School of Engineering
Brown University

EN40: Dynamics and Vibrations

Midterm Examination
Thursday March 7 2019

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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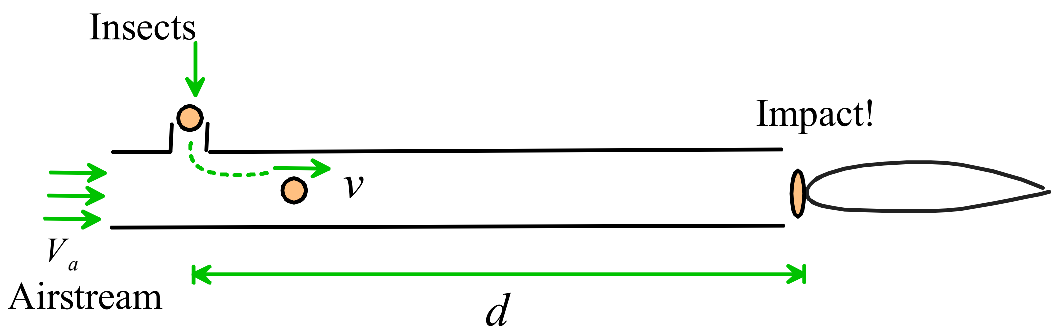
1 (8 points) ______________

2. (10 points) ______________

3. (10 points) ______________

4. (7 points) ______________

TOTAL (35 points)____________



1. In an experiment to study the effects of insect debris on lift generated by aircraft wings, insects are injected into an air-stream that flows with constant speed V_a . The insects are at rest at time *t=0* and have a speed dependent acceleration

$$a = \omega(V_a - v)$$

where ω is a constant (with dimensions of 1/time), and v is the insects speed. The insects collide with a stationary airfoil a distance d downstream.

1.1 Find an expression for the speed of an insect as a function of time

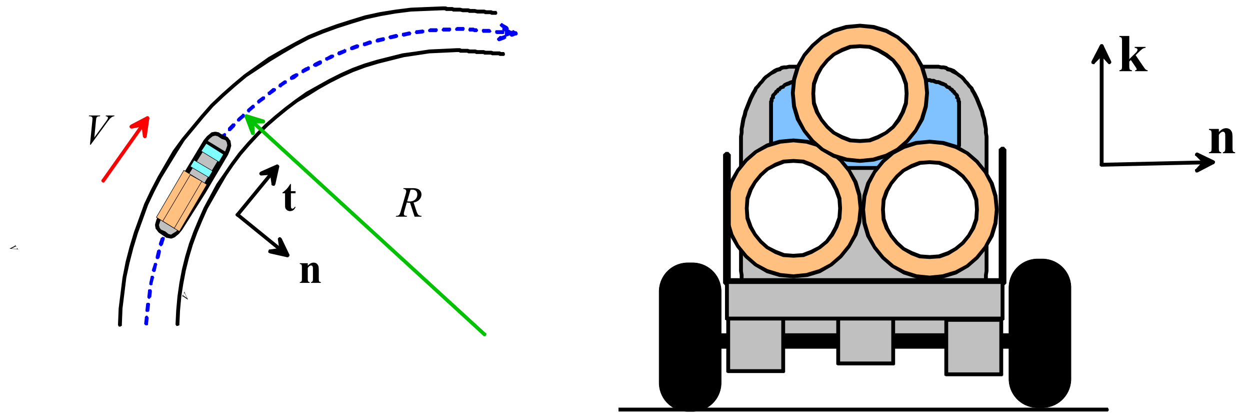
[3 POINTS]

1.2 Find an expression for the distance traveled by the insect as a function of time

[2 POINTS]

1.3. The insects must reach the airfoil with speed exceeding $V_a / 2$. Find a formula for the minimum value of the distance *d* necessary to reach the reach this speed, in terms of V_a and ω .

[3 POINTS]

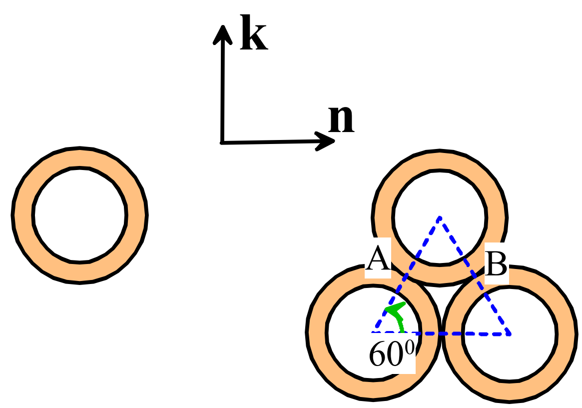


2. A flatbed truck carrying three long cylindrical water-pipes travels around a curved road with radius R at constant speed V. The pipes are arranged on the truckbed as shown in the figure. The goal of this problem is to determine the maximum allowable speed that will ensure that the **topmost** cylinder does not roll off the truck as it negotiates the bend.

2.1 Write down the acceleration of the vehicle in terms of V and R, using the normal-tangential coordinate system shown in the figure.

[1 POINT]

2.2 Draw the forces acting on the topmost cylinder on the diagram on the left below (the figure on the right is provided for information). **Neglect friction** (but do include gravity!)



[2 POINTS]

2.3 Write down Newton's law in the normal-tangential-vertical coordinate system, and hence find formulas for the (normal) reaction forces acting at the two contact points A and B shown in Fig 2.2

[3 POINTS]

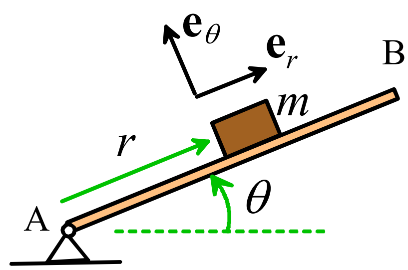
2.4 Hence, find the critical speed at which the topmost cylinder will just start to roll off the truck

[2 POINTS]

2.5 If friction acts at the contact points, will this change the critical speed? Please explain your reasoning with the aid of relevant calculations (and possibly a new free body diagram).

[2 POINTS]

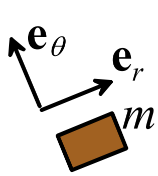
3. The figure shows an idealized model of the centrifugal pump discussed in class. The bar AB has negligible mass, and rotates in the horizontal plane about A. The rotation of the bar is driven by a motor that supplies a constant power P to the system (**NB: the bar does not rotate at constant angular speed**). A particle with mass m slides up the bar. **Friction between the bar and mass can be neglected.**



3.1 Write down formulas for the velocity and acceleration vectors of the mass, in terms of $\theta, \frac{d\theta}{dt}, \frac{d^2\theta}{dt^2}, r, \frac{dr}{dt}, \frac{d^2 r}{dt^2}$, using the polar coordinate system shown.

[2 POINTS]

3.2 Draw a free body diagram showing the forces acting on the mass m. (Neglect friction and gravity).



[2 POINTS]

3.3 Write down Newton's law **F** = m**a** for the mass using the polar coordinate system.

[1 POINT]

3.4 Hence, show that the rate of work done by the force (i.e. the power of the force) acting on the mass is related to r, θ by

$$P = m\left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt} \right) r\frac{d\theta}{dt}$$

[1 POINT]

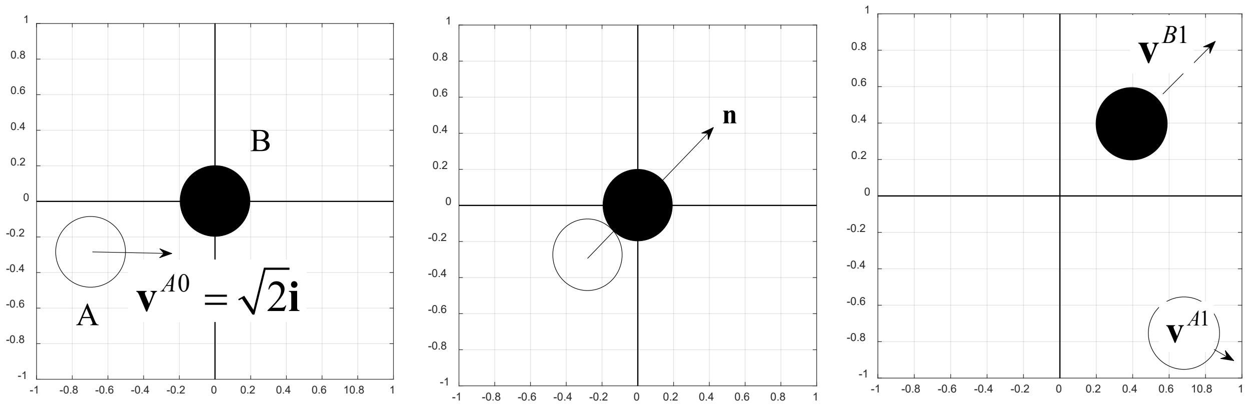
3.5 The work done by the forces on the mass must equal the constant power P supplied by the motor. Use this (and 3.3) to show that θ and r must satisfy

$$\frac{d^2\theta}{dt^2} = \frac{P}{mr^2\left(\frac{d\theta}{dt}\right)} - \frac{2}{r}\frac{dr}{dt}\frac{d\theta}{dt} \qquad\qquad \frac{d^2 r}{dt^2} = r\left(\frac{d\theta}{dt}\right)^2$$

[2 POINTS]

3.6 Re-write the equations in part (3.5) into a form that can be solved for r and θ using the MATLAB ode45 function.

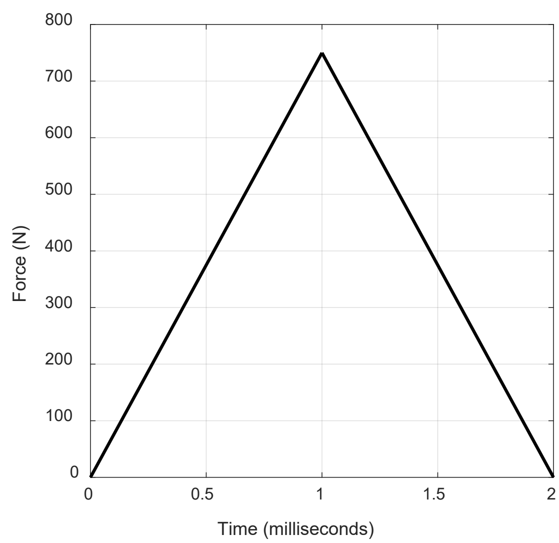
[2 POINTS]



Before impact At impact After impact

4. The figure shows a frictionless oblique impact between two spheres. The spheres both have mass 1 kg. At time $t=0$ sphere B is at rest, and sphere A has velocity vector $\mathbf{v}^{A0} = \sqrt{2}\mathbf{i}$ m/s. The collision direction $\mathbf{n}$ is at 45 degrees to the $\mathbf{i}$ and $\mathbf{j}$ directions.

4.1 The figure shows the magnitude of the force acting on the particles at the point of contact during the collision. Calculate the magnitude of the impulse.



[1 POINT]

4.2 Explain why the direction of the impulse on each individual particle during the collision must be parallel to $(\mathbf{i}+\mathbf{j})$

[1 POINT]

4.3 Hence, calculate the velocity vectors for particles A and B after the collision

[3 POINTS]

4.4 Calculate the restitution coefficient for the collision.

[2 POINTS]