



School of Engineering  
Brown University

## EN40: Dynamics and Vibrations

**Midterm Examination**  
**Thursday March 8 2018**

**NAME:** \_\_\_\_\_

### General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

### Please initial the statement below to show that you have read it

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

\_\_\_\_\_

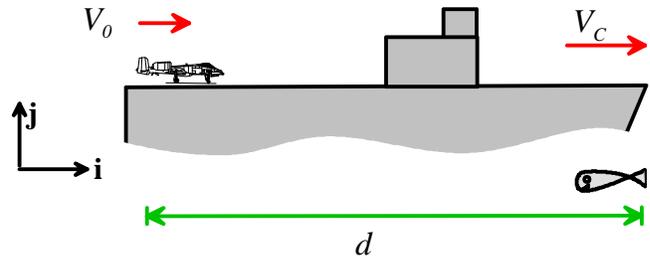
**1 (10 points)** \_\_\_\_\_

**2. (11 points)** \_\_\_\_\_

**3. (9 points)** \_\_\_\_\_

**TOTAL (30 points)** \_\_\_\_\_

1. An aircraft carrier travels with constant speed  $V_c$ . At time  $t=0$  an aircraft with mass  $m$  traveling with initial speed  $V_0$  (relative to a stationary origin) touches down on its deck. After touchdown, the aircraft is slowed by a drag force  $\mathbf{F}_D = -cv^2\mathbf{i}$ , where  $v$  is the aircraft's speed (relative to a stationary origin).



1.1 Write down a formula for the distance  $x_c$  traveled by the aircraft carrier for time  $t>0$ , in terms of  $V_c$

[1 POINT]

1.2 Find a formula for the speed of the aircraft (relative to a stationary origin) as a function of time (for  $t>0$ ), in terms of  $c, m, V_0$ .

[3 POINTS]

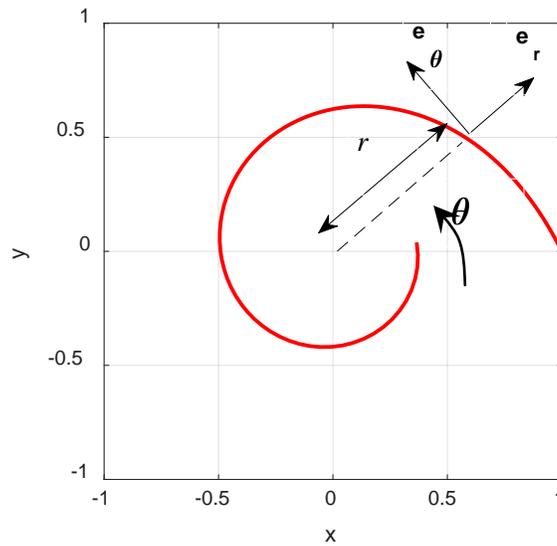
1.3 Find a formula for the distance traveled by the aircraft (relative to a stationary point) as a function of time. The following integral may be helpful

$$\int_0^t \frac{dt}{a+bt} = \frac{1}{b} \log\left(1 + \frac{b}{a}t\right)$$

**[2 POINTS]**

1.3 Hence, or otherwise, find formula for the minimum length  $d$  required for the aircraft to land without falling off the end of the deck, in terms of  $m, c, V_0, V_c$

**[4 POINTS]**



2. The figure shows the trajectory of a particle with mass  $m$  in an electric field. The polar coordinates of the particle vary with time  $t$  according to the formulas

$$r = e^{-t} \quad \theta = e^{2t} - 1$$

2.1 Find a formula for the velocity vector of the particle as a function of time, as components in the  $\{\mathbf{e}_r, \mathbf{e}_\theta\}$  basis

[1 POINT]

2.2 Find a formula for the acceleration vector of the particle as a function of time, as components in the  $\{\mathbf{e}_r, \mathbf{e}_\theta\}$  basis

[2 POINTS]

2.3 Show that the force on the particle acts in the  $\mathbf{e}_r$  direction, and show that the magnitude of the force as a function of  $r$  and  $m$  is

$$|\mathbf{F}| = m \left( r - \frac{4}{r^3} \right)$$

**[2 POINTS]**

2.4 Find a formula for the potential energy of the force acting on the particle (your answer may include an arbitrary constant if you wish)

**[2 POINTS]**

2.5 Find unit vectors normal and tangent to the path **at time  $t=0$** , as components in the  $\{\mathbf{e}_r, \mathbf{e}_\theta\}$  basis

**[2 POINTS]**

2.6 Find the normal and tangent components of acceleration **at time  $t=0$** .

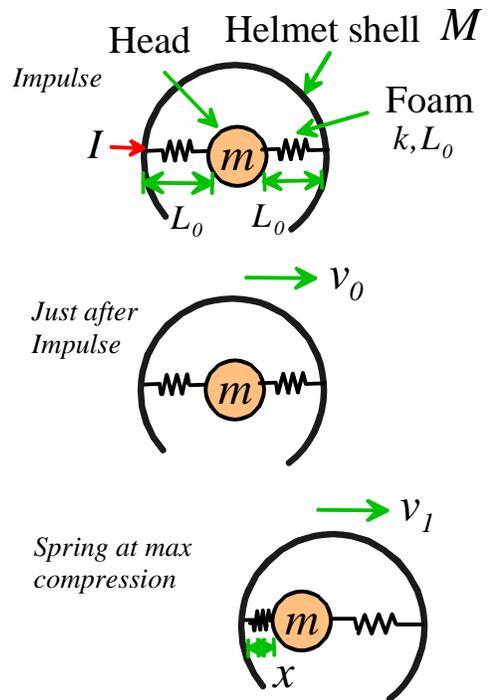
**[2 POINTS]**

3. The figure shows an idealization of a head inside a protective helmet. The helmet shell has mass  $M$ , and the protective foam inside the shell is idealized as springs with stiffness  $k$  and unstretched length  $L_0$

At time  $t=0$  the helmet and head are both at rest, and the length of the springs is equal to their unstretched length.

The helmet shell is then subjected to a horizontal impulse  $I$ , which causes it to move at speed  $v_0$  just after the impulse.

The springs exert no force on either the head or the helmet casing during the impulse. Gravity and vertical motion of the head or helmet may be neglected.



3.1. Find a formula for  $v_0$  in terms of  $I$  and  $M$ .

[1 POINT]

3.2. Find expressions for the total linear momentum and total kinetic energy of the system just after the impulse, in terms of  $I$  and the mass  $M$  of the shell

[2 POINTS]

3.3. Consider the system at the instant when the foam on the left of the head is compressed to its smallest thickness (i.e.  $x$  is a minimum). Using energy and/or momentum conservation, find a formula for  $x$  at this instant. Assume that the foam remains in contact with the head on both sides (so one spring is stretched), as indicated in the figure.

**[3 POINTS]**

3.4. Hence, find a formula for the minimum foam thickness  $L_0$  necessary to prevent the head from striking the helmet shell (i.e to ensure  $x > 0$ ), in terms of  $m, M, I, k$

**[1 POINT]**

3.5. Find a formula for the total impulse exerted on the head between  $t=0$  and the instant of maximum foam compression, in terms of  $I, M, m$ .

**[2 POINTS]**