School of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Sat May 12 2018: 9am-12:00pm

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1-20 [40 points] ______________

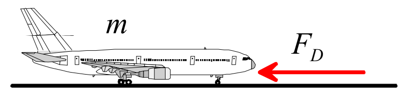
21 [10 POINTS] ______________

22 [10 POINTS] ______________

TOTAL [60 POINTS] ______________

FOR PROBLEMS 1-20 WRITE YOUR ANSWER IN THE SPACE PROVIDED. ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. The aircraft shown in the figure has mass m. While landing, it touches down with speed v_0 and it is slowed by a drag force with magnitude $F_D = cv^2$, where c is a constant and v is the aircraft's speed. The engine thrust is negligible. The time required for the speed to decrease to $v_0 / 2$ is given by:



(a) $m/(cv_0)$
(b) $m/(2cv_0)$
(c) $2m/(cv_0)$
(d) None of the above

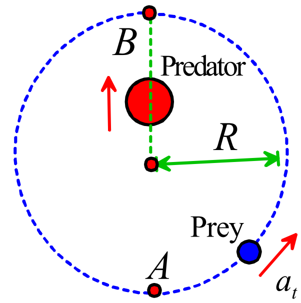
ANSWER______________ **(2 POINTS)**

2. An aircraft with mass 1000kg gliding without engine power at constant airspeed of 50m/s takes 1 hour to descend 3600m. The drag force on the aircraft is (approximately):

(a) $20N$
(b) $200N$
(c) $10000N$
(d) None of the above

ANSWER______________ **(2 POINTS)**

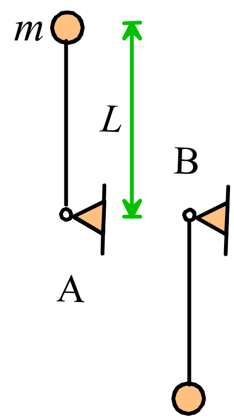
3. A 'prey' particle starts at rest at A at time *t=0* and for *t>0* moves around a circular path with constant tangential acceleration a_t. A 'predator' starts at rest at the center of the circle at *t=0* and for *t>0* moves with constant acceleration along a vertical straight-line path. To catch the prey (i.e. to arrive at B at the same time as the prey), the predator's acceleration must be



(a) $a_t / 2$
(b) a_t
(c) a_t / π
(d) $a_t / (2\pi)$
(e) None of the above

ANSWER______________ **(2 POINTS)**

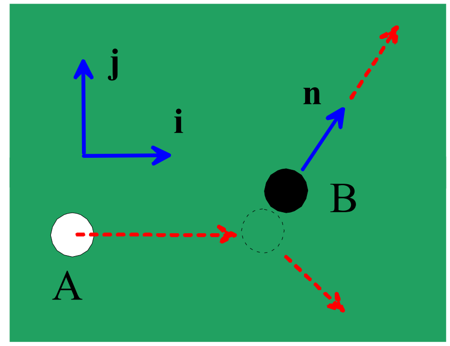
4. The pendulum shown in the figure starts at rest in the inverted vertical configuration (A) at time *t*=0. Following a small disturbance it falls over. At the instant when it swings through the vertical configuration (B), the magnitude of the acceleration of the mass *m* is (in terms of gravitational acceleration g)



(a) zero
(b) g
(c) 1.5g
(d) 2g
(e) None of the above

ANSWER______________ **(2 POINTS)**

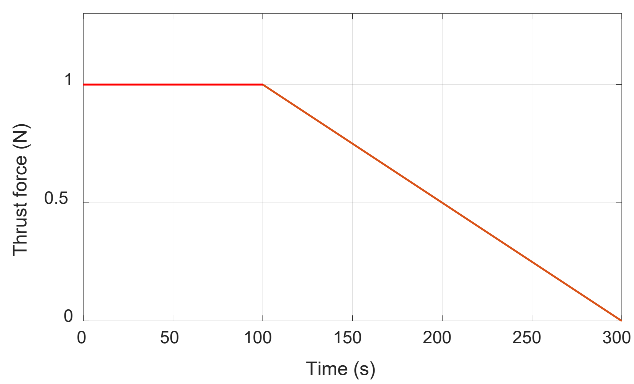
5. A frictionless collision takes place between two billiard balls. Before impact the black ball (B) is stationary and the white one (A) has velocity $V_0\mathbf{i}$. The normal vector parallel to the collision direction is $\mathbf{n} = (\mathbf{i} + \mathbf{j})/\sqrt{2}$ and the restitution coefficient for the collision is $e = 1$. After impact, the black ball has speed $V_0/\sqrt{2}$. The velocity vector of A after impact is



(a) Zero
(b) $V_0(\mathbf{i} - \mathbf{j})/\sqrt{2}$
(c) $V_0(\mathbf{i} - \mathbf{j})/2$
(d) $V_0\left(\mathbf{i}\left\{\sqrt{2} - 1\right\} - \mathbf{j}\right)/\sqrt{2}$
(e) None of the above

ANSWER_______________ **(2 POINTS)**

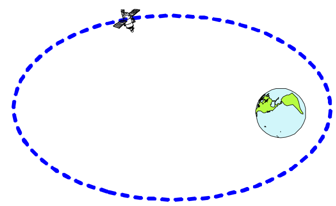
6. The figure shows the thrust force exerted by a model rocket motor as a function of time. The total impulse exerted by the motor is



(a) 300 Ns
(b) 250 Ns
(c) 200 Ns
(d) 150 Ns
(e) None of the above

ANSWER_______________ **(2 POINTS)**

7. The figure shows a satellite in orbit around the earth. **For a system consisting of only the satellite** (idealized as a particle), identify whether each statement below is true or false



(a) Linear momentum is conserved

TRUE FALSE

(b) Kinetic energy is conserved

TRUE FALSE

(c) Potential energy is conserved

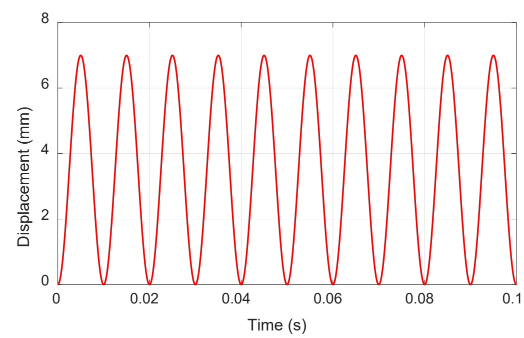
TRUE FALSE

(d) Angular momentum about the earth's center is conserved

TRUE FALSE

(2 POINTS)

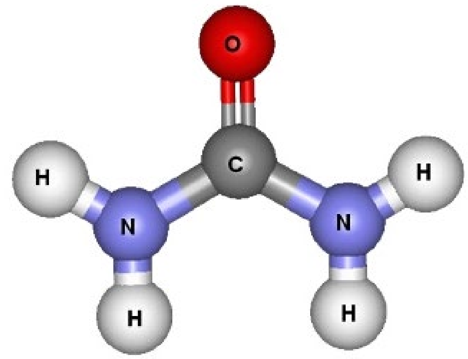
8. The figure shows a vibration signal from a displacement transducer. The amplitude of the acceleration is



(a) 2760 m/s^2
(b) 1380 m/s^2
(c) 4.39 m/s^2
(d) 2.20 m/s^2
(e) None of the above

ANSWER______________ **(2 POINTS)**

9. How many vibration modes does the urea molecule shown in the figure have?

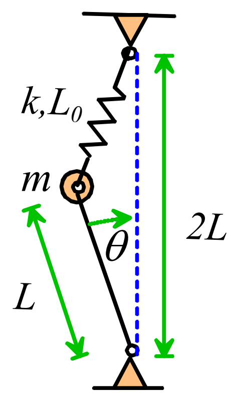


(a) 10
(b) 18
(c) 23
(d) 24
(e) None of the above

ANSWER_______________ **(2 POINTS)**

10. The equation of motion for the system shown in the figure is

$$mL^2 \frac{d^2\theta}{dt^2} + kL^2\left(1 - \frac{L_0}{\sqrt{5L^2 - 4L^2\cos\theta}}\right)\sin 2\theta - mgL\sin\theta = 0$$

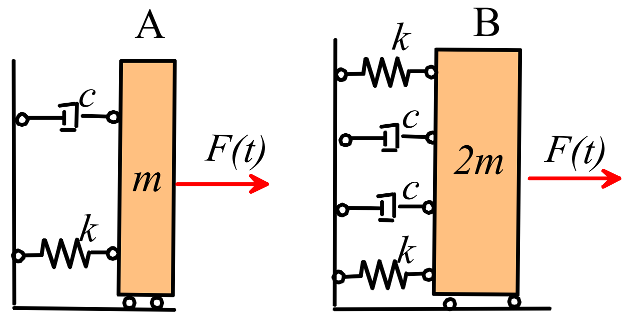


The natural frequency of small amplitude vibrations of the system is

(a) $\sqrt{\frac{k}{m}\left(1-\frac{L_0}{L}\right)-\frac{g}{L}}$
(b) $\sqrt{\frac{2k}{m}\left(1-\frac{L_0}{L}\right)-\frac{g}{L}}$
(c) $\sqrt{\frac{2k}{m}\left(1-\frac{L_0}{5L}\right)-\frac{g}{L}}$
(d) $\sqrt{\frac{k}{m}\left(1-\frac{L_0}{5L}\right)-\frac{g}{L}}$
(e) None of the above.

ANSWER_______________ **(2 POINTS)**

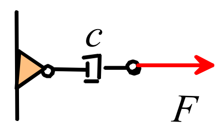
11. Systems A and B in the figure shown are subjected to the same harmonic force *F(t)*. The steady state amplitude of vibration of system A is measured to be 1mm. The steady-state vibration amplitude of system B is



(a) 0.5mm
(b) 1mm
(c) 2mm
(d) Cannot be determined without more information
(e) None of the above

ANSWER_______________ **(2 POINTS)**

12. A dashpot is stretched by a constant force F as shown in the figure. The rate of work done by the force is



(a) cF
(b) F / c
(c) F^2 / c
(d) cF^2
(e) None of the above

ANSWER_______________ **(2 POINTS)**

13. A rigid body is subjected to a sequence of two rotations.

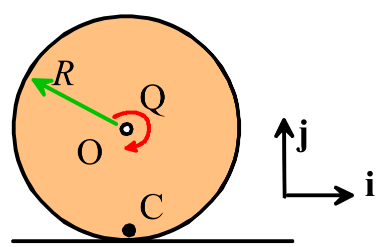
$$\mathbf{R}^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \mathbf{R}^{(2)} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The sequence of rotations $\mathbf{R} = \mathbf{R}^{(2)}\mathbf{R}^{(1)}$ represents

(a) A 120 degree rotation about an axis parallel to $\mathbf{n} = (-\mathbf{i} + \mathbf{j} - \mathbf{k}) / \sqrt{3}$
(b) A 30 degree rotation about an axis parallel to $\mathbf{n} = (-\mathbf{i} + \mathbf{j} - \mathbf{k}) / \sqrt{3}$
(c) A 45 degree rotation about an axis parallel to $\mathbf{k}$
(d) A 45 degree rotation about an axis parallel to $\mathbf{i}$
(e) None of the above.

ANSWER_______________ **(2 POINTS)**

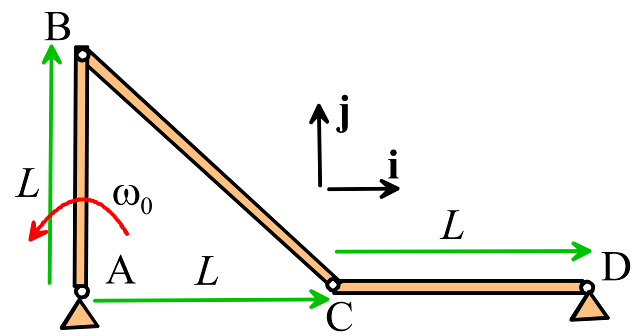
14. The center of the wheel moves with (instantaneous) velocity $\mathbf{v}_O = V\mathbf{i}$ and the wheel rolls without slip. A torque $\mathbf{Q} = -Q\mathbf{k}$ is exerted on the wheel by an axle attached to O. The rate of work done by the torque on the wheel is:



(a) Zero
(b) QV
(c) QV/R
(d) QVR
(e) None of the above

ANSWER_______________ **(2 POINTS)**

15. Member AB in the four-bar mechanism shown in the figure rotates with **constant** angular velocity $\boldsymbol{\omega}_{AB} = \omega_0 \mathbf{k}$. At the instant shown, the angular velocity and angular acceleration of member BC are



(a) $\boldsymbol{\omega}_{BC} = \omega_0 \mathbf{k}$ $\boldsymbol{\alpha}_{BC} = 2\omega_0^2 \mathbf{k}$

(b) $\boldsymbol{\omega}_{BC} = \omega_0 \mathbf{k}$ $\boldsymbol{\alpha}_{BC} = -2\omega_0^2 \mathbf{k}$

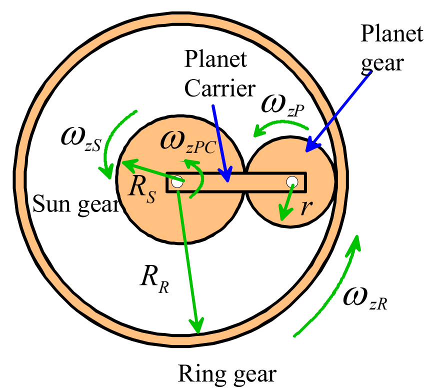
(c) $\boldsymbol{\omega}_{BC} = -\omega_0 \mathbf{k}$ $\boldsymbol{\alpha}_{BC} = \omega_0^2 \mathbf{k}$

(d) $\boldsymbol{\omega}_{BC} = \omega_0 \mathbf{k}$ $\boldsymbol{\alpha}_{BC} = \mathbf{0}$

(e) None of the above

ANSWER_______________ **(2 POINTS)**

16. With the sun gear fixed, the planet carrier and ring gears in the epicyclic gear system shown have a ratio $\omega_{zR} / \omega_{zPC} = 3/2$. If the sun gear has 20 teeth, the ring and planet gears have



(a) $N_R = 40, \quad N_P = 10$ teeth

(b) $N_R = 40, \quad N_P = 20$ teeth

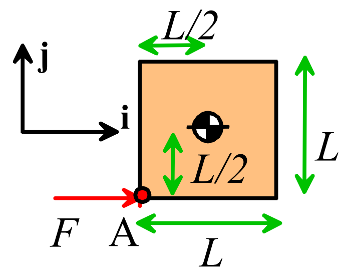
(c) $N_R = 80, \quad N_P = 20$ teeth

(d) $N_R = 80, \quad N_P = 30$ teeth

(e) None of the above

ANSWER_______________ **(2 POINTS)**

17. A square plate with mass m, side length L, and mass moment of inertia $mL^2/12$ is at rest for time $t<0$. At time $t=0$ a force F is applied at the corner located at point A in the figure. At the instant t=0 the acceleration vector of point A is



(a) $\mathbf{a} = (F/m)\mathbf{i}$

(b) $\mathbf{a} = 3(F/m)\mathbf{i}$

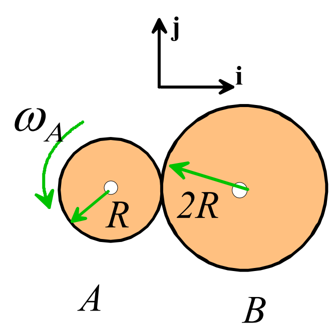
(c) $\mathbf{a} = 4(F/m)\mathbf{i} - 3(F/m)\mathbf{j}$

(d) $\mathbf{a} = 3(F/m)\mathbf{i} - 3(F/m)\mathbf{j}$

(e) None of the above

ANSWER_______________ **(2 POINTS)**

18. The two gears A and B in the figure have radii R and $2R$, and mass moments of inertia $mR^2/2$ and $2mR^2$, respectively. Their centers are stationary. If gear A rotates counterclockwise at angular speed ω_A, the total angular momentum of the system (including both gears) is



(a) zero

(b) $(mR^2)\omega_A\mathbf{k}$

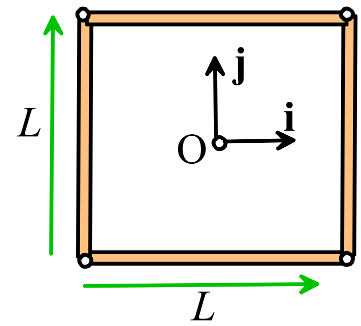
(c) $-(mR^2/2)\omega_A\mathbf{k}$

(d) $(3mR^2/2)\omega_A\mathbf{k}$

(e) None of the above

ANSWER_______________ **(2 POINTS)**

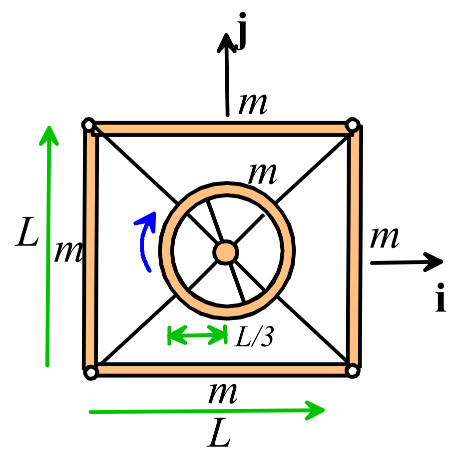
19. Four rods with mass m and length L are connected to form a square frame as shown in the figure. The mass moment of inertia of the frame about its center is



(a) $4mL^2/3$
(b) $5mL^2/6$
(c) $3mL^2/2$
(d) $5mL^2/6$
(e) None of the above

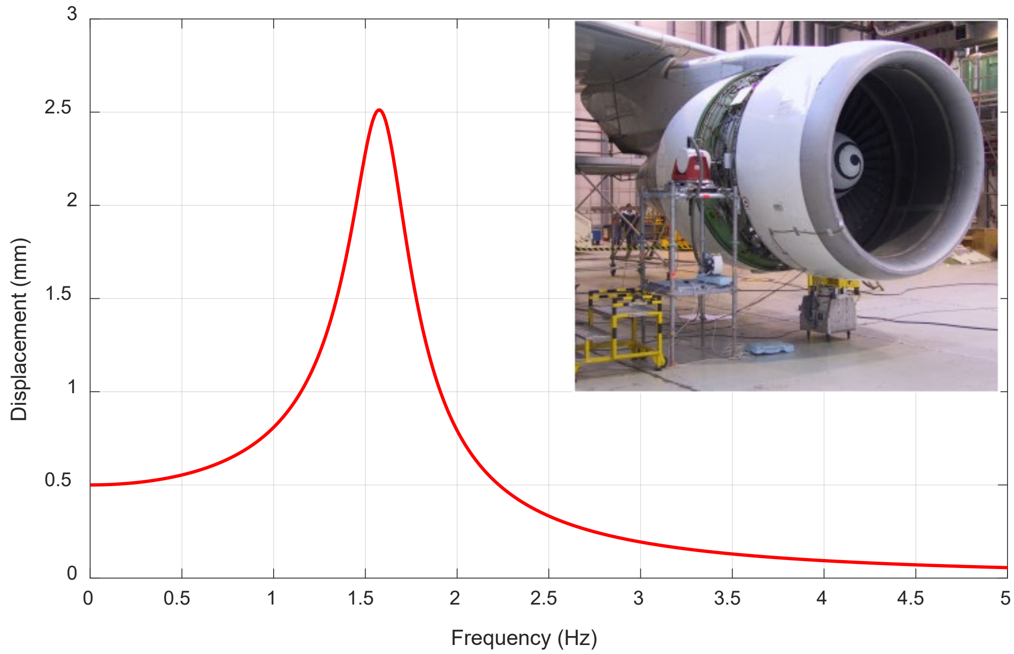
ANSWER_______________ **(2 POINTS)**

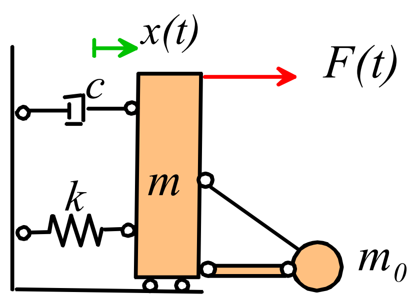
20. A spacecraft is idealized as a frame made up of 4 bars with length L and mass m. The orientation of the spacecraft is controlled by a momentum wheel that can be idealized as a thin ring with mass m and radius $R=L/3$. At time $t=0$ the spacecraft is at rest (in space) and the momentum wheel is stationary. A motor then spins up the momentum wheel to an angular velocity $-\omega_0\mathbf{k}$ **(relative to the frame)**. The angular velocity of the frame is



(a) $\omega_f = (\omega_0/10)\mathbf{k}$
(b) $\omega_f = (\omega_0/11)\mathbf{k}$
(c) $\omega_f = (\omega_0/12)\mathbf{k}$
(d) $\omega_f = (\omega_0/13)\mathbf{k}$
(e) None of the above

ANSWER_______________ **(2 POINTS)**





Rotor (prevented from rotating)

21 As part of the airworthiness certification process, the rotating parts of a jet engine are prevented from turning, and the engine is subjected to an external horizontal harmonic force $F(t) = F_0 \sin \omega t$ with amplitude $F_0 =$ 250N. The amplitude X_0 of the steady-state horizontal vibration $x(t) = X_0 \sin(\omega t + \phi)$ of the engine is measured.

The measured displacement amplitude X_0 is shown in the figure as a function of frequency (in cycles/sec).

21.1 Assuming that the engine and its mounting are idealized as a spring-mass-damper system (with light damping), use the graph provided to estimate values for the following quantities

(a) The natural frequency of vibration of the engine (give both the frequency in cycles per second and the angular frequency in rad/s)

[2 POINTS]

(b) The damping factor ζ. (Use the peak. Note that the graph shows the displacement amplitude, not magnification *M*)

[2 POINTS]

(c) The spring stiffness (use the displacement at very low frequency)

[1 POINT]

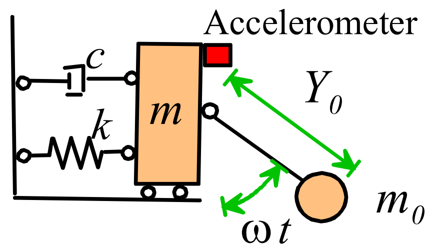
(d) The total mass $m + m_0$

[1 POINT]

(e) The dashpot coefficient c

[1 POINT]

21.2 During operation, the engine spins at 9550 rpm. An accelerometer mounted on the outside of the engine measures a harmonic acceleration with amplitude 10 m/s^2. What is the amplitude of the displacement?



[1 POINT]

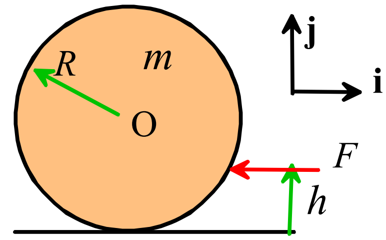
21.3 What is the engine speed (in rpm) at which the steady-state displacement amplitude will be a maximum?

[1 POINT]

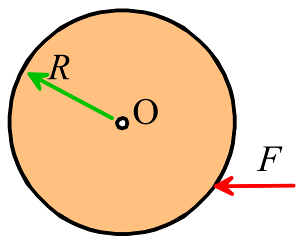
21.4 What is the steady-state displacement amplitude when the engine runs at the speed in 21.3?

[1 POINT]

22. The figure shows a spherical billiard ball with mass *m*, radius *R* and mass moment of inertia $I_{Gzz} = 2mR^2 / 5$ at rest on a pool table. It is subjected to a horizontal force *F* by a cue at a height *h* above the table.



2.1 Draw a free body diagram showing all the forces acting on the ball (include gravity and assume no slip at the contact)



[2 POINTS]

2.2 Write down the equations of motion relating the forces to the (unknown) linear and angular accelerations of the spheres. Please state what points you take moments about for the equation for rotational motion.

[2 POINTS]

2.3 Write down the kinematics equation relating the acceleration of the center at O and the angular acceleration of the sphere

[1 POINT]

2.4 Hence, find a formula for the acceleration of the center of the sphere in terms of F,h,R,m.

[2 POINTS]

2.5 Find a formula for the reaction forces at the contact in terms of *m,g,h,R* and *F*. If the coefficient of friction at the contact is μ , find a formula for the critical value of *F* that will cause slip at the contact, in terms of *m,g,h,R*.

[2 POINTS]

2.6 Where should the ball be struck to guarantee no slip for any value of *F*?

[1 POINT]