School of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Friday May 8 2015: 2pm-5pm

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1-20 [40 points] ______________

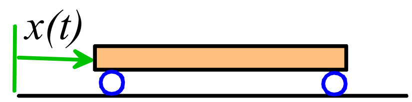
21 [12 POINTS] ______________

22 [10 POINTS] ______________

TOTAL [62 POINTS] ________________

FOR PROBLEMS 1-20 WRITE YOUR ANSWER IN THE SPACE PROVIDED. ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. The platform shown in the figure vibrates horizontally with a displacement $x(t) = X_0(1-\cos\omega t)$. Its horizontal acceleration is



(a) $a(t) = -X_0\omega^2(1-\cos\omega t)$

(b) $a(t) = -X_0\omega^2 \sin\omega t$

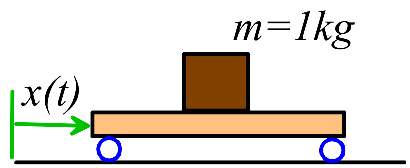
(c) $a(t) = X_0\omega^2 \cos\omega t$

(d) $a(t) = X_0\omega \sin\omega t$

Straight line motion formulas give $a(t) = d^2x/dt^2 = X_0\omega^2\cos\omega t$

ANSWER_______C_______ **(2 POINTS)**

2. A 1 kg mass rests on a horizontal surface with friction coefficient 0.5. At time $t=0$ the surface begins to vibrate horizontally with a displacement $x(t) = X_0(1-\cos\omega t)$. The frequency of vibration is $\omega = 10$ rad/sec. For each value of X_0 listed below, state whether or not slip will occur at the contact between the mass and the surface.

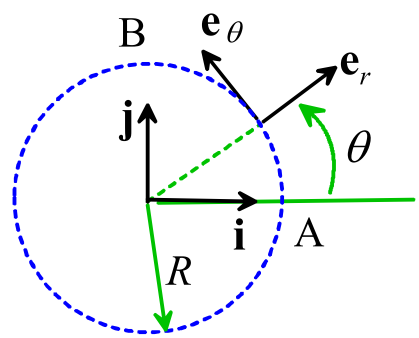


(a) $X_0 = 1cm$	SLIP	[NO SLIP]
(b) $X_0 = 2cm$	SLIP	[NO SLIP]
(c) $X_0 = 4cm$	SLIP	[NO SLIP]
(d) $X_0 = 8cm$	[SLIP]	NO SLIP

If no slip occurs the block has the same acceleration as the surface. F=ma gives the normal force at the contact as $N=mg$ and the tangential force as $T = mX_0\omega^2\cos\omega t$. Slip occurs if the maximum value of T exceeds $\mu N \Rightarrow X_0 > \mu g/\omega^2 = 5/100m = 5cm$

(2 POINTS)

3. A particle starts at rest at point A and travels with constant tangential acceleration around a circular path with radius R=4m. After 2 seconds it has a speed of 4 m/s. The acceleration at time t=2 sec is



(a) $\mathbf{a} = 2\mathbf{e}_r + 4\mathbf{e}_\theta \quad m/s^2$

(b) $\mathbf{a} = -4\mathbf{e}_r + 2\mathbf{e}_\theta \quad m/s^2$

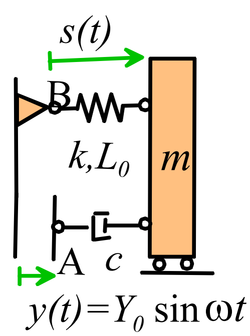
(c) $\mathbf{a} = -2\mathbf{e}_r + 4\mathbf{e}_\theta \quad m/s^2$

(d) $\mathbf{a} = 4\mathbf{e}_r + 2\mathbf{e}_\theta \quad m/s^2$

The tangential acceleration is $a_\theta = V/t = 4/2 = 2m/s^2$. The normal acceleration is $V^2/R = 16/4 = 4 \ m/s^2$. Note that the normal acceleration is towards the center of the circle so in the negative $\mathbf{e}_r$ direction.

ANSWER________B______ **(2 POINTS)**

4. The end of the dashpot at A moves with a prescribed time dependent displacement $y(t)$. With the definitions $\omega_n = \sqrt{k/m}, \ \zeta = c/2\sqrt{km} \quad K = 1$ the equation of motion for $s(t)$ is



(a) $\dfrac{1}{\omega_n^2}\dfrac{d^2s}{dt^2} + \dfrac{2\zeta}{\omega_n}\dfrac{ds}{dt} + s = L_0 + Ky(t)$

(b) $\dfrac{1}{\omega_n^2}\dfrac{d^2s}{dt^2} + \dfrac{2\zeta}{\omega_n}\dfrac{ds}{dt} + s = L_0 + K\dfrac{2\zeta}{\omega_n}\dfrac{dy}{dt}$

(c) $\dfrac{1}{\omega_n^2}\dfrac{d^2s}{dt^2} + \dfrac{2\zeta}{\omega_n}\dfrac{ds}{dt} + s = L_0 + K\left\{y(t) + \dfrac{2\zeta}{\omega_n}\dfrac{dy}{dt}\right\}$

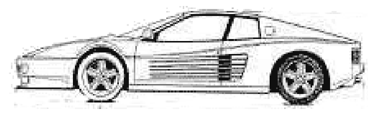
(d) $\dfrac{1}{\omega_n^2}\dfrac{d^2s}{dt^2} + \dfrac{2\zeta}{\omega_n}\dfrac{ds}{dt} + s = L_0 + \dfrac{K}{\omega_n^2}\dfrac{d^2y}{dt^2}$

Draw a FBD, use F=ma and the spring/damper force equations to see that

$m\dfrac{d^2s}{dt^2} + c\dfrac{ds}{dt} + ks = kL_0 + c\dfrac{dy}{dt}$. Divide through by k to see that the equation looks like (B)

ANSWER_______B_______ **(2 POINTS)**

5. An electric vehicle with mass 2000 kg is powered by a battery that is capable of developing a maximum power of 200kW. Neglecting air resistance, the shortest possible time for the vehicle to accelerate from rest to 20m/s while traveling on level ground is approximately



(a) 0.5 sec

(b) 1 sec

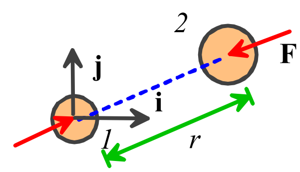
(c) 2 sec

(d) 4 sec

Use the work-KE relation: Power*time=(change in KE) so time = (change in KE)/Power $=(2000\times 20^2/2)/200000=2$ s

ANSWER_______C_______ **(2 POINTS)**

6. The bond in a diatomic molecule has a potential energy that is approximated using the function $V(r)=-E_0(r/d)\exp(-r/d)$, where r is the distance between atoms and E_0 and d are constants. When the atoms are separated by a distance *r=2d*, the force of attraction between them is



(a) $F=E_0\exp(-2)/(2d)$

(b) $F=-2E_0\exp(-2)$

(c) $F=E_0\exp(-2)/d$

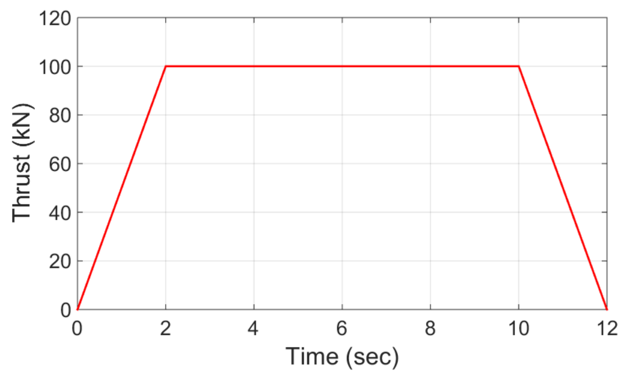
(d) $F=2E_0\exp(-2)/d$

The force-potential energy formula gives

$$F=dV/dr=-E_0(\exp(-r/d)/d-r\exp(-r/d)/d^2)=E_0\exp(-2)/d$$

ANSWER_______C_______ **(2 POINTS)**

7. A satellite with mass 500kg in a circular low earth orbit has velocity 8 km/s. A rocket is fired that exerts a thrust on the satellite (directed parallel to the velocity of the satellite), which varies with time as shown in the figure. Neglecting the impulse exerted by gravity on the satellite, the velocity of the satellite just after the rocket is fired (at $t=12$ sec) is



(a) 6 km/s

(b) 9 km/s

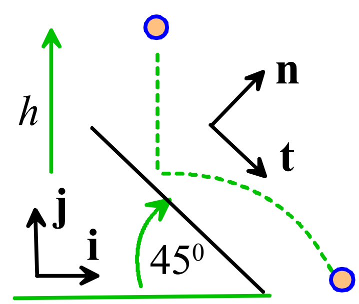
(c) 10 km/s

(d) 12 km/s

The impulse of the force is the area under the thrust-time curve, i.e. 1000000Ns. The impulse-momentum equation gives the change in velocity as $\Delta v = I/m = 1000000/500 = 2000 m/s = 2 km/s$

ANSWER_______C_______ **(2 POINTS)**

8. A spherical rock sample is dropped from rest from height h onto a 45 degree slope. The collision has a restitution coefficient e and is frictionless. The normal and tangential velocities of the sphere after impact are



(a) $v_n = e\sqrt{2gh} \qquad v_t = \sqrt{2gh}$

(b) $v_n = e\sqrt{gh} \qquad v_t = \sqrt{gh}$

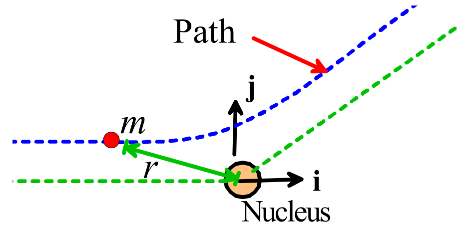
(c) $v_n = e\sqrt{gh} \qquad v_t = e\sqrt{gh}$

(d) $v_n = e\sqrt{2gh} \qquad v_t = e\sqrt{2gh}$

Energy conservation gives the velocity of the sample just before the impact as $\mathbf{v} = -\sqrt{2gh}\mathbf{j} = -\sqrt{2gh}(\mathbf{n}-\mathbf{t})/\sqrt{2}$. Since the collision is frictionless the tangential velocity does not change. The restitution formula gives the normal velocity after collision as $v_{n1} = -e v_{n0}$. Therefore $v_t = \sqrt{gh} \quad v_n = e\sqrt{gh}$

ANSWER______B________ **(2 POINTS)**

9. The figure shows the path of a charged particle with mass m in a scattering experiment. The nucleus can be assumed to be stationary, and exerts a repulsive radial force on the charged particle. Identify whether the statements below are true or false



(a) Linear momentum of the charged particle is conserved T [F]

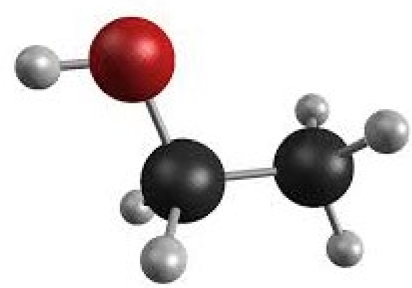
(b) The total energy of the system is conserved [T] F

(c) Angular momentum of the charged particle is conserved about the origin [T] F

(d) The forces acting on the charged particle do no work T [F]

(2 POINTS)

10. In the ethyl alcohol molecule shown in the figure, the atoms can be idealized as particles and the bonds between atoms as springs. The molecule has



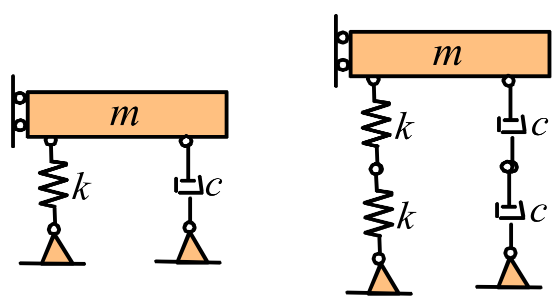
(a) 18 degrees of freedom and 12 vibration modes
(b) 18 degrees of freedom and 18 vibration modes
(c) 27 degrees of freedom and 21 vibration modes
(d) 27 degrees of freedom and 27 vibration modes

9 particles, so 27 DOF. There are 6 rigid body modes, so 21 vibration modes

ANSWER________C______ **(2 POINTS)**

11. The system shown in figure (a) is critically damped. The system in figure (b) must therefore have a damping factor

(a) $\zeta = 1/\sqrt{2}$
(b) $\zeta = 1/2\sqrt{2}$
(c) $\zeta = \sqrt{2}$
(d) $\zeta = 1$
(e) None of the above.

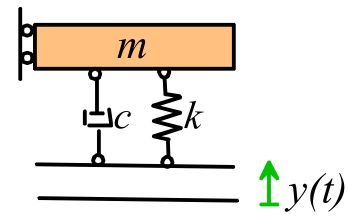


(a) (b)

Since the first system is critically damped $c/2\sqrt{km} = 1$. The effective spring stiffness and dashpot coefficient for the second system are $k/2$ and $c/2$. Therefore $\zeta = (c/2)/2\sqrt{km/2} = (c/2\sqrt{km})/\sqrt{2} = 1/\sqrt{2}$

ANSWER______A________**(2 POINTS)**

12. A vibration isolation platform can be idealized as a spring-mass system. It has a natural frequency of 2Hz. The base is excited at a frequency 4Hz . Identify whether each of the changes to the system listed below will increase or decrease the steady-state vibration amplitude of the system:



(a) Double the dashpot coefficient *c* **INCREASE** DECREASE

(b) Double the spring stiffness *k* **INCREASE** DECREASE

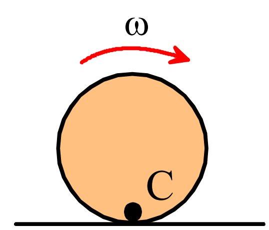
(c) Double the mass *m* INCREASE **DECREASE**

(d) Double the frequency of the base excitation to 8Hz. INCREASE **DECREASE**

Since the excitation frequency is twice the natural frequency the original system is in the vibration isolation regime. In this regime increasing *c* will increase the vibration amplitude. Doubling the spring stiffness will increase the natural frequency by a factor of $\sqrt{2}$ and so increases the amplitude. Doubling the mass decreases the natural frequency by the same factor and so reduces the amplitude. Doubling the frequency reduces the vibration amplitude (check the graphs of *M* for the base excited system to see these)

(2 POINTS)

13. The disk shown in the figure rolls without slip, with a clockwise angular velocity. From the list below, pick the vector that best describes the acceleration of point C

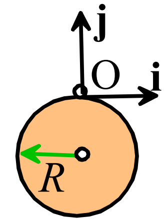


(a) Zero
(b) →
(c) ←
(d) ↑
(e) ↓

Point C has a zero tangential acceleration and a vertical acceleration. If you don't remember this you can use the fact that the acceleration of the center is $-\alpha R\mathbf{i}$ and then use the rigid body acceleration formula to see that $\mathbf{a}_C = -\alpha R\mathbf{i} + \alpha\mathbf{k}\times(-R\mathbf{j}) + \omega\mathbf{k}\times\omega\mathbf{k}\times(-R\mathbf{j})$

ANSWER_______D_______ **(2 POINTS)**

14. The mass moment of inertia of the thin disk about an axis parallel to the **k** direction passing through O is



(a) $I_0 = mR^2/2$

(b) $I_0 = mR^2$

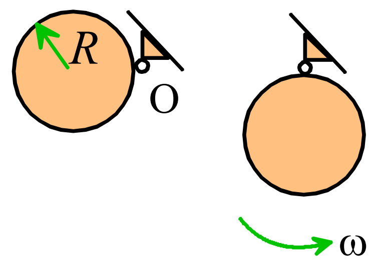
(c) $I_0 = 3mR^2/2$

(d) $I_0 = 2mR^2$

The mass moment of inertia about the center is $mR^2/2$ The parallel axis theorem gives $mR^2/2 + mR^2 = 3mR^2/2$

ANSWER_______C_______ **(2 POINTS)**

15. The disk shown in the figure swings freely about O. At time *t=0* the center of the disk is level with O, and the disk is stationary. When the center of the disk is immediately below O, its angular speed is



(a) $\omega = 2\sqrt{\dfrac{g}{3R}}$

(b) $\omega = \sqrt{\dfrac{2g}{3R}}$

(c) $\omega = \sqrt{\dfrac{2g}{R}}$

(d) $\omega = 2\sqrt{\dfrac{g}{R}}$

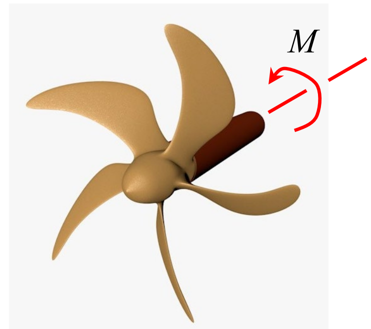
(e) None of the above

Energy conservation gives

$PE + KE = const \Rightarrow -mgR + I_0\omega^2/2 = 0 \Rightarrow -mgR + 3mR^2\omega^2/4 = 0 \Rightarrow \omega = 2\sqrt{g/3R}$

ANSWER_______A_______**(2 POINTS)**

16. The figure shows a propeller with total mass moment of inertia 150 kg m^2. The propeller is driven by a motor (not shown) that exerts a moment M on the shaft. At the instant shown, the propeller has zero angular velocity and has an angular acceleration of 10 rad s^{-2}. At this instant, the rate of work done by the moment on the propeller is



(a) Zero

(b) 1500 Watts

(c) 7500 Watts

(d) 75000 Watts

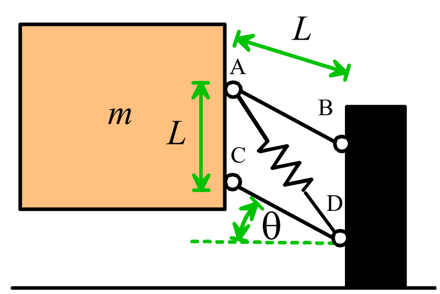
Power = torque x angular speed so zero

ANSWER______A________**(2 POINTS)**

17. The figure shows a proposed design for a suspension. The links AB and CD are rigid, and the spring has stiffness k. The static equilibrium configuration is $\theta = 0$, and the equation of motion for the system is

$$mL\frac{d^2\theta}{dt^2} + (mg + kL)\left[\cos\theta - \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right] = 0$$

The natural (angular) frequency for *small amplitude oscillations of* θ is



(a) $\omega_n = \sqrt{\dfrac{mg + kL}{2mL}}$

(b) $\omega_n = \sqrt{\dfrac{mg + kL}{mL}}$

(c) $\omega_n = \dfrac{1}{2}\sqrt{\dfrac{(mg + kL)}{mL}}$

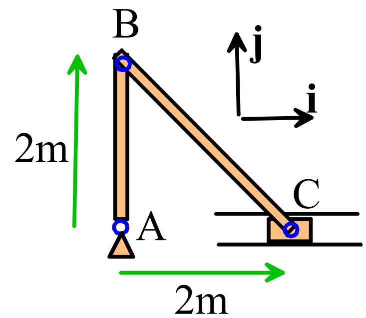
(d) $\omega_n = \sqrt{\dfrac{2(mg + kL)}{mL}}$

Linearize ($\cos\theta \approx 1, \ \sin\theta \approx \theta$) and rearrange the equation into standard form $\dfrac{2mL}{(mg + kL)}\dfrac{d^2\theta}{dt^2} + \theta = 0$

The coefficient of the first term is $1/\omega_n^2$

ANSWER________A______**(2 POINTS)**

18. In the figure shown the link AB rotates counter-clockwise with constant angular speed 4 rad/s. The velocity of C and the angular velocity of link BC are



(a) $\mathbf{v}_C = \mathbf{0},\ \omega_{BC} = 4\mathbf{k}\ rad/s$

(b) $\mathbf{v}_C = \mathbf{0},\ \omega_{BC} = -4\mathbf{k}\ rad/s$

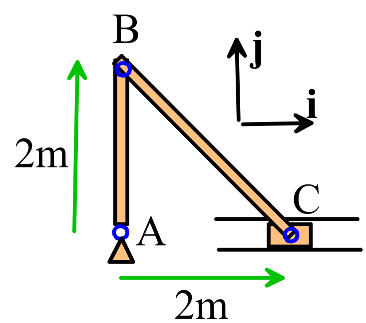
(c) $\mathbf{v}_C = 8\mathbf{i}\ m/s,\ \ \omega_{BC} = 0$

(d) $\mathbf{v}_C = -8\mathbf{i}\ m/s,\ \ \omega_{BC} = 0$

$\mathbf{v}_B = 4\mathbf{k}\times 2\mathbf{j} = -8\mathbf{i}$ $\quad \mathbf{v}_C = -8\mathbf{i} + \omega_{BC}\mathbf{k}\times(2\mathbf{i}-2\mathbf{j}) = (2\omega_{BC}-8)\mathbf{i} + 2\omega_{BC}\mathbf{j}$. Since C must move in the **i** direction, $\omega_{BC} = 0,\ \mathbf{v}_C = -8\mathbf{i}$

ANSWER______D________**(2 POINTS)**

19. In the figure shown the link AB rotates counter-clockwise with constant angular speed 4 rad/s. The acceleration of C and the angular acceleration of link BC are



(a) $\mathbf{a}_C = -32\mathbf{j} m/s^2,\ \alpha_{BC} = 0$

(b) $\mathbf{a}_C = 32\mathbf{i} m/s^2,\ \alpha_{BC} = 16\mathbf{k}\ rad/s^2$

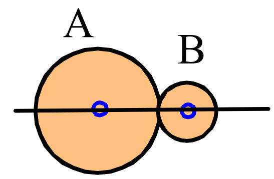
(c) $\mathbf{a}_C = -32\mathbf{i} m/s^2,\ \alpha_{BC} = -16\mathbf{k}\ rad/s^2$

(d) $\mathbf{a}_C = -32\mathbf{i}\ m/s^2,\ \alpha_{BC} = 0$

$\mathbf{a}_B = -32\mathbf{j} \ \Rightarrow\ \mathbf{a}_C = -32\mathbf{j} + \alpha_{BC}\mathbf{k}\times(2\mathbf{i}-2\mathbf{j}) - \omega_{BC}^2(2\mathbf{i}-2\mathbf{j}) = 2\alpha_{BC}\mathbf{i} + (2\alpha_{BC}-32)\mathbf{j}$. The acceleration of C is zero in the **j** direction and therefore $\mathbf{a}_C = 32\mathbf{i} m/s^2,\ \alpha_{BC} = 16\mathbf{k}\ rad/s^2$

ANSWER_______B_______**(2 POINTS)**

20. In the figure shown gear A has 32 teeth and gear B has 24 teeth. Gear A rotates clockwise at 4 rad/s. Gear B rotates



(a) clockwise at 16/3 rad/s

(b) clockwise at 3/16 rad/s

(c) counterclockwise at 16/3 rad/s

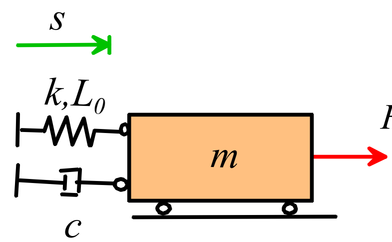
(d) counterclockwise at 3 rad/s

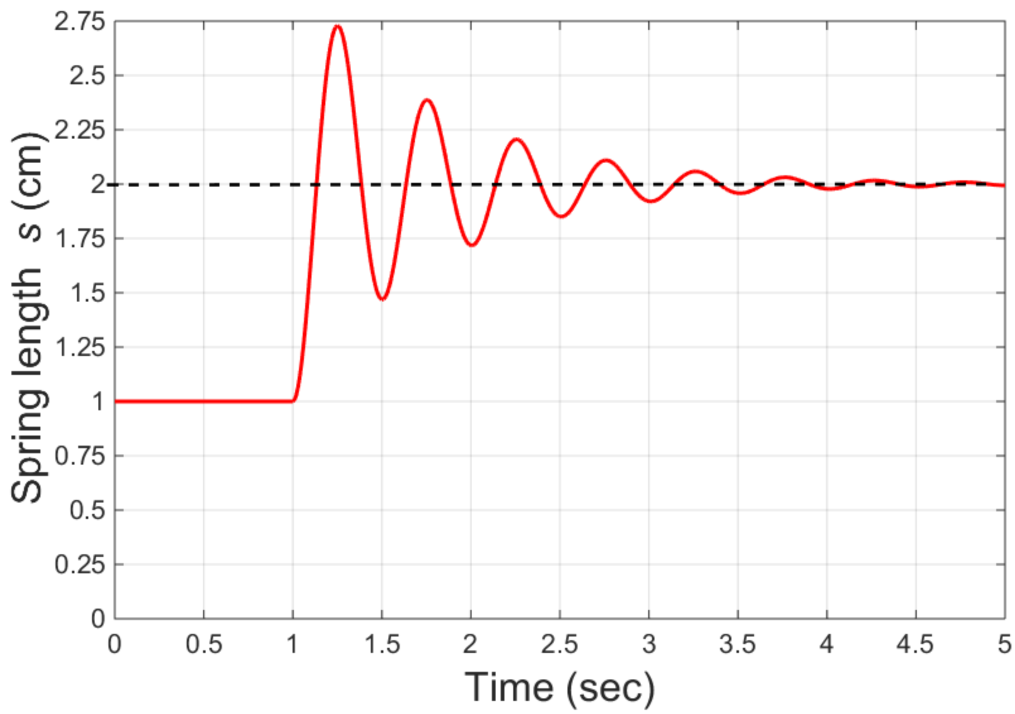
From HW6 we know that the gear radius is proportional to the numbe of teeth. $\omega_B r_B = -\omega_A r_A \Rightarrow \omega_B = 4 \times 32/24 = 16/3$ rad/s. (The positive sign means counterclockwise)

ANSWER_______C_______**(2 POINTS)**

21 The figure shows a simple idealization of a force sensor. Its purpose is to measure the force *F*, by providing an electrical signal that is proportional to the length *s* of the spring.

At time *t=0* the system is at rest, and *F=0*. At time *t=1s* a constant force of *F*=100*N* is applied to the mass. The figure below shows the variation of *s* with time for *0<t*<5s.





21.1 Using the graph provided, calculate values for the following quantities.

(a) The period of vibration (**1 POINT)**

2 cycles takes 1 sec so *T=0.5*s.

(b) The damped natural frequency ω_d (**1 POINT)**

$\omega_d = 2\pi / T = 4\pi rad / s$

(c) The log decrement of the vibration δ (be careful to use the correct origin) (**1 POINT)**

The first peak has amplitude 0.7; the third has amplitude 0.2 so the formula for log decrement gives $\delta = \frac{1}{2}\log(0.7/0.2) = 0.626$

(d) The damping factor of the system ζ (**1 POINT)**

From the formula $\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = 0.1$

(e) The undamped natural frequency of the system ω_n (**1 POINT)**

From the formula $\omega_n = \frac{\sqrt{4\pi^2 + \delta^2}}{T} = 12.62 rad/s$

(f) The un-stretched length of the spring L_0 (**1 POINT)**

The length of the spring must be equal to its unstretched length before the force is applied, so $L_0 = 1cm$

(g) The spring stiffness k (**1 POINT)**

After the oscillations die out, the spring has stretched by 1cm after the 100N force is applied. Therefore $k = 100/0.01 = 10000 N/m$

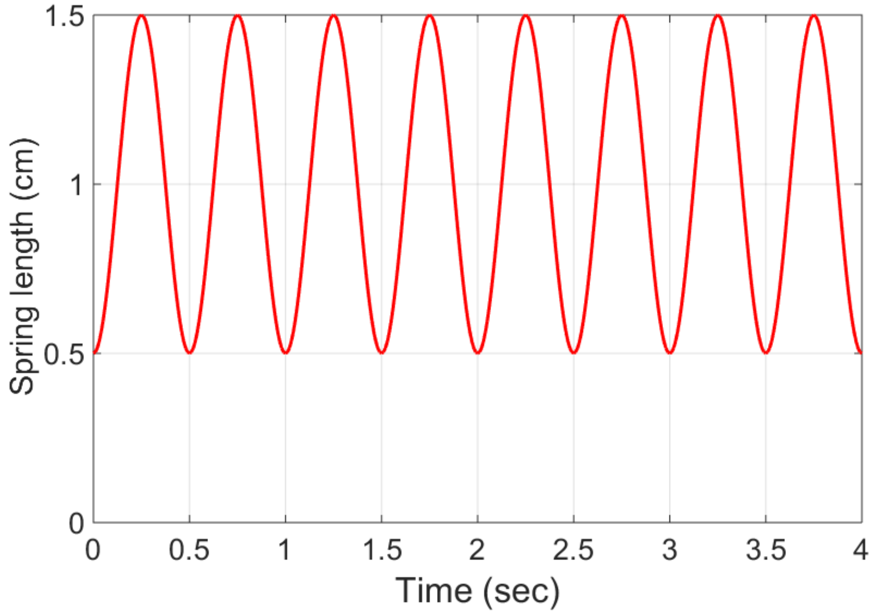
(h) The mass m (**1 POINT)**

We know that $\omega_n = \sqrt{k/m} \Rightarrow m = k/\omega_n^2 = 10000/(12.62)^2 = 62.8kg$ (this must be the world's heaviest force sensor – don't design one like this!)

(i) The dashpot coefficient c. (**1 POINT)**

We have $\zeta = c/2\sqrt{km} \Rightarrow c = 2\zeta\sqrt{km} = 158 Ns/m$

21.2 The sensor is now used to measure a force that vibrates harmonically $F(t) = F_0 \sin \omega t$. The figure below shows the steady-state variation of the spring length s with time. Calculate the amplitude of the force F_0 .

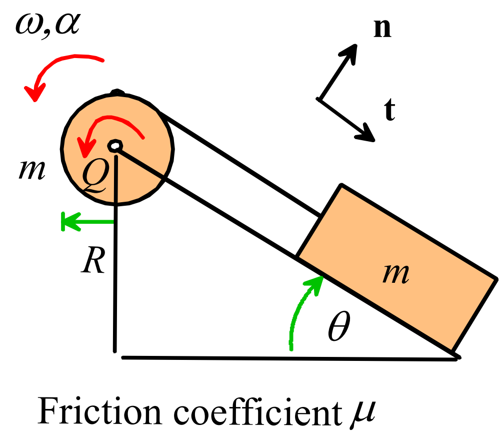


Note that the frequency of the force is equal to the natural frequency (the period of vibration is equal to the period in the first figure). This means the system is at resonance, and we can use the formula for the amplitude at resonance

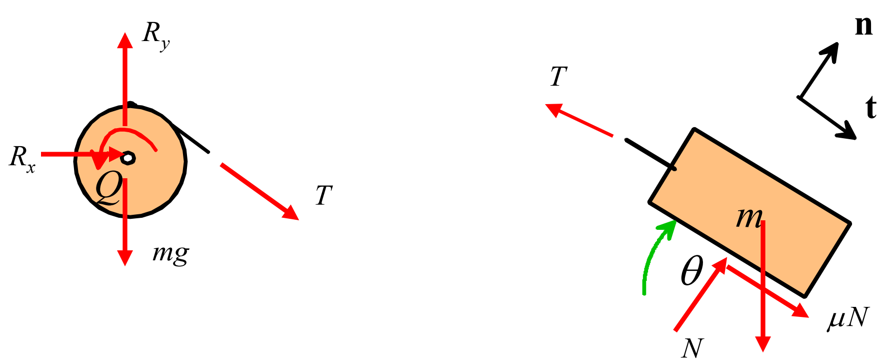
$$X_0 = KF_0 M_{\max} \approx \frac{1}{k} F_0 \frac{1}{2\zeta} \Rightarrow F_0 = 2\zeta k X_0 = 2 \times 0.1 \times 10000 \times 0.005 = 10N$$

(3 POINTS)

22 A crate of mass m is pulled up a slope with angle θ by an inextensible cable that is wrapped around a pulley. The contact between the crate and slope has friction coefficient μ . The pulley has mass m and radius R and mass moment of inertia $mR^2/2$. It is rotated counterclockwise by a motor attached to an axle at its center, which exerts a moment (torque) with magnitude Q on the pulley. The bearings supporting the axle of the pulley are frictionless. The goal of this problem is to find a formula for the angular acceleration α of the pulley.



22.1 On the figures provided below, draw free body diagrams for the pulley and crate.



(3 POINTS)

22.2 Write down $\mathbf{F}=m\mathbf{a}$ for the crate, expressing both forces and accelerations as components in the $\{\mathbf{n},\mathbf{t}\}$ basis shown in the figure.

$$(N-mg\cos\theta)\mathbf{n}+(mg\sin\theta+\mu N-T)\mathbf{t}=ma_t\mathbf{t}$$

(2 POINTS)

22.3 Write down $\sum \mathbf{r}\times\mathbf{F}+\sum\mathbf{Q}=\mathbf{r}_G\times m\mathbf{a}_G+I_G\boldsymbol{\alpha}$ for the pulley, stating what point you take moments about

Moments about the center of the pulley gives $(Q-TR)\mathbf{k}=\frac{1}{2}mR^2\alpha\mathbf{k}$

(1 POINT)

22.4 Write down a kinematics equation relating the angular acceleration of the pulley α to the acceleration of the crate.

$a_t = -R\alpha$

(1 POINT)

22.5 Hence, show that

$$\alpha = \frac{2}{3}\left(\frac{Q}{mR^2} - \frac{g}{R}(\sin\theta + \mu\cos\theta)\right)$$

The equation system from the preceding parts can be solved to see that

$$N = mg\cos\theta$$
$$T = \frac{Q}{R} - \frac{1}{2}mR\alpha$$
$$ma_t = -mR\alpha = mg\sin\theta\mu N - T = \frac{1}{2}mR\alpha + mg\sin\theta + \mu mg\cos\theta - \frac{Q}{R}$$
$$\Rightarrow \frac{3}{2}mR\alpha = \frac{Q}{R} - mg\left(\sin\theta + \mu\cos\theta\right)$$

which reduces to the stated answer.

(3 POINTS)