2)/(2d)$

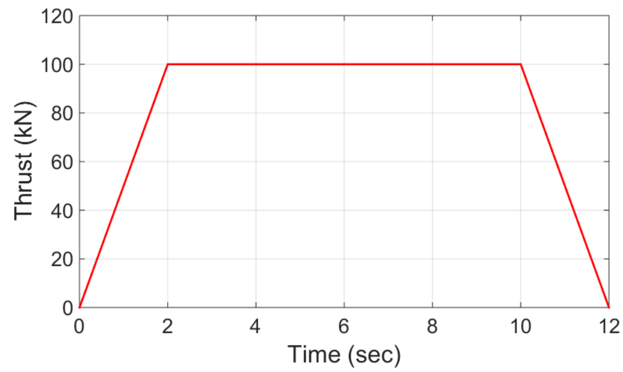
(b) $F = -2E_0 \exp(-2)$

(c) $F = E_0 \exp(-2)/d$

(d) $F = 2E_0 \exp(-2)/d$

ANSWER_______________ **(2 POINTS)**

7. A satellite with mass 500kg has velocity 8 km/s. A rocket is fired that exerts a thrust on the satellite (directed parallel to the velocity of the satellite), which varies with time as shown in the figure. Neglecting the impulse exerted by gravity on the satellite, the velocity of the satellite just after the rocket is fired (at *t=12* sec) is



(a) 6 km/s

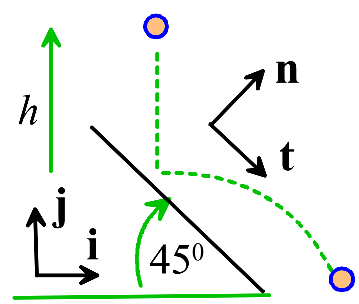
(b) 9 km/s

(c) 10 km/s

(d) 12 km/s

ANSWER_______________ **(2 POINTS)**

8. A spherical rock sample is dropped from rest from height h onto a 45 degree slope. The collision has a restitution coefficient e and is frictionless. The normal and tangential velocities of the sphere after impact are



(a) $v_n = e\sqrt{2gh}$ $\quad v_t = \sqrt{2gh}$

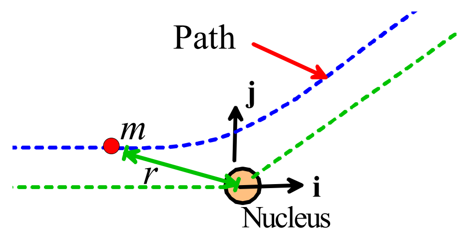
(b) $v_n = e\sqrt{gh}$ $\quad v_t = \sqrt{gh}$

(c) $v_n = e\sqrt{gh}$ $\quad v_t = e\sqrt{gh}$

(d) $v_n = e\sqrt{2gh}$ $\quad v_t = e\sqrt{2gh}$

ANSWER_______________ **(2 POINTS)**

9. The figure shows the path of a charged particle with mass m in a scattering experiment. The nucleus can be assumed to be stationary, and exerts a repulsive radial force on the charged particle. Identify whether the statements below are true or false



(a) Linear momentum of the charged particle is conserved T F

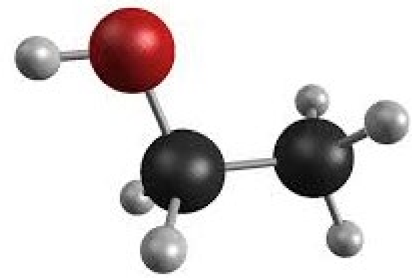
(b) The total energy of the system is conserved T F

(c) Angular momentum of the charged particle is conserved about the origin T F

(d) The forces acting on the charged particle do no work T F

(2 POINTS)

10. In the ethyl alcohol molecule shown in the figure, the atoms can be idealized as particles and the bonds between atoms as springs. The molecule has

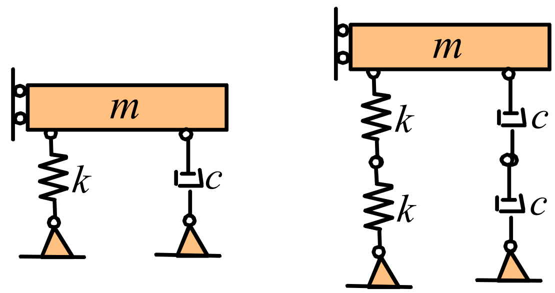


(a) 18 degrees of freedom and 12 vibration modes
(b) 18 degrees of freedom and 18 vibration modes
(c) 27 degrees of freedom and 21 vibration modes
(d) 27 degrees of freedom and 27 vibration modes

ANSWER_______________ **(2 POINTS)**

11. The system shown in figure (a) is critically damped. The system in figure (b) must therefore have a damping factor

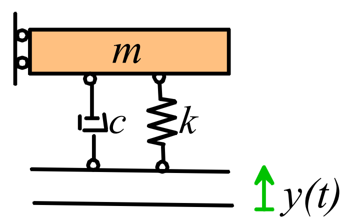
(a) $\zeta = 1/\sqrt{2}$
(b) $\zeta = 1/2\sqrt{2}$
(c) $\zeta = \sqrt{2}$
(d) $\zeta = 1$
(e) None of the above.



(a) (b)

ANSWER______________**(2 POINTS)**

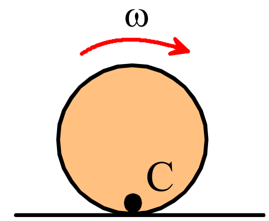
12. A vibration isolation platform can be idealized as a spring-mass system. It has a natural frequency of 2Hz. The base is excited at a frequency 4Hz . Identify whether each of the changes to the system listed below will increase or decrease the steady-state vibration amplitude of the system:



(a) Double the dashpot coefficient c	INCREASE	DECREASE
(b) Double the spring stiffness k	INCREASE	DECREASE
(c) Double the mass m	INCREASE	DECREASE
(d) Double the frequency of the base excitation to 8Hz.	INCREASE	DECREASE

(2 POINTS)

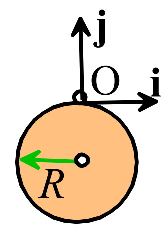
13. The disk shown in the figure rolls without slip, with a clockwise angular velocity. From the list below, pick the vector that best describes the acceleration of point C



(a) Zero
(b) →
(c) ←
(d) ↑
(e) ↓

ANSWER______________ **(2 POINTS)**

14. The mass moment of inertia of the thin disk of mass *m* and uniform density about an axis parallel to the **k** direction passing through O is



(a) $I_0 = mR^2/2$

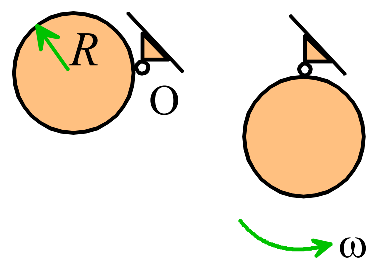
(b) $I_0 = mR^2$

(c) $I_0 = 3mR^2/2$

(d) $I_0 = 2mR^2$

ANSWER______________ **(2 POINTS)**

15. The disk shown in the figure swings freely about O. At time *t=0* the center of the disk is level with O, and the disk is stationary. When the center of the disk is immediately below O, its angular speed is



(a) $\omega = 2\sqrt{\frac{g}{3R}}$

(b) $\omega = \sqrt{\frac{2g}{3R}}$

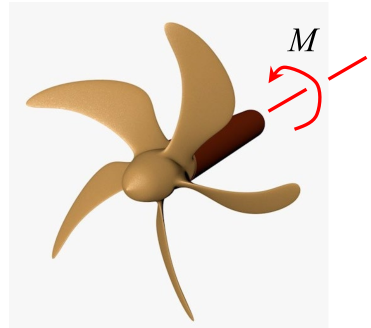
(c) $\omega = \sqrt{\frac{2g}{R}}$

(d) $\omega = 2\sqrt{\frac{g}{R}}$

(e) None of the above

ANSWER______________**(2 POINTS)**

16. The figure shows a propeller with total mass moment of inertia 150 kg m^2. The propeller is driven by a motor (not shown) that exerts a moment M on the shaft. At the instant shown, the propeller has zero angular velocity and has an angular acceleration of 10 rad s^{-2}. At this instant, the rate of work done by the moment on the propeller is



(a) Zero

(b) 1500 Watts

(c) 7500 Watts

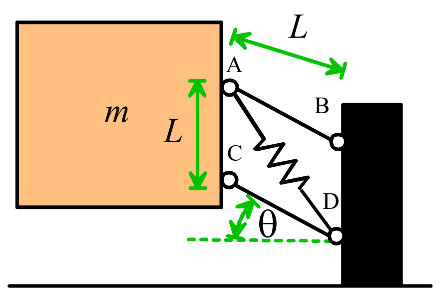
(d) 75000 Watts

ANSWER_____________**(2 POINTS)**

17. The figure shows a proposed design for a suspension. The links AB and CD are rigid, and the spring has stiffness k. The static equilibrium configuration is $\theta = 0$, and the equation of motion for the system is

$$mL\frac{d^2\theta}{dt^2} + (mg + kL)\left[\cos\theta - \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right] = 0$$

The natural (angular) frequency for *small amplitude oscillations of* θ is



(a) $\omega_n = \sqrt{\dfrac{mg + kL}{2mL}}$

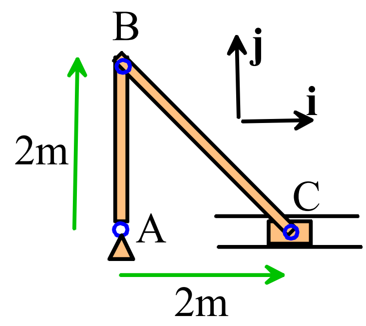
(b) $\omega_n = \sqrt{\dfrac{mg + kL}{mL}}$

(c) $\omega_n = \dfrac{1}{2}\sqrt{\dfrac{(mg + kL)}{mL}}$

(d) $\omega_n = \sqrt{\dfrac{2(mg + kL)}{mL}}$

ANSWER_____________**(2 POINTS)**

18. In the figure shown the link AB rotates counter-clockwise with constant angular speed 4 rad/s. The velocity of C and the angular velocity of link BC are



(a) $\mathbf{v}_C = \mathbf{0},\ \omega_{BC} = 4\mathbf{k}\ rad/s$

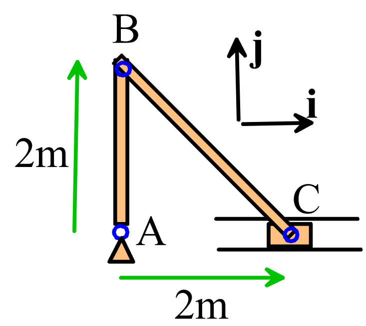
(b) $\mathbf{v}_C = \mathbf{0},\ \omega_{BC} = -4\mathbf{k}\ rad/s$

(c) $\mathbf{v}_C = 8\mathbf{i}\ m/s,\ \ \omega_{BC} = 0$

(d) $\mathbf{v}_C = -8\mathbf{i}\ m/s,\ \ \omega_{BC} = 0$

ANSWER______________**(2 POINTS)**

19. In the figure shown the link AB rotates counter-clockwise with constant angular speed 4 rad/s. The acceleration of C and the angular acceleration of link BC are



(a) $\mathbf{a}_C = -32\mathbf{j} m/s^2,\ \alpha_{BC} = 0$

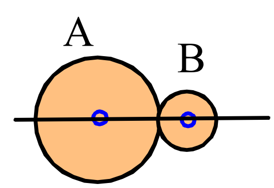
(b) $\mathbf{a}_C = 32\mathbf{i} m/s^2,\ \alpha_{BC} = 16\mathbf{k}\ rad/s^2$

(c) $\mathbf{a}_C = -32\mathbf{i} m/s^2,\ \alpha_{BC} = -16\mathbf{k}\ rad/s^2$

(d) $\mathbf{a}_C = -32\mathbf{i}\ m/s^2,\ \alpha_{BC} = 0$

ANSWER______________**(2 POINTS)**

20. In the figure shown gear A has 32 teeth and gear B has 24 teeth. Gear A rotates clockwise at 4 rad/s. Gear B rotates



(a) clockwise at 16/3 rad/s

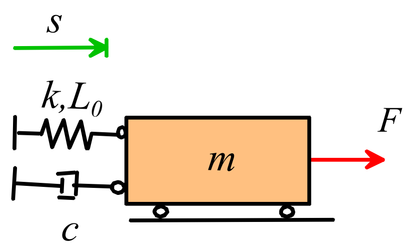
(b) clockwise at 3/16 rad/s

(c) counterclockwise at 16/3 rad/s

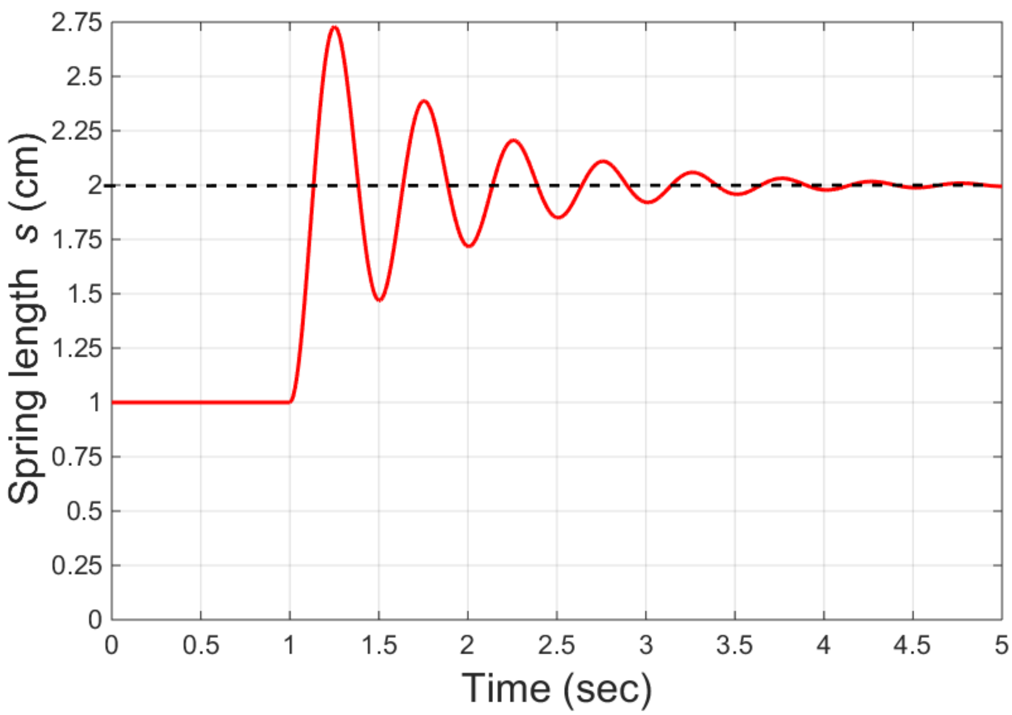
(d) counterclockwise at 3 rad/s

ANSWER______________**(2 POINTS)**

21 The figure shows a simple idealization of a force sensor. Its purpose is to measure the force F, by providing an electrical signal that is proportional to the length s of the spring.



At time $t=0$ the system is at rest, and $F=0$. At time $t=1s$ a constant force of $F=100N$ is applied to the mass. The figure below shows the variation of s with time for $0<t<5$s.



21.1 Using the graph provided, calculate values for the following quantities. Be sure to indicate your answers to each part clearly

(a) The period of vibration (**1 POINT)**

(b) The log decrement of the vibration δ (be careful to use the correct origin) (**1 POINT)**

(c) The damped natural frequency ω_d (**1 POINT)**

(d) The damping factor of the system ζ (**1 POINT)**

(e) The undamped natural frequency of the system ω_n (**1 POINT)**

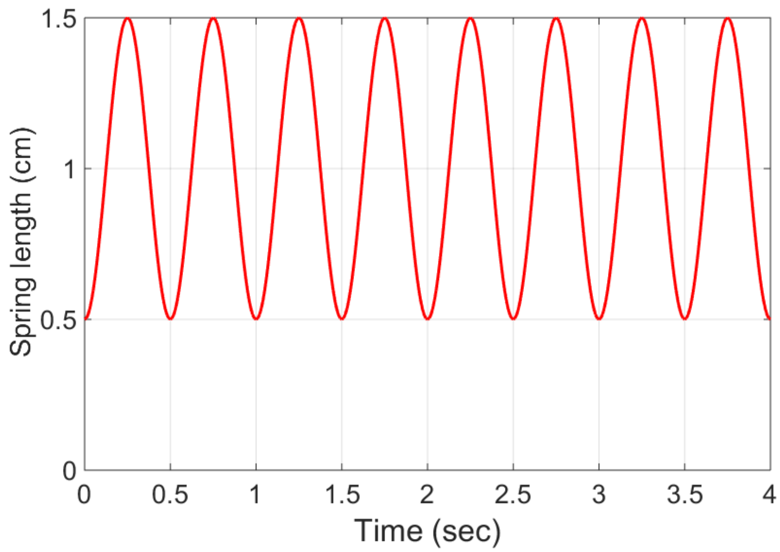
(f) The un-stretched length of the spring L_0 (**1 POINT)**

(g) The spring stiffness k (**1 POINT)**

(h) The mass m (**1 POINT)**

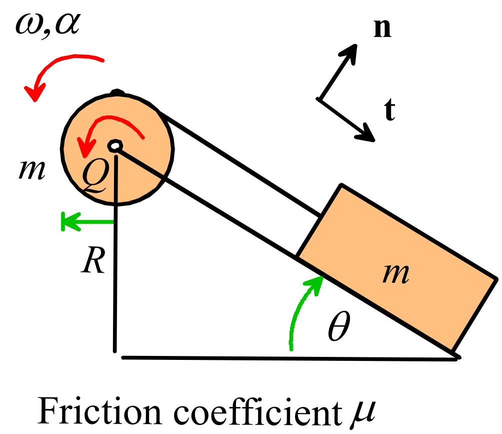
(i) The dashpot coefficient c. (**1 POINT**)

21.2 The sensor is now used to measure a force that vibrates harmonically $F(t) = F_0 \sin \omega t$. The figure below shows the steady-state variation of the spring length s with time. Calculate the amplitude of the force F_0 .

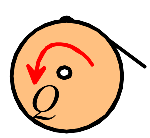


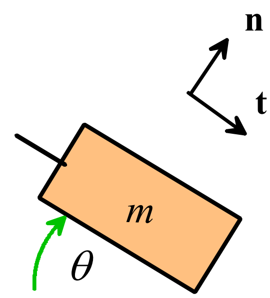
(3 POINTS)

22 A crate of mass m is pulled up a slope with angle θ by an inextensible cable that is wrapped around a pulley. The contact between the crate and slope has friction coefficient μ . The pulley has mass m and radius R and mass moment of inertia $mR^2/2$. It is rotated counterclockwise by a motor attached to an axle at its center, which exerts a moment (torque) with magnitude Q on the pulley. The bearings supporting the axle of the pulley are frictionless. The goal of this problem is to find a formula for the angular acceleration α of the pulley.



22.1 On the figures provided below, draw free body diagrams for the pulley and crate.





(3 POINTS)

22.2 Write down $\mathbf{F} = m\mathbf{a}$ for the crate, expressing both forces and accelerations as components in the $\{\mathbf{n}, \mathbf{t}\}$ basis shown in the figure. (You will need to introduce suitable variables to denote unknown reactions and acceleration components)

(2 POINTS)

22.3 Write down $\sum \mathbf{r} \times \mathbf{F} + \sum \mathbf{Q} = \mathbf{r}_G \times m\mathbf{a}_G + I_G \boldsymbol{\alpha}$ for the pulley, stating what point you take moments about

(1 POINT)

22.4 Write down a kinematics equation relating the angular acceleration of the pulley α to the acceleration of the crate.

(1 POINT)

22.5 Hence, show that $\alpha = \frac{2}{3}\left(\frac{Q}{mR^2} - \frac{g}{R}(\sin\theta + \mu\cos\theta) \right)$

(3 POINTS)