School of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Wed May 10 2017: 2pm-5pm

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1-20 [41 points] _____________

21 [10 POINTS] _____________

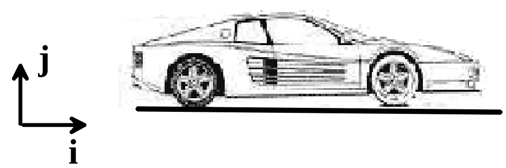
22 [12 POINTS] _____________

TOTAL [63 POINTS] _______________

FOR PROBLEMS 1-20 WRITE YOUR ANSWER IN THE SPACE PROVIDED. ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. The vehicle shown in the figure starts from rest and accelerates (with constant acceleration) to 10m/s over a distance of 25m. Its acceleration is

(a) 0.5 m/s^2
(b) 1 m/s^2
(c) 2 m/s^2
(d) None of the above



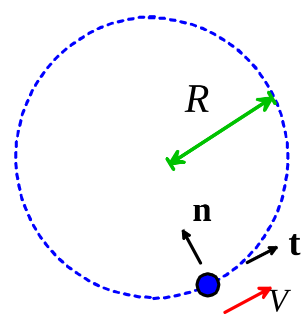
ANSWER______________ **(2 POINTS)**

2. The vehicle in problem 1 has four wheel drive and each wheel is at the point of slip (so $T = \mu N$ for each wheel). The coefficient of friction at the contact between the tires and road is approximately:

(a) ≈ 0.1
(b) ≈ 0.2
(c) ≈ 0.5
(d) None of the above

ANSWER______________ **(2 POINTS)**

3. A 'prey' particle with mass 30 grams travels ***at constant speed*** of 1m/s around a circular path with radius 10m. It is subjected to a viscous drag force with magnitude 0.004N and direction opposite to the direction of motion, as well as a propulsive force. The magnitude of the propulsive force on the particle is

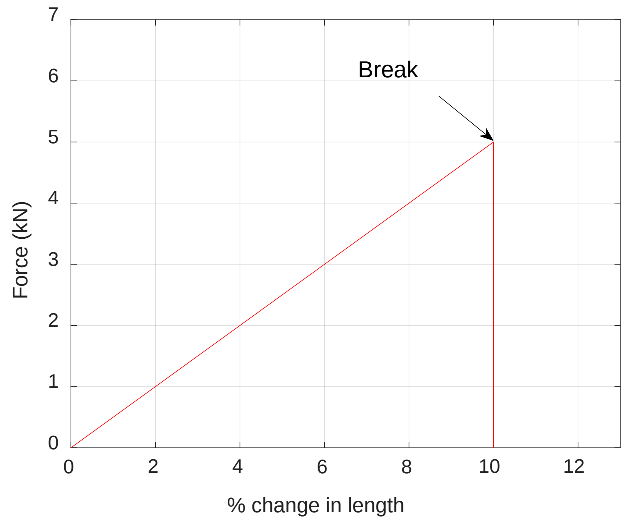


(a) 0.003N
(b) 0.004N
(c) 0.005N
(d) 0.01N
(e) None of the above

ANSWER______________ **(2 POINTS)**

4. The figure shows a force-extension curve for a length of climbing rope. The energy (or work) required to break a 10m length of rope is

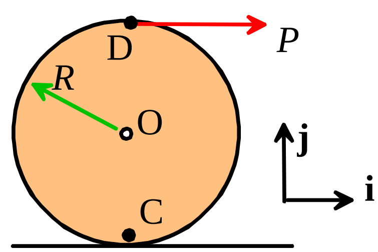
(a) 1.25 kJ
(b) 2.5 kJ
(c) 5 kJ
(d) 10 kJ
(e) None of the above, or cannot be determined



ANSWER_______________ **(2 POINTS)**

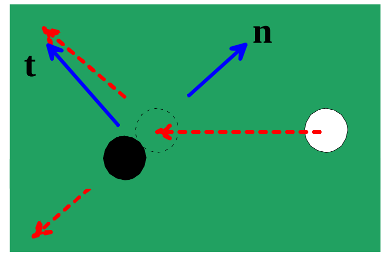
5. The center of the wheel moves with (instantaneous) velocity $\mathbf{v}_O = V\mathbf{i}$ and the wheel rolls without slip. The rate of work done by the force *P* acting at point D on the wheel is:

(a) zero
(b) *PV/2*
(c) *PV*
(d) *2PV*
(e) None of the above



ANSWER________________ **(2 POINTS)**

6. A frictionless collision takes place between two billiard balls. $\{\mathbf{n}, \mathbf{t}\}$ denote unit vectors parallel and perpendicular to the line connecting the centers of the spheres at the instant of collision.



Please circle the answers to indicate whether the following statements are true or false:

(a) Momentum of the entire system in the **n** direction is conserved during the collision

TRUE FALSE

(b) Momentum of the entire system in the **t** direction is conserved during the collision

TRUE FALSE

(c) Momentum of the black ball in the **n** direction is conserved during the collision

TRUE FALSE

(d) Momentum of the white ball in the **n** direction is conserved during the collision

TRUE FALSE

(e) Momentum of the black ball in the **t** direction is conserved during the collision

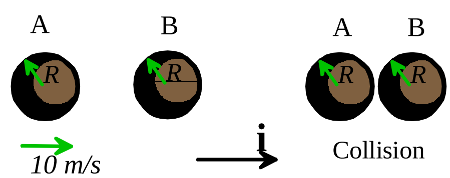
TRUE FALSE

(f) Momentum of the white ball in the **t** direction is conserved during the collision

TRUE FALSE

(3 POINTS)

7. The two masses in the figure both have masses 1 kg. At time t=0 sphere A has velocity 10**i** m/s and B is stationary. They collide with a restitution coefficient $e=0$. The impulses exerted on the two particles during the collision are:

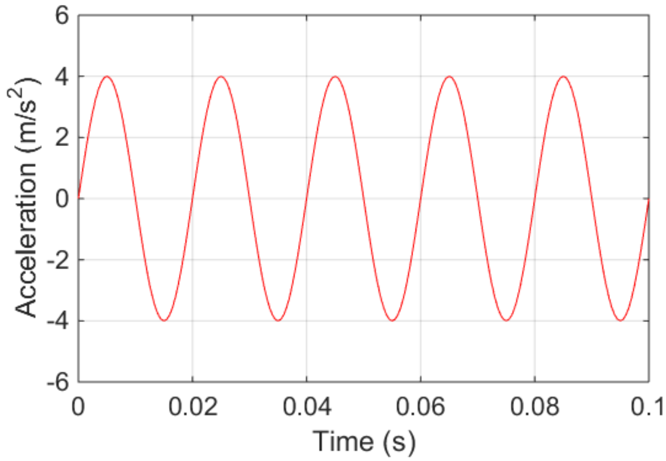


(a) $\mathbf{I}_A = -10\mathbf{i}$ $\quad \mathbf{I}_B = 10\mathbf{i}$ Ns
(b) $\mathbf{I}_A = -5\mathbf{i}$ $\quad \mathbf{I}_B = 10\mathbf{i}$ Ns
(c) $\mathbf{I}_A = -5\mathbf{i}$ $\quad \mathbf{I}_B = 5\mathbf{i}$ Ns
(d) $\mathbf{I}_A = 0\mathbf{i}$ $\quad \mathbf{I}_B = 0\mathbf{i}$ Ns
(e) None of the above

ANSWER_______________ **(2 POINTS)**

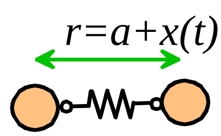
8. The figure shows a vibration measurement from an accelerometer. The amplitude of the displacement is

(a) 12.7mm
(b) 40mm
(c) 12.7 μm
(d) 41μm
(e) None of the above



ANSWER_______________ **(2 POINTS)**

9. The figure shows a diatomic molecule. In static equilibrium, the distance between the atoms is $r{=}a$. When the atoms vibrate, the distance between them changes by a small amount x, so that $r(t)=a+x(t)$. The equation of motion for x is



$$m\frac{d^2x}{dt^2}=-16F_0\left(\left(\frac{a}{a+x}\right)^5-\left(\frac{a}{a+x}\right)^9\right)$$

where F_0 is a constant and m is the mass of one atom. The natural frequency of small amplitude vibrations of the molecule is

(a) $\omega_n=8\sqrt{\frac{F_0}{ma}}$

(b) $\omega_n=4\sqrt{\frac{F_0}{m}}$

(c) $\omega_n=\frac{1}{4}\sqrt{\frac{m}{F_0}}$

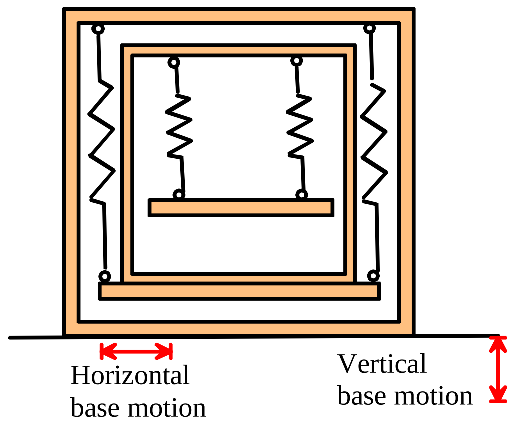
(d) $\omega_n=\frac{1}{8}\sqrt{\frac{ma}{F_0}}$

(e) None of the above

ANSWER_______________ **(2 POINTS)**

10. The figure shows a 2D idealization of a vibration isolation system. There is no slip at any of the contacts. The system has

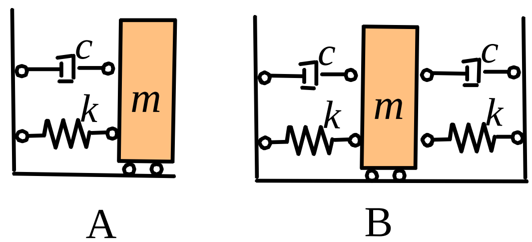
(a) 2 degrees of freedom and 2 vibration modes
(b) 4 degrees of freedom and 4 vibration modes
(c) 6 degrees of freedom and 6 vibration modes
(d) 6 degrees of freedom and 4 vibration modes
(e) None of the above



ANSWER_______________ **(2 POINTS)**

11. System A in the figure is critically damped. System B is

(a) Overdamped
(b) Critically damped
(c) Underdamped
(d) Favorably dry
(e) None of the above



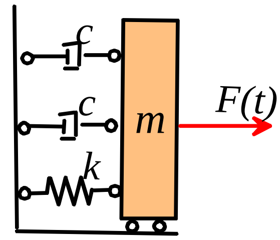
ANSWER_______________ **(2 POINTS)**

12. For the most effective base isolation, a vibration isolation system should have

(a) A high natural frequency and high damping
(b) A low natural frequency and low damping
(c) A low natural frequency and high damping
(d) A high natural frequency and low damping
(e) None of the above

ANSWER_______________ **(2 POINTS)**

13. The spring-mass system shown in the figure is subjected to a harmonic force. The amplitude of the force is 1 kN and the frequency is equal to the undamped natural frequency of the system. The amplitude of vibration is 1mm. If one dashpot is removed, and the system is subjected to the same force (i.e the amplitude and frequency of the force is the same), the steady-state amplitude of vibration will be:



(a) 2mm
(b) 1mm
(c) 0.5mm
(d) 0.25mm
(e) None of the above

ANSWER_______________ **(2 POINTS)**

14. The rotation tensor

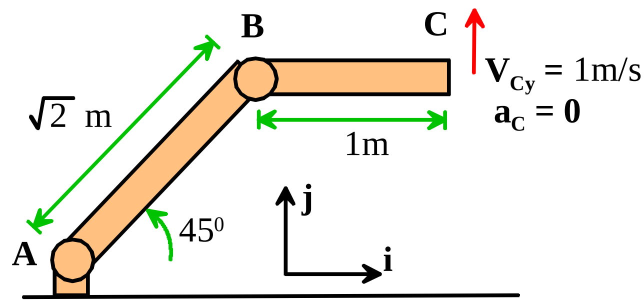
$$\mathbf{R} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

represents

(a) A 90 degree rotation about an axis parallel to $\mathbf{n} = (\mathbf{i} + \mathbf{j}) / \sqrt{2}$
(b) A 90 degree rotation about an axis parallel to $\mathbf{n} = (\mathbf{i} - \mathbf{j}) / \sqrt{2}$
(c) A 45 degree rotation about an axis parallel to **k**
(d) A 45 degree rotation about an axis parallel to **i**
(e) None of the above.

ANSWER_______________ **(2 POINTS)**

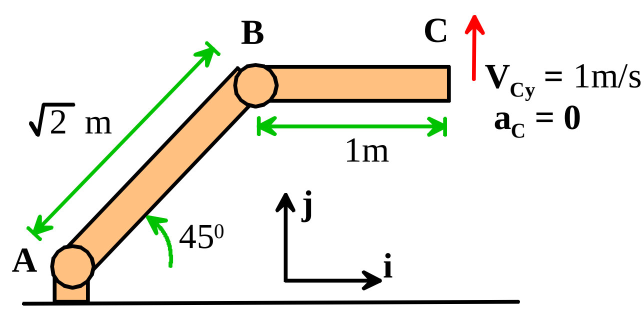
15 Point C on the robotic actuator shown in the figure moves vertically with velocity $\mathbf{v}_C = 1\mathbf{j}$ m/s. The angular velocities of members AB and BC are



(a) $\omega_{AB} = 0,\ \ \omega_{BC} = 1$ rad/s
(b) $\omega_{AB} = -1,\ \ \omega_{BC} = 1$ rad/s
(c) $\omega_{AB} = 1,\ \ \omega_{BC} = -1$ rad/s
(d) $\omega_{AB} = 1,\ \ \omega_{BC} = 0$ rad/s
(e) None of the above

ANSWER_______________ **(2 POINTS)**

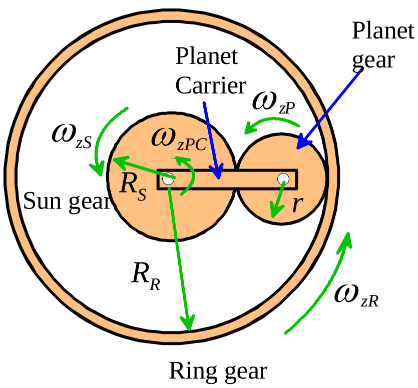
16 Point C on the robotic actuator shown in the figure moves vertically with velocity $\mathbf{v}_C = 1\mathbf{j}$ m/s and zero acceleration. The angular accelerations of members AB and BC are



(a) $\alpha_{AB} = 0,\ \ \alpha_{BC} = 1$ rad/s^2
(b) $\alpha_{AB} = -1,\ \ \alpha_{BC} = 1$ rad/s^2
(c) $\alpha_{AB} = 1,\ \ \alpha_{BC} = -1$ rad/s^2
(d) $\alpha_{AB} = 1,\ \ \alpha_{BC} = 0$ rad/s^2
(e) None of the above

ANSWER_______________ **(2 POINTS)**

17. With the ring gear fixed, the planet carrier and sun gears in the epicyclic gear system shown have a ratio $\omega_{zS} / \omega_{zPC} = 3$. If the sun gear has 20 teeth, the ring and planet gears have

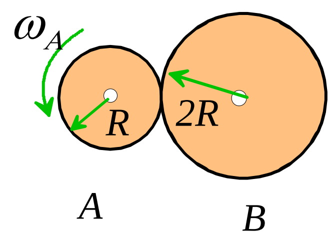


(a) $N_R = 40, \quad N_P = 10$ teeth
(b) $N_R = 40, \quad N_P = 20$ teeth
(c) $N_R = 80, \quad N_P = 20$ teeth
(d) $N_R = 80, \quad N_P = 30$ teeth
(e) None of the above

ANSWER_______________ **(2 POINTS)**

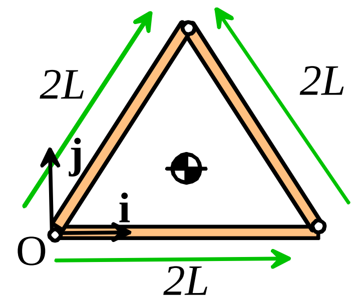
18. The two gears A and B in the figure have radii *R* and *2R*, and mass moments of inertia $mR^2 / 2$ and $2mR^2$, respectively. Their centers are stationary. If gear A rotates at angular speed ω_A , the total kinetic energy of the two gears is

(a) $4mR^2\omega_A^2$
(b) $3mR^2\omega_A^2$
(c) $mR^2\omega_A^2$
(d) $mR^2\omega_A^2 / 2$
(e) None of the above



ANSWER_______________ **(2 POINTS)**

19. Three bars with mass m and length $2L$ are connected to form an equilateral triangle as shown in the figure. The position of the center of mass of the triangle with respect to the origin at O is



(a) $L\mathbf{i}+L\mathbf{j}$

(b) $L\mathbf{i}+\frac{1}{3}L\mathbf{j}$

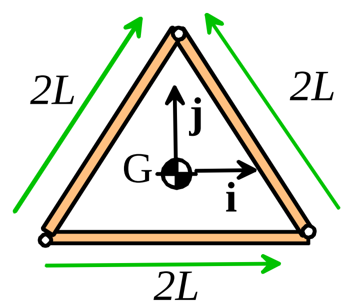
(c) $L\mathbf{i}+\frac{1}{\sqrt{3}}L\mathbf{j}$

(d) $L\mathbf{i}+\frac{2}{\sqrt{3}}L\mathbf{j}$

(e) None of the above

ANSWER_______________ **(2 POINTS)**

20. Three bars with mass m and length $2L$ are connected to form an equilateral triangle as shown in the figure. The (2D) mass moment of inertia of the assembly about the center of mass is



(a) $I_{Gzz}=mL^2/4$

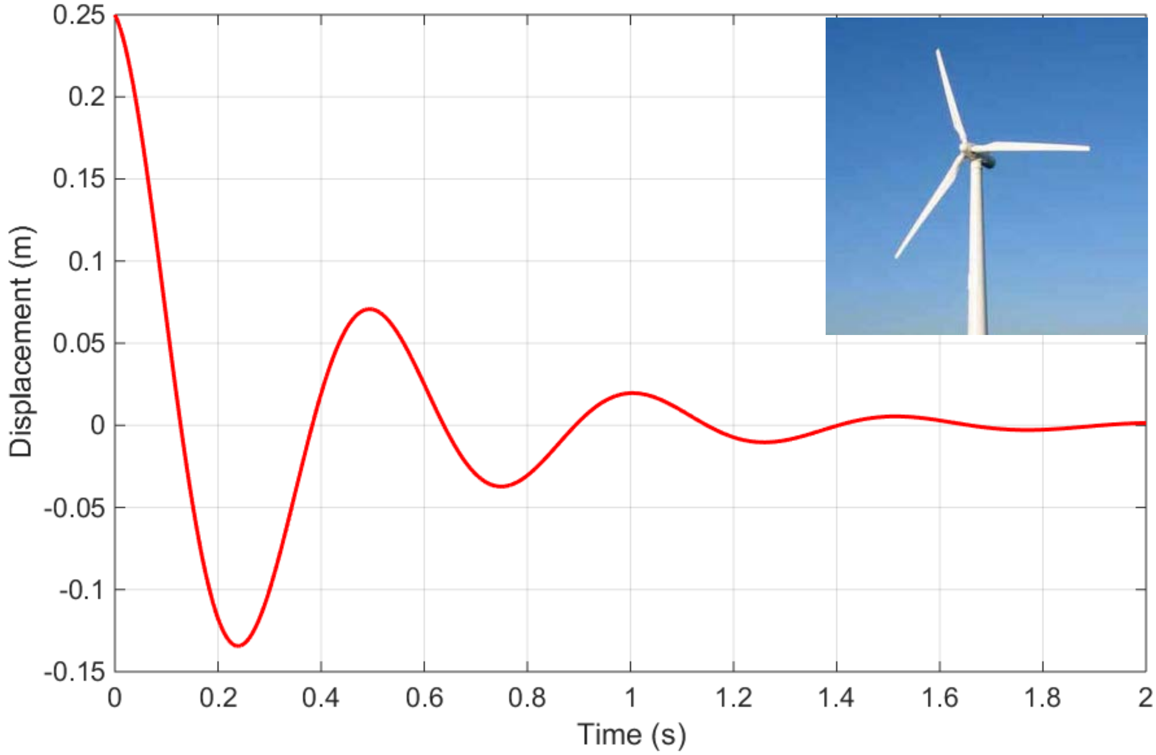
(b) $I_{Gzz}=mL^2$

(c) $I_{Gzz}=2mL^2$

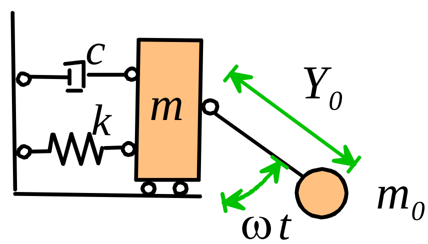
(d) $I_{Gzz}=mL^2/2$

(e) None of the above

ANSWER_______________ **(2 POINTS)**



21 An unbalanced wind-turbine is idealized as a rotor-excited spring-mass system as shown in the figure. The mass m represents the tower, and m_0 represents the combined mass of the three rotor blades. The spring and damper represent the stiffness and energy dissipation in the tower. The rotor is 'unbalanced' because its center of mass is a small distance Y_0 away from the axle. The total mass $(m + m_0)$ of the system is 25000kg.



The figure shows the results of a free vibration experiment on the turbine.

21.1 Use the data provided to determine the following quantities:

(a) The vibration period

[1 POINT]

(b) The log decrement

[1 POINT]

(c) The undamped natural frequency

[1 POINT]

(d) The damping factor

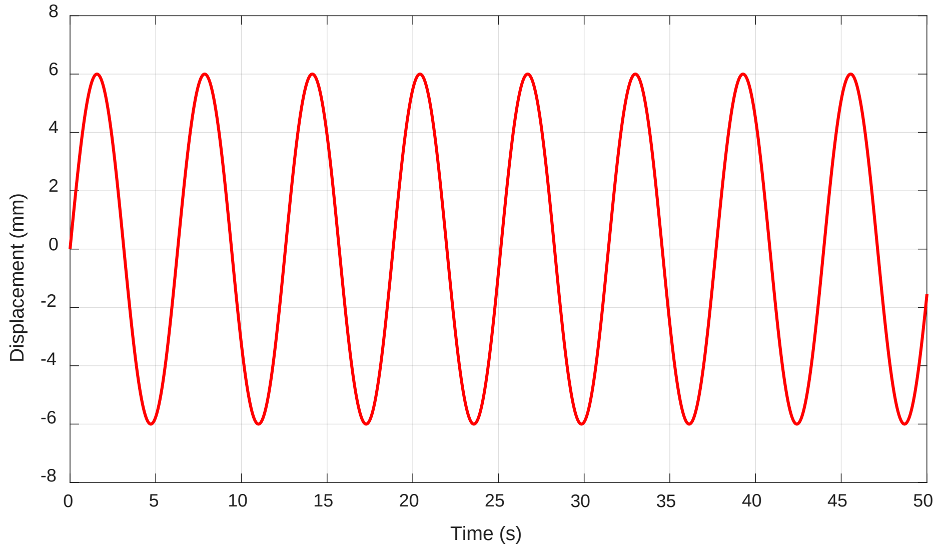
[1 POINT]

(e) The spring stiffness

[1 POINT]

(f) The dashpot coefficient

[1 POINT]

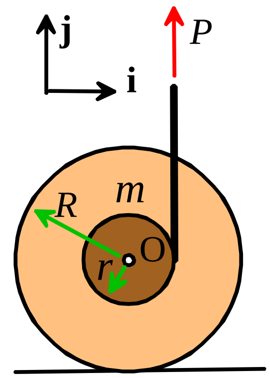


21.2 The figure shows the measured displacement of the system during operation. The blades have a radius of 40m, and the total mass of the system $(m + m_0)$ is 25000kg. Assuming that the rotor can be balanced by adding mass to the tip of one blade, estimate the mass that must be added to balance the rotor.

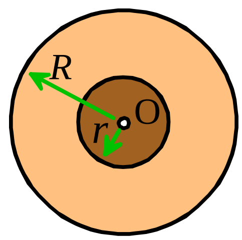
[4 POINTS]

22. The figure shows a spool (e.g. a yo-yo) with outer radius R, mass m and (2D) mass moment of inertia $I_{Gzz} = mR^2 / 2$ resting on a table. The hub has radius r. A constant vertical force P is applied to the yo-yo string.

The goal of this problem is to (i) find a formula for the (horizontal) acceleration of the spool, and (ii) find a formula for the critical value of P that will cause slip at the contact between the spool and the table



22.1 Draw a free body diagram showing the forces acting on the spool (please use the figure provided). Assume that the spool remains in contact with the surface, and that no slip occurs at the contact.



[3 POINTS]

22.2 Write down the equations of linear (Newton's law) and rotational (the moment-angular acceleration relation) motion. Your equation should include forces from 22.1, and the linear and angular acceleration of the spool. Please state which point you are taking moments about for the moment equation.

[2 POINTS]

22.3 Write down a relationship between the angular acceleration α_z and linear acceleration $\mathbf{a}_G$ of the center of mass of the spool

[1 POINT]

22.4 Use 22.2 and 22.3 to find formulas for (a) the angular acceleration and (b) the linear acceleration of the spool in terms of *P*, and other relevant variables.

[2 POINTS]

22.5 Find formulas for the reaction forces at the contact, in terms of *P*, *m*, *g*, *R* and *r*

[2 POINTS]

22.6 The contact has a friction coefficient μ. Find a formula for the critical value of P at the point where the contact begins to slip, in terms of μ, m, g and other relevant variables.

[2 POINTS]