School of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Thurs May 12 2015: 2pm-5pm

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

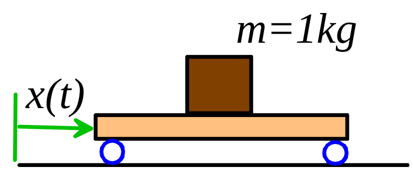
Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1-20 [40 points]	_____________
21 [10 POINTS]	_____________
22 [15 POINTS]	_____________
TOTAL [65 POINTS]	_______________

FOR PROBLEMS 1-20 WRITE YOUR ANSWER IN THE SPACE PROVIDED. ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.



1. A mass *m* rests on a horizontal surface with friction coefficient μ. At time *t=0* the mass and surface are both at rest. The surface then begins to accelerate to the right with acceleration $3\mu g$. The acceleration of the mass is

(a) $a = \mu g$

(b) $a = 2\mu g$

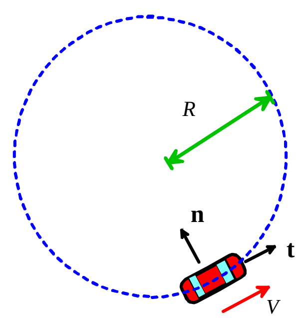
(c) $a = 3\mu g$

(d) $a = \mu g / 2$

Start by guessing no slip between the surfaces. A FBD and **F**=m**a** then shows that the normal and tangential forces are $T = 3m\mu g \quad N = mg$. Since $T > \mu N$ the guess is incorrect and the contact must slip. If slip occurs $T = \mu N = \mu mg \Rightarrow a = \mu g$

ANSWER________A______ **(2 POINTS)**

2. A car travels around a circular track with radius *R*. To ensure the car does not skid on the track, its acceleration must be less than μg. The shortest possible time to complete a full lap is



(a) $t = 2\sqrt{\pi R / (\mu g)}$

(b) $t = 2\pi\sqrt{R / (\mu g)}$

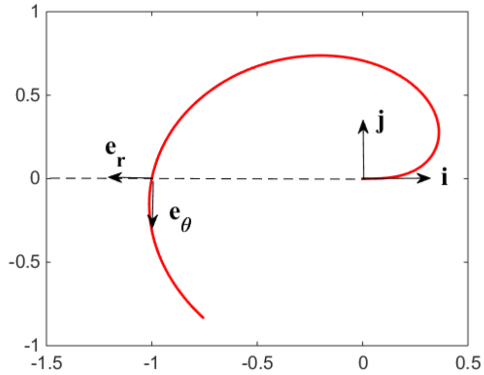
(c) $t = 2\pi\sqrt{\mu g / R}$

(d) $t = \sqrt{R / (\mu g)}$

The shortest time will occur when *V* is constant. In this case the acceleration is $(V^2 / R)\mathbf{n}$. The max possible speed for no slip is $V = \sqrt{\mu g R}$. The time to complete a lap is $2\pi R / V = 2\pi\sqrt{R / \mu g}$

ANSWER________B______ **(2 POINTS)**

3. The polar coordinates (in meters and radians) of a particle vary with time according to the formulas $r(t) = t/\sqrt{\pi}$, $\theta(t) = t^2$. At the instant when $\theta = \pi$ the velocity of the particle is



(a) $\mathbf{v} = 2\sqrt{\pi}\mathbf{e}_\theta \quad m/s$

(b) $\mathbf{v} = \mathbf{e}_r + 2\sqrt{\pi}\mathbf{e}_\theta \quad m/s$

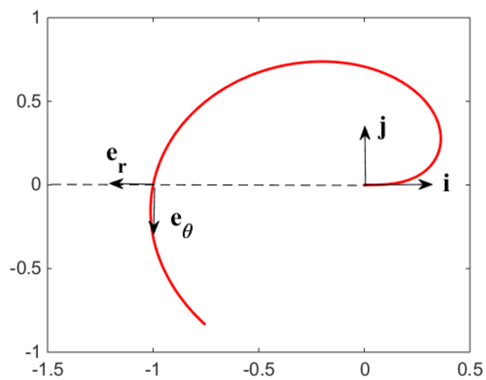
(c) $\mathbf{a} = \sqrt{\pi}\mathbf{e}_r + \pi\mathbf{e}_\theta \quad m/s^2$

(d) $\mathbf{a} = \frac{1}{\sqrt{\pi}}\mathbf{e}_r + 2\sqrt{\pi}\mathbf{e}_\theta \quad m/s$

The velocity is $\mathbf{v} = \frac{dr}{dt}\mathbf{e}_r + r\frac{d\theta}{dt}\mathbf{e}_\theta = \frac{1}{\sqrt{\pi}}\mathbf{e}_r + 2tr\mathbf{e}_\theta$. Since $\theta = \pi \Rightarrow t = \sqrt{\pi} \Rightarrow r = 1$ and so $\mathbf{v} = \frac{dr}{dt}\mathbf{e}_r + r\frac{d\theta}{dt}\mathbf{e}_\theta = \frac{1}{\sqrt{\pi}}\mathbf{e}_r + 2\sqrt{\pi}\mathbf{e}_\theta$.

ANSWER________D______ **(2 POINTS)**

4. The polar coordinates (in meters and radians) of a particle vary with time according to the formulas $r(t) = t/\sqrt{\pi}$, $\theta(t) = t^2$. At the instant when $\theta = \pi$ the acceleration of the particle is



(a) $\mathbf{a} = -4\pi\mathbf{e}_r + 2\mathbf{e}_\theta$

(b) $\mathbf{v} = 6\mathbf{e}_\theta$

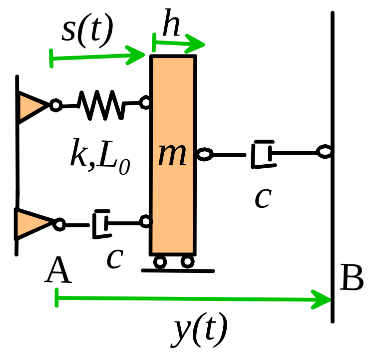
(c) $\mathbf{a} = -4\pi\mathbf{e}_r + 6\mathbf{e}_\theta$

(d) None of the above

The acceleration is $\mathbf{a} = \left(\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)\mathbf{e}_r + \left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\mathbf{e}_\theta = -r(2t)^2\mathbf{e}_r + \left(2r + 2\frac{1}{\sqrt{\pi}}2t\right)\mathbf{e}_\theta$. Since $\theta = \pi \Rightarrow t = \sqrt{\pi} \Rightarrow r = 1$ and so $\mathbf{a} = -4\pi\mathbf{e}_r + 6\mathbf{e}_\theta$.

ANSWER________C______ **(2 POINTS)**

5. The distance AB is a prescribed time-dependent function $y(t)$. The equation of motion for $s(t)$ is



(a) $m\frac{d^2s}{dt^2}+c\frac{ds}{dt}+ks=kL_0+c\frac{dy}{dt}$

(b) $m\frac{d^2s}{dt^2}+c\frac{ds}{dt}+ks=kL_0+cy(t)$

(c) $m\frac{d^2s}{dt^2}+2c\frac{ds}{dt}+ks=kL_0+c\frac{dy}{dt}$

(d) $m\frac{d^2s}{dt^2}+c\frac{ds}{dt}+ks=kL_0-kh+cy(t)$

Draw a FBD, use F=ma and the spring/damper force equations to see that $m\frac{d^2s}{dt^2}=-c\frac{ds}{dt}-k(s-L_0)+c\frac{d}{dt}(y-s-h)$. Rearrange and see the answer is (c)

ANSWER_______C_______ **(2 POINTS)**

6. An aircraft with total mass 600kg and lift-drag ratio 10:1 is in level flight with horizontal speed of 50 m/s and horizontal acceleration of 1 m/s^2. The engine power is approximately

(a) 500 kW

(b) 10 kW

(c) 60 kW

(d) 50 kW

The power-energy equation gives $P_{Thrust}+P_{Drag}=\frac{d}{dt}\left(\frac{1}{2}mv^2\right)=mv\frac{dv}{dt}$. The drag power is $-F_{drag}v$ so $P_{Thrust}=mva+\frac{mg}{10}v=600\times 50\times(1+9.81/10)\approx 60kW$

ANSWER______C________ **(2 POINTS)**

7. A 5 kg bowling ball is dropped on the ground from a height of 0.8m. The restitution coefficient for the collision with the ground is 0.5. The speed of the ball just before and just after impact is approximately

(a) 6m/s and 4 m/s

(b) 4 m/s and 2 m/s

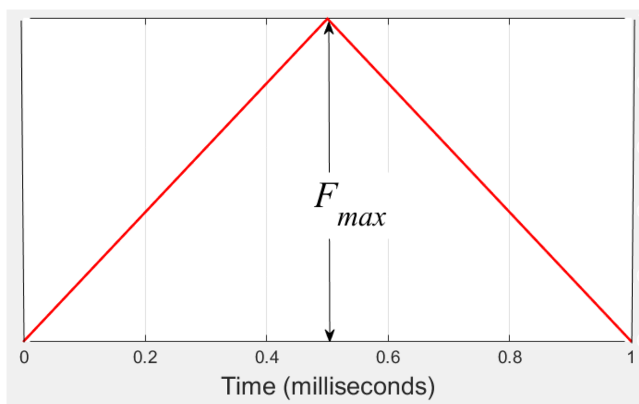
(c) 2m/s and 1 m/s

(d) None of the above

For the free fall portion use energy or straight line motion formulas to find $v = \sqrt{2gh} \approx \sqrt{2 \times 0.8 \times 10} = 4m/s$. The restitution formula gives $v_1 = ev_0$ after impact so 2 m/s.

ANSWER______B________ **(2 POINTS)**

8. A 5 kg bowling ball is dropped on the ground from a height of 0.8m. The restitution coefficient for the collision with the ground is 0.5. The figure shows the force acting on the ball during impact as a function of time. The maximum force is



(a) 60 kN

(b) 30 kN

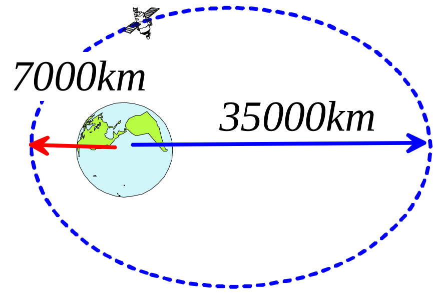
(c) 20 kN

(d) None of the above

The impulse extered by the collision force is $m(v_1 - v_0) = 5 \times (4 - (-2)) = 30Ns$. The impulse is the area under the force-v-time curve, so the max force must be 60 kN.

ANSWER______A________ **(2 POINTS)**

9. A satellite in orbit around a planet has a speed of 10km/s at its perigee (the closest point to the planet), which is 7000km from the center of the planet. At apogee (the furthest point from the planet), the satellite is 35000km from the center of the planet. The speed of the satellite at apogee is



(a) 2 km/s

(b) 4 km/s

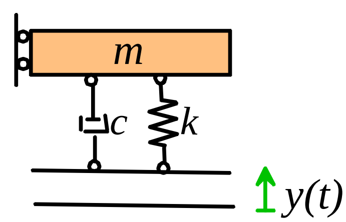
(c) 1 km/s

(d) None of the above

Angular momentum is conserved about the center of the planet so $rmv = 10 \times 7000m = v \times 35000m \Rightarrow v = 10/5 = 2km/s$

ANSWER______A________ **(2 POINTS)**

10. A vibration isolation system can be idealized as a base excited spring-mass system. The damping is small enough to be neglected when calculating the steady-state vibration amplitude. When the base vibrates harmonically with frequency 10rad/s and *displacement* amplitude 0.99mm, an accelerometer mounted on the isolation platform records a steady-state harmonic *acceleration* with steady-state amplitude 1 mm/s^2. The natural frequency of the vibration isolation system is approximately



(a) 1 rad/s

(b) $10/\sqrt{2}$ rad/s

(c) 2 rad/s

(d) None of the above

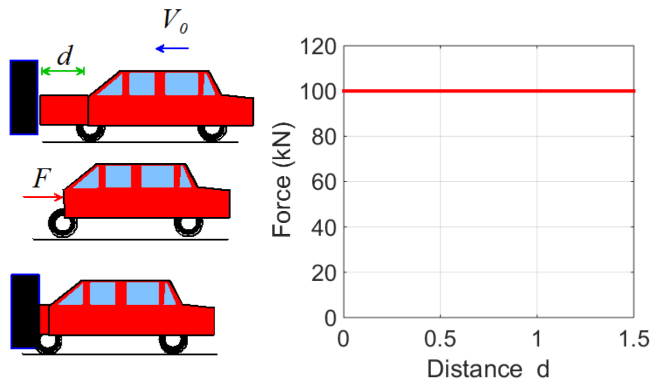
Recall that acceleration amplitude is related to displacement amplitude by $A = \omega^2 X$. The ratio of platform amplitude to base amplitude is therefore $1/10^2 / 0.99 = 1/99$. The magnification factor for a lightly damped vibration isolator is $1/\left|(1-\omega^2/\omega_n^2)\right|$ so

$1/(\omega^2/\omega_n^2 - 1) = 1/99 \Rightarrow \omega^2/\omega_n^2 = 100 \Rightarrow \omega_n = \omega/10 = 1$ rad/s

ANSWER_______A_______ **(2 POINTS)**

11. The figure shows a force-v-deflection relation for one of the two crash-rails at the front of a vehicle. The vehicle has mass 2000kg. In a head-on collision with a stationary object at 10 m/s, the rails will be compressed by a distance

(a) 1m
(b) 2m
(c) 0.5m
(d) 0.25m

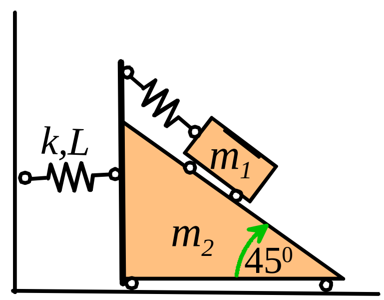


Work-energy gives $T_1 - T_0 = W \Rightarrow 0 - mv^2/2 = \int_0^d -F dx$. Since F is constant

$200000d = 2000 \times 100/2 \Rightarrow d = 0.5m$ (the factor of 2 is because there are two crash rails.

ANSWER_______C_______ **(2 POINTS)**

12. Give the number of degrees of freedom and the number of vibration modes for the system shown in the figure.

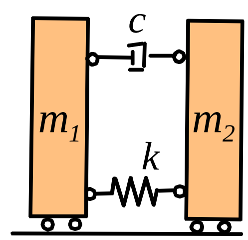


There are two masses, which do not rotate and hence can be idealized as particles. There are two constraints (a normal reaction between the wedge and ground; and a normal force acting between the masses. This gives DOF= 2x2-2=2. There are no rigid body modes so 2 vibration modes

NUMBER OF DOF_______2_______ **(1 POINT)**

NUMBER OF VIBRATION MODES_______2_______ **(1 POINT)**

13. When mass m_1 is held fixed, and mass m_2 vibrates, the system shown in the figure has a natural frequency ω_n and damping factor ζ. When mass m_2 is held fixed, and m_1 vibrates, the natural frequency and damping factor are



(a) $\omega_n\sqrt{\frac{m_1}{m_2}}, \quad \zeta\sqrt{\frac{m_1}{m_2}}$

(b) $\omega_n\sqrt{\frac{m_2}{m_1}}, \quad \zeta\sqrt{\frac{m_1}{m_2}}$

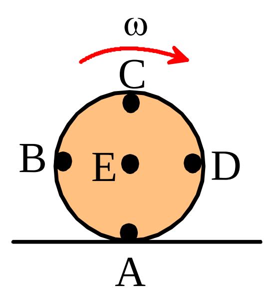
(c) $\omega_n\sqrt{\frac{m_1}{m_2}}, \quad \zeta\sqrt{\frac{m_2}{m_1}}$

(d) $\omega_n\sqrt{\frac{m_2}{m_1}}, \quad \zeta\sqrt{\frac{m_2}{m_1}}$

Natural frequency is $\sqrt{k/m}$ and damping factor is $c/(2\sqrt{km})$ so both are changed by the ratio $\sqrt{m_2/m_1}$

ANSWER______D________**(2 POINTS)**

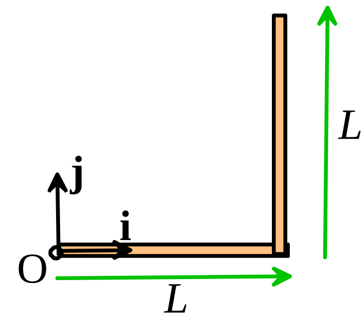
14. The disk shown in the figure rolls without slip. Which point on the disk has the greatest speed?



The wheel is instantaneously rotating about A so the point furthest from A has the greatest speed.

ANSWER_______C_______ **(2 POINTS)**

15. Two rigid slender rods with mass *m* and length *L* are rigidly connected as shown in the figure. Their combined mass moment of inertia about an axis parallel to the **k** direction passing through O is



(a) $I_0 = 2mL^2/3$

(b) $I_0 = mL^2/6$

(c) $I_0 = 5mL^2/3$

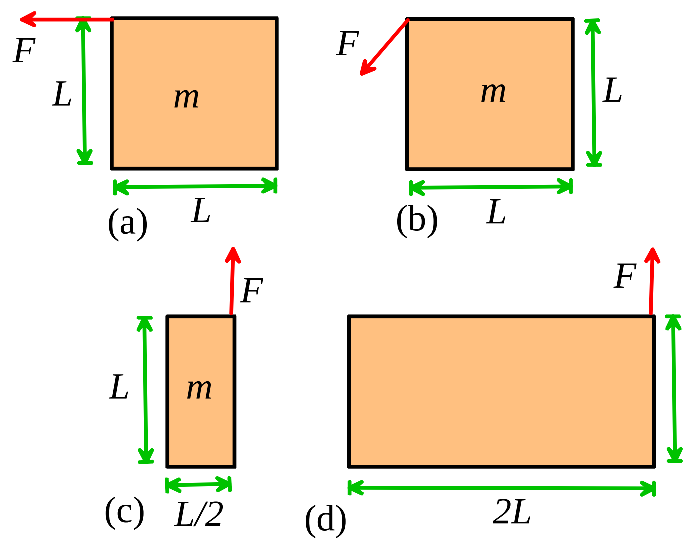
(d) $I_0 = 7mL^2/24$

The mass moment of inertia of a rod about its center is $mL^2/12$ The parallel axis theorem gives

$$I_0 = 2\frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2 + m\left\{\left(\frac{L}{2}\right)^2 + L^2\right\} = m\left(\frac{1}{6}+\frac{2}{4}+1\right)L^2 = 5mL^2/3$$

ANSWER_______C_______ **(2 POINTS)**

16. In the figures shown, the length *L*, mass *m* and force *F* have the same value. Other than the force *F*, no other forces act on the objects. Which of the objects shown has the greatest angular acceleration?



The angular acceleration is $\alpha = M_G / I_G$. The mass moments of inertia of a rectangle with dimensions $a \times b$ is $m(a^2+b^2)/12$. The angular accelerations of the objects shown are therefore

(a) $\alpha = (LF/2)/(mL^2/6) = 3F/(mL^2)$

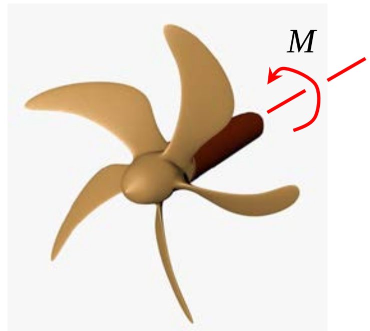
(b) $\alpha = (\sqrt{2}LF/2)/(mL^2/6) = 3\sqrt{2}F/(mL^2)$

(c) $\alpha = (LF/4)/\{(m(L^2+L^2/4)/12\} = 12F/(5mL^2)$

(d) $\alpha = (LF)/\{(m(L^2+4L^2)/12\} = 12F/(5mL^2)$

ANSWER_______B_______**(2 POINTS)**

17. The figure shows a propeller with total mass moment of inertia 200 kg m^2. The propeller is driven by a motor (not shown) that exerts a moment *M* on the shaft, which induces a constant angular acceleration of 10 rad s^{-2}. The propeller is at rest at time t=0. After 1 second, the total work done by the moment on the propeller is



(a) 5 kJ

(b) 10 kJ

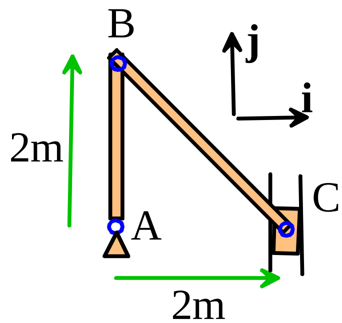
(c) 20 kJ

(d) None of the above

After 1 sec the angular speed is 10 rad/s. The work done by the moment is equal to the change in KE of the propeller ie $I\omega^2/2 = 200 \times 100/2 = 10kJ$

ANSWER_____B________**(2 POINTS)**

18. In the figure shown the link AB rotates counter-clockwise with constant angular speed 4 rad/s. C can only move in the **j** direction. The velocity of C and the angular velocity of link BC are



(a) $\mathbf{v}_C = \mathbf{0}, \quad \omega_{BC} = 4\mathbf{k} \ rad/s$

(b) $\mathbf{v}_C = \mathbf{0}, \quad \omega_{BC} = -4\mathbf{k} \ rad/s$

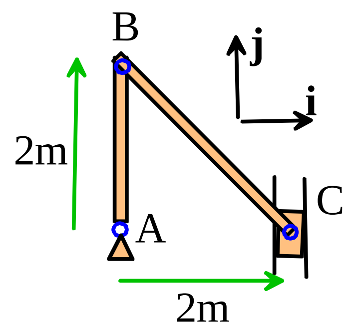
(c) $\mathbf{v}_C = 8\mathbf{j} m/s, \quad \omega_{BC} = 4\mathbf{k} rad/s$

(d) $\mathbf{v}_C = -8\mathbf{j} m/s, \quad \omega_{BC} = -4\mathbf{k} rad/s$

$\mathbf{v}_B = 4\mathbf{k} \times 2\mathbf{j} = -8\mathbf{i} \quad \mathbf{v}_C = -8\mathbf{i} + \omega_{BC}\mathbf{k} \times (2\mathbf{i} - 2\mathbf{j}) = (2\omega_{BC} - 8)\mathbf{i} + 2\omega_{BC}\mathbf{j}$. Since C must move in the **j** direction, $\omega_{BC} = 4, \ \mathbf{v}_C = 8\mathbf{j}$

ANSWER______C________**(2 POINTS)**

19. In the figure shown the link AB rotates counter-clockwise with constant angular speed 4 rad/s. The acceleration of C and the angular acceleration of link BC are



(a) $\mathbf{a}_C = 32\mathbf{j} m/s^2,\ \alpha_{BC} = 0\ rad/s^2$

(b) $\mathbf{a}_C = 16\mathbf{j} m/s^2,\ \alpha_{BC} = 8\mathbf{k}\ rad/s^2$

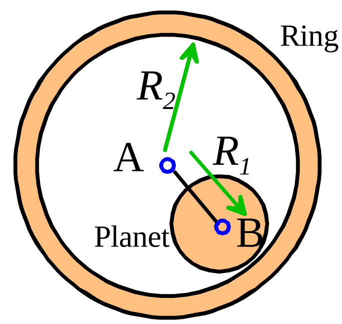
(c) $\mathbf{a}_C = -32\mathbf{j} m/s^2,\ \alpha_{BC} = -16\mathbf{k}\ rad/s^2$

(d) $\mathbf{a}_C = 32\mathbf{j} m/s^2,\ \alpha_{BC} = 16\mathbf{k}\ rad/s^2$

$\mathbf{a}_B = -32\mathbf{j} \;\Rightarrow\; \mathbf{a}_C = -32\mathbf{j} + \alpha_{BC}\mathbf{k}\times(2\mathbf{i}-2\mathbf{j}) - \omega_{BC}^2(2\mathbf{i}-2\mathbf{j}) = (2\alpha_{BC}-32)\mathbf{i} + (2\alpha_{BC}-32+32)\mathbf{j}$. The acceleration of C is zero in the **i** direction and therefore $\mathbf{a}_C = 32\mathbf{j} m/s^2,\ \alpha_{BC} = 16\mathbf{k}\ rad/s^2$

ANSWER_______D_______**(2 POINTS)**

20. In the figure shown the ring gear has radius R_2 and the planet carrier bar AB has radius R_1. If the ring gear is stationary and the planet gear rotates at angular speed ω, bar AB has angular speed



(a) $\omega\dfrac{R_1}{R_2}$

(b) $\omega\dfrac{R_2}{R_1}$

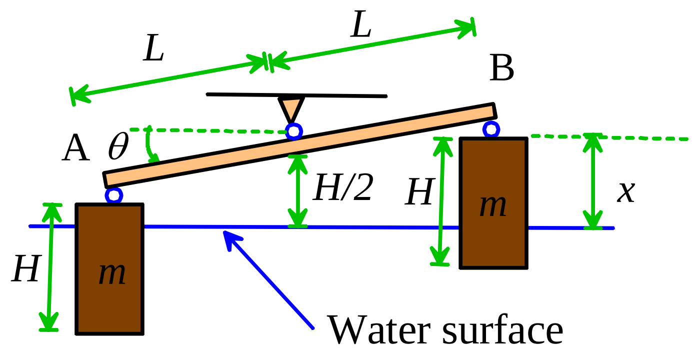
(c) $\omega\left(1-\dfrac{R_1}{R_2}\right)$

(d) $\omega\left(\dfrac{R_2}{R_1}-1\right)$

The planet gear has radius $R_2 - R_1$. It rolls without slip on the ring gear so its center (at B) has speed $v = \omega(R_2 - R_1)$. B is in circular motion about A so bar AB must rotate with angular speed $v/R_1 = \omega(\dfrac{R_2}{R_1}-1)$

ANSWER_______D_______**(2 POINTS)**

21. The figure shows a schematic diagram of a wave energy harvesting device. It consists of two cylindrical buoys with cross-sectional area A , height H and mass m, attached to a bar with total length $2L$ and negligible mass. The buoys are immersed in water with mass density ρ .



During operation waves cause the bar AB to rock through a time-varying angle θ as shown. When $\theta = 0$ both buoys are half submerged $(x = H/2)$.

The goal of this problem is to analyze oscillation of the device in still water (no waves). No torque or friction acts at the pivot. Neglect the mass of AB and viscous drag of the water.

21.1 The buoyancy force acting (upwards) on one cylinder can be expressed in terms of its height x above the water surface as

$$F_B = \rho g A(H - x)$$

Show that (for $0 < x < H$) the potential energy of the buoyancy force is $V = -\rho g A\left(Hx - \frac{x^2}{2}\right) + C$ where C is a constant

From the definition of potential energy $V = -\int_0^x F_B dx + C = -\int_0^x \rho g A(H - x)dx + C = -\rho g A(Hx - \frac{x^2}{2}) + C$

[1 POINT]

21.2 Write down a formula for the speed of the masses (which is equal to the speed of ends A and B of the bar) in terms of $L, d\theta / dt$

A and B describe circular motion about the pivot, therefore $v = L\frac{d\theta}{dt}$

[1 POINT]

21.3 Hence, write down the total potential and kinetic energy of the system, in terms of ρ, A, L, m, g and θ and its time derivatives (assume that $0 < x < H$ at all times).

The height of the mass attached to B above the water is $x_B = H/2 + L\sin\theta$; the height of the mass attached to A is $x_A = H/2 - L\sin\theta$. The total potential energy is

$$
\begin{aligned}
V &= -\rho g A\left(H x_A - \frac{x_A^2}{2} \right) - \rho g A\left(H x_B - \frac{x_B^2}{2} \right) + 2C + mgx_A + mgx_B \\
&= 2C + (mg - \rho g A H)(x_A + x_B) + \frac{1}{2}\rho g A (x_A^2 + x_B^2) \\
&= 2C + (mg - \rho g A H)H + \frac{1}{2}\rho g A\left(H/2 + L\sin^2\theta \right)^2 + \frac{1}{2}\rho g A\left(H/2 - L\sin^2\theta \right)^2 \\
&= 2C + (mg - \rho g A H)H + \rho g A H^2/4 + \rho g A L^2 \sin^2\theta
\end{aligned}
$$

(not necessary to fully simplify solution to receive credit, and also OK to drop all the constant terms since they are arbitrary. Also OK to use some other datum for gravity)

The total kinetic energy is $T = \frac{1}{2} 2m \left(L \frac{d\theta}{dt} \right)^2$

[2 POINTS]

21.4 Hence, show that θ satisfies the equation of motion

$$
m\frac{d^2\theta}{dt^2} + \rho g A \sin\theta\cos\theta = 0
$$

We can use energy conservation to derive the EOM:

$$
\begin{aligned}
&\frac{d}{dt}\left(T + V\right) = 0 \Rightarrow 2mL^2 \frac{d\theta}{dt}\frac{d^2\theta}{dt^2} + \rho g A L^2 2\sin\theta\cos\theta\frac{d\theta}{dt} = 0 \\
&\Rightarrow m\frac{d^2\theta}{dt^2} + \rho g A \sin\theta\cos\theta = 0
\end{aligned}
$$

[2 POINTS]

21.5 Linearize the equation of motion in 21.4 for small θ and hence find a formula for the natural frequency of vibration of the energy harvester, in terms of ρ, g, A, m .

We can use $\sin\theta \approx \theta \quad \cos\theta \approx 1$ to see that

$$m\frac{d^2\theta}{dt^2} + \rho g A\theta = 0$$
$$\Rightarrow \frac{m}{\rho g A}\frac{d^2\theta}{dt^2} + \theta = 0$$
$$\Rightarrow \omega_n = \sqrt{\frac{\rho g A}{m}}$$

[2 POINTS]

21.6 Rearrange the equation in 21.4 into a form that MATLAB could solve using the ode45 function.

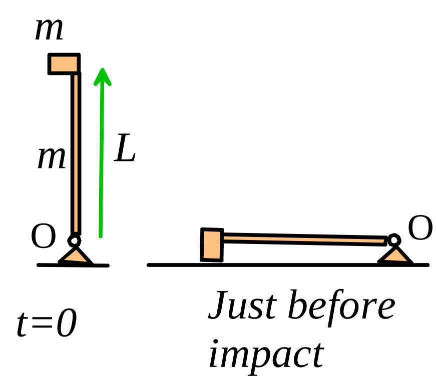
Following the usual procedure we introduce $\omega = \frac{d\theta}{dt}$ as an additional variable. The EOM can therefore be rearranged as

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \omega\end{bmatrix} = \begin{bmatrix}\omega \\ -(\rho g A / m)\sin\theta\cos\theta\end{bmatrix}$$

[2 POINTS]

22. The figure shows an impulse hammer that is used to strike the ground in a seismic experiment. It consists of a slender rod with a hammer-head at its end. The rod has mass m, length L and mass moment of inertia $mL^2/12$ about its center of mass. The hammer head has mass m and has negligible mass moment of inertia about its center of mass. The rod pivots freely at O. It is released from rest with the slender rod vertical.

The goal of this problem is to calculate (i) the speed of the hammer-head when it just hits the ground and (ii) the reaction forces acting at O at the instant just before the hammer-head strikes the ground.



22.1 Find the total mass moment of inertia of the system (the rod together with the hammer-head) about O

The total mass moment of inertia about O is $mL^2/12 + m(L/2)^2 + mL^2 = 4mL^2/3$

[2 POINTS]

22.2 Using energy conservation, show that the angular speed of the rod just before the hammer-head strikes the ground is $\omega = \frac{3}{2}\sqrt{g/L}$

$$PE + KE = const \Rightarrow \sum mgh = \frac{1}{2} I_O \omega^2 \Rightarrow \frac{2}{3} mL^2 \omega^2 = mgL + mgL/2 \Rightarrow \omega = \frac{3}{2}\sqrt{g/L}$$

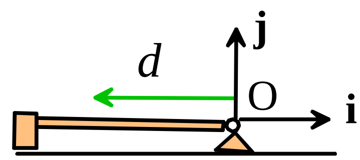
[2 POINTS]

22.3 Hence, find the speed of the hammer head just before it hits the ground

$$v = \omega L = \frac{3}{2}\sqrt{gL}$$

[1 POINT]

22.4 Find the distance *d* of the center of mass of the system from O, in terms of *L*.



$$d = (mL + mL/2)/(2m) = 3L/4$$

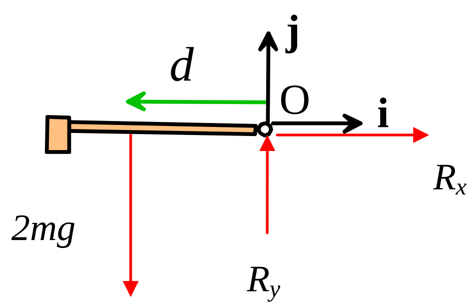
[1 POINT]

22.5 Find a formula for the acceleration of the center of mass of the system at the instant when the shaft is horizontal in terms of d, the angular velocity ω and acceleration α. Express your answer as components in the **i,j** basis shown.

The rigid body kinematics formula gives $\mathbf{a}_G = \alpha\mathbf{k}\times\mathbf{r}_{G/O} + \omega\mathbf{k}\times\omega\mathbf{k}\times\mathbf{r}_{G/O} = -d\alpha\,\mathbf{j} + \omega^2 d\,\mathbf{i}$

[2 POINTS]

22.6 Draw a free body diagram showing the forces acting on the system on the figure provided below. Assume that the hammer-head has not yet hit the ground.



(also OK to draw gravity on the head and shaft separately)

[2 POINTS]

22.7 Using the rigid body dynamics equations (i.e. the equations relating angular accelerations and moments and/or Newton's laws) show that the angular acceleration of the hammer at the instant just before it strikes the ground is $\alpha = \frac{9}{8}\frac{g}{L}$

$$I_O \alpha \mathbf{k} = \sum \mathbf{M}_O \Rightarrow \frac{4}{3} m L^2 \alpha = 2mg\frac{3}{4}L \Rightarrow \alpha = \frac{9}{8}\frac{g}{L}$$

[2 POINTS]

22.8 Find a formula for the reaction forces acting at O at the instant just before the hammer-head hits the ground, in terms of *m* and *g* (no other variables should appear in your solution). Express your answer as components in the **i,j** basis.

$$\mathbf{F} = m\mathbf{a}_G \Rightarrow R_x \mathbf{i} + (R_y - 2mg)\mathbf{j} = 2m\left(-d\alpha\mathbf{j} + \omega^2 d\mathbf{i}\right) = 2m\left\{-\frac{3}{4}L\frac{9}{8}\frac{g}{L}\mathbf{j} + \left(\frac{3}{2}\sqrt{g/L}\right)^2 \frac{3}{4}L\mathbf{i}\right\}$$

$$\Rightarrow R_x = \frac{27}{8}mg \qquad R_y = 2\left(1 - \frac{27}{32}\right)mg = \frac{5}{16}mg$$

[3 POINTS]