



School of Engineering  
Brown University

## EN40: Dynamics and Vibrations

**Midterm Examination**  
**Tuesday March 8 2016**

**NAME:** \_\_\_\_\_

### General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

### Please initial the statement below to show that you have read it

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

\_\_\_\_\_

**1 (19 points)** \_\_\_\_\_

**2. (11 points)** \_\_\_\_\_

**3. (10 points)** \_\_\_\_\_

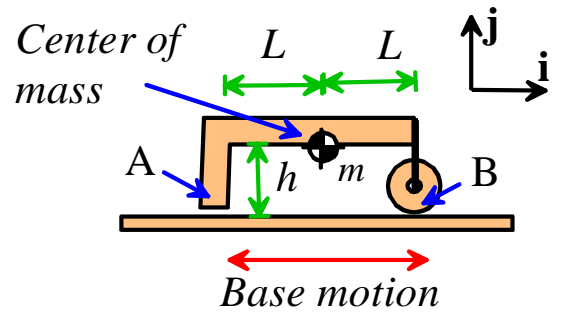
**TOTAL (40 points)** \_\_\_\_\_

1. The figure shows an ‘inertial crawler’ that will spontaneously translate to the right over a vibrating surface. Assume that

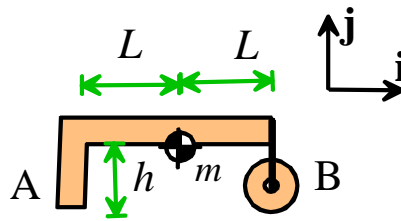
- The device and surface are both at rest at time  $t=0$ .
- For the time interval  $0 < t < T$  (where  $T$  is a constant) the surface has velocity

$$\mathbf{v} = \begin{cases} +V_0 \mathbf{i} & 0 < t < T/2 \\ -V_0 \mathbf{i} & T/2 < t < T \end{cases}$$

- $V_0$  is sufficiently large that slip occurs at A throughout the interval  $0 < t < T$ .
- The contact at A has a friction coefficient  $\mu$ .
- The wheel at B is massless and rolls freely (so the contact at B is subjected only to a normal force).
- The center of mass of the device is midway between A and B and a height  $h$  above the surface.



1. Draw a free body diagram showing the forces acting on the crawler during the period  $0 < t < T/2$  on the figure provided below. Note that during this phase of the motion the surface moves to the right, and is moving faster than the crawler.



[3 POINTS]

2. Write down Newton’s law and the equation of rotational motion (i.e. moments) for  $0 < t < T/2$

[3 POINTS]

3. Hence, show that the acceleration of the crawler for  $0 < t < T/2$  is given by

$$a = \frac{\mu g}{2 - \mu(h/L)} \mathbf{i}$$

**[3 POINTS]**

4. Repeat steps 1-3 for the period  $T/2 < t < T$  (note that during this phase of the motion the surface moves to the left) to show that during this time interval

$$a = \frac{-\mu g}{2 + \mu(h/L)} \mathbf{i}$$

**[4 POINTS]**

5. Use the results of (3) and (4) to find a formula for the distance that the crawler moves during the time period  $0 < t < T$ . Assume that the crawler is at rest at time  $t=0$ .

**[4 POINTS]**

6. Find a formula for the minimum value of  $V_0$  for slip to occur at the contact point A between the crawler and the surface in the time interval  $0 < t < T$ .

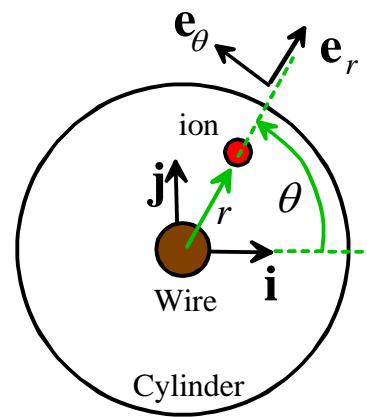
**[2 POINTS]**

2. The figure shows a charged particle with mass  $m$  in a static 'Kingdon trap' that uses an electric field to confine the particle within a cylinder. The electric field subjects the particle to a radial force

$$\mathbf{F} = -\frac{F_0}{r}\mathbf{e}_r$$

where  $F_0$  is a constant (which depends on the electric field in the cylinder).

At time  $t=0$  the particle is located at position  $\theta = 0, r = R_0$  and has velocity vector  $\mathbf{v} = V_0\mathbf{e}_\theta$



2.1 Write down the formula for the acceleration of the particle in polar coordinates, in terms of time derivatives of  $r, \theta$ . (Do not assume circular motion).

[2 POINTS]

2.2 Hence, write down Newton's law  $\mathbf{F} = m\mathbf{a}$  for the particle, using polar coordinates.

[1 POINT]

2.3 Write down the angular momentum vector of the particle at time  $t=0$ , in terms of  $V_0$ ,  $m$  and  $R_0$

**[1 POINT]**

2.4 Use angular momentum, or otherwise, show that  $\theta$  and  $r$  are related by

$$\frac{d\theta}{dt} = \frac{V_0 R_0}{r^2}$$

**[2 POINTS]**

2.5 Use the answers to 2.2 and 2.4 to show that the coordinate  $r$  satisfies the differential equation

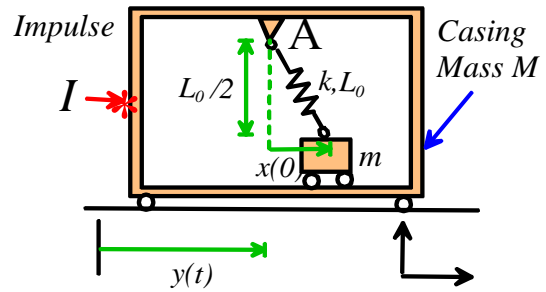
$$\frac{d^2 r}{dt^2} - \frac{(V_0 R_0)^2}{r^3} + \frac{F_0}{mr} = 0$$

**[3 POINTS]**

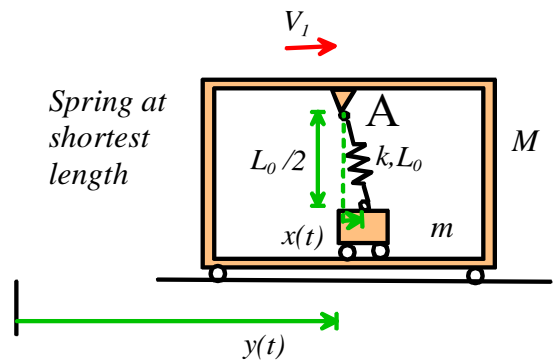
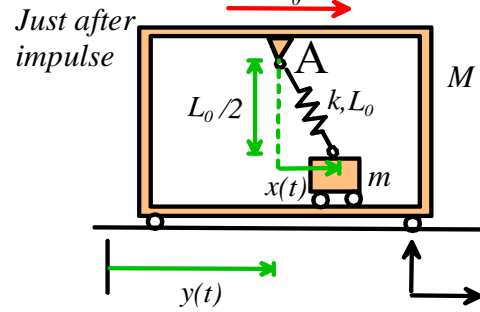
2.6 Rearrange the equation of motion for  $r$  in part 3.3 into a form that MATLAB can solve.

**[2 POINTS]**

3. The figure shows a device that is intended to detect an impulse. It consists of a casing, mass  $M$ , a proof mass,  $m$ , and a spring with stiffness  $k$  and unstretched length  $L_0$ . If the casing is subjected to an impulse that exceeds a critical magnitude, the small mass  $m$  will flip from its initial position to the right of the pivot to a new stable position to the left of the pivot at A. The goal of this problem is to calculate the critical value of impulse for which this will occur.



3.1 Suppose that at time  $t=0$  the system is at rest and the spring is un-stretched. The casing is then subjected to a horizontal impulse  $I$ . Write down a formula for the speed of the casing just after the impulse. Note that the spring exerts no force on either the casing or the mass  $m$  during the impulse.



[1 POINT]

3.2 Find expressions for the total linear momentum and total kinetic energy of the system just after the impulse, in terms of  $I$  and the mass  $M$  of the casing.

[2 POINTS]



3.3 Consider the system at the instant when the spring reaches its shortest length (assume  $x > 0$ ). Using energy and/or momentum conservation show that at this instant

$$x = \sqrt{\left( L_0 - I \sqrt{\frac{m}{kM(M+m)}} \right)^2 - \frac{L_0^2}{4}}$$

(You can find the spring length using Pythagoras' theorem. Note that  $L_0$  is the un-stretched length of the spring)

**[5 POINTS]**

3.4 Hence, find a formula for the critical value of  $I$  that will flip the mass past  $x=0$ .

**[2 POINTS]**