School of Engineering
Brown University

EN40: Dynamics and Vibrations

Midterm Examination
Thursday March 9 2017

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1. (10 points) _____________

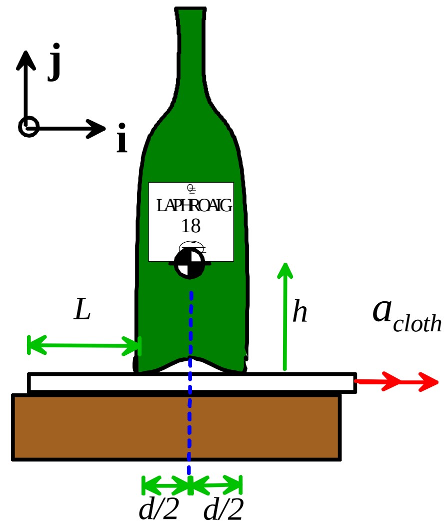
2. (12 points) _____________

3. (3 points) _____________

4. (10 points) _____________

TOTAL (35 points) ___________

1. A bottle with mass m rests on a table-cloth. The contact between them has friction coefficient μ . At time $t=0$ the object and cloth are both stationary. For time $t>0$, the cloth is pulled with constant acceleration $a_{cloth} = 5\mu g$ the right. Note that since $a_{cloth} > \mu g$, slip must occur at the contact just after $t=0$.



1.1 Draw a free body diagram showing the forces acting on the object on the figure below.

[3 POINTS]

1.2 Find a formula for the horizontal acceleration of the bottle. Assume that the bottle does not tip over.

[2 POINTS]

1.3 Find a formula for the horizontal distance moved by the bottle as a function of time.

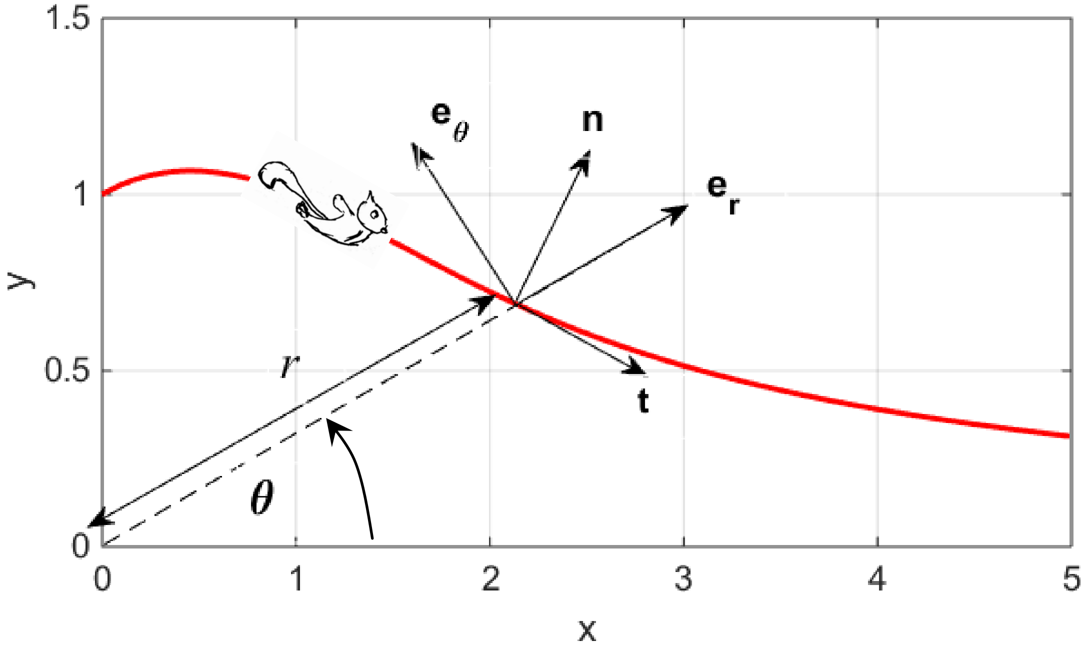
[1 POINT]

1.4 Find a formula for the distance moved by the cloth as a function of time

[1 POINT]

1.5 At time $t=0$ the bottle is a distance L from the edge of the cloth. Calculate the distance that the bottle has moved in the **i** direction at the instant that the edge of the cloth reaches the edge of the bottle ($L=0$ at this instant). Express your answer in terms of L and any other parameters you think are relevant.

[3 POINTS]



2. The figure shows the trajectory of a flying squirrel, which is measured (in polar coordinates) to be

$$r = (1+t) \text{ (meters)} \qquad \theta = \frac{\pi}{2(1+t)^2} \quad (radians)$$

(Note that $\theta = \pi/2$ at time $t=0$, and decreases as t increases, so the squirrel flies to the right).

2.1 Find a formula for the velocity vector of the squirrel (in polar coordinates $\{\mathbf{e}_r, \mathbf{e}_\theta\}$) terms of time

[1 POINT]

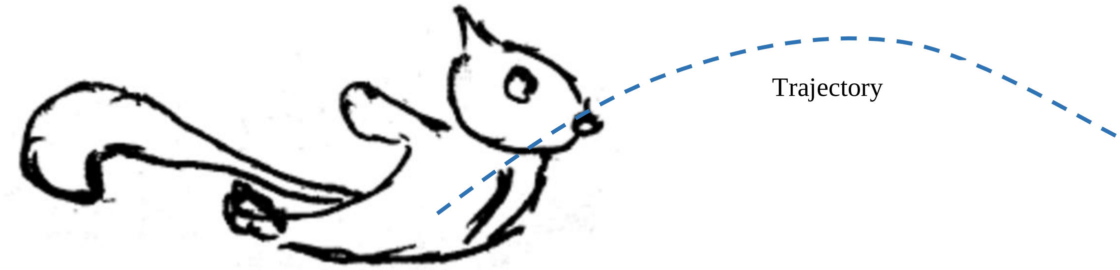
2.2 Find a formula for the acceleration vector in polar coordinates $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ as a function of time

[2 POINTS]

2.3 Find unit vectors normal and tangent to the path of the squirrel **AT TIME t=0**, in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis. Choose the sign of **n** so that the **n** has a positive $\mathbf{e}_r$ component.

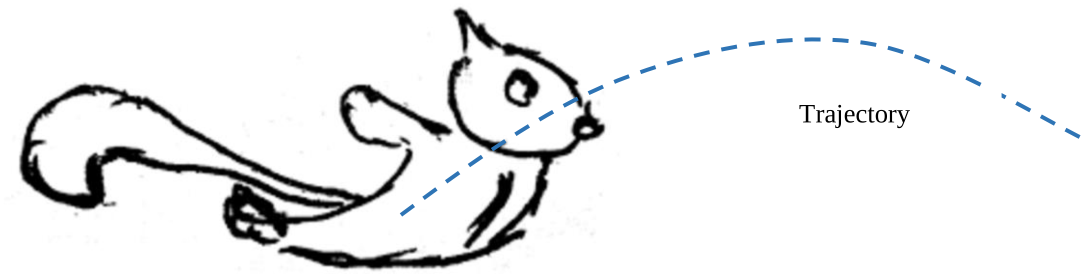
[2 POINTS]

2.4 Draw the unit vectors $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ **AT TIME t=0** on the figure provided below



[1 POINT]

2.5 Draw a free body diagram showing the forces acting on the squirrel **at time *t=0*** on the figure provided. Include lift and drag forces, and any other forces you think are relevant.



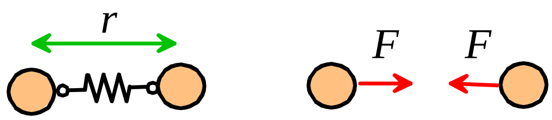
[2 POINTS]

2.6 If the squirrel has mass 100 grams, calculate the magnitude of the lift and drag force acting on the squirrel **at time *t=0*** (you can use the approximation $g \approx 10 m/s^2$ and leave your answer in terms of π - there is no need to get a number for the answer)

[4 POINTS]

3. The attractive force between two atoms in a diatomic molecule is related to the distance *r* between them by

$$F = 8F_0\left(\left(\frac{a}{r}\right)^5 - \left(\frac{a}{r}\right)^9\right)$$



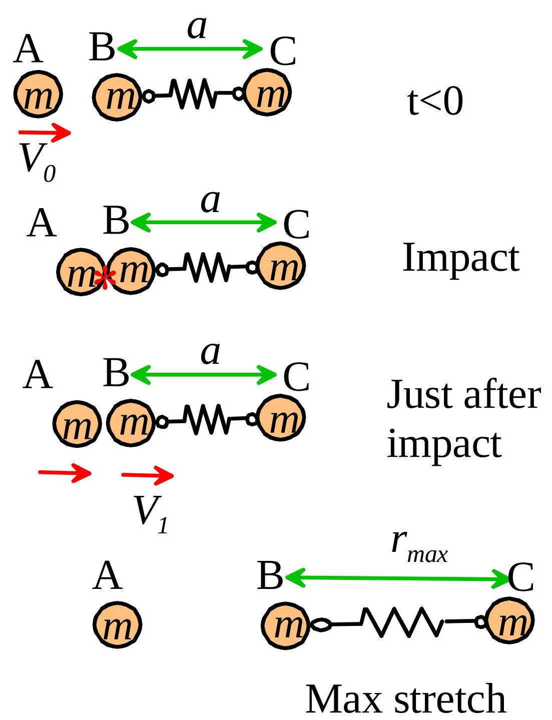
where F_0 and *a* are constants. (**NB:** the spring in the figure is used to indicate the atomic bond – it does not obey the usual force-v-length relation for a linear spring)

Show that the potential energy of the interatomic bond is (to within an arbitrary constant)

$$V(r) = F_0 a\left(1 - 2\left(\frac{a}{r}\right)^4 + \left(\frac{a}{r}\right)^8\right)$$

[3 POINTS]

4. The figure shows a straight-line collision between an ion (A) and a (neutral) diatomic molecule (B-C)



- The ion and each atom are idealized as hard spheres with mass *m*.
- **The bond between atoms B and C has potential energy given in problem 3**
$$V(r) = F_0 a\left(1 - 2\left(\frac{a}{r}\right)^4 + \left(\frac{a}{r}\right)^8\right)$$
- For time $t<0$ atoms the bond between B and C has zero force (i.e. the distance between atoms $r=a$); and the ion A is moving to the right with speed V_0 .
- The ion A collides with atom B at time $t=0$. The collision has restitution coefficient $e=1$.

NB: the spring in the figure is used to indicate the atomic bond – it does not obey the usual force-v-length relation for a linear spring. Use the potential energy relation from problem 3, not the potential energy for a spring!)

4.1 Calculate the velocity V_1 of atom B just after the collision, in terms of V_0 (note that the bond between B and C exerts no forces during the collision, and recall that $e=1$ and A and B have the same mass)

[2 POINTS]

4.2 What is the total linear momentum of the molecule with atoms (BC) just after the impact, in terms of *m* and V_0 ?

[1 POINT]

4.3 What is the total kinetic energy of the molecule with atoms (BC) just after the impact, in terms of *m* and V_0 ?

[1 POINT]

4.4 Using energy and momentum conservation, show that the maximum separation between atoms in molecule BC following the collision is

$$r = a\frac{1}{\left(1-\frac{1}{2}\sqrt{\frac{mV_0^2}{F_0 a}}\right)^{1/4}}$$

(Hint: at the instant of maximum separation both atoms B and C have the same velocity).

[4 POINTS]

4.5 Determine the critical value of V_0 that will just break the bond between the atoms (assume that the bond breaks if $r \to \infty$ after the collision).

[2 POINTS]