School of Engineering
Brown University

EN40: Dynamics and Vibrations

Midterm Examination
Thursday March 7 2019

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

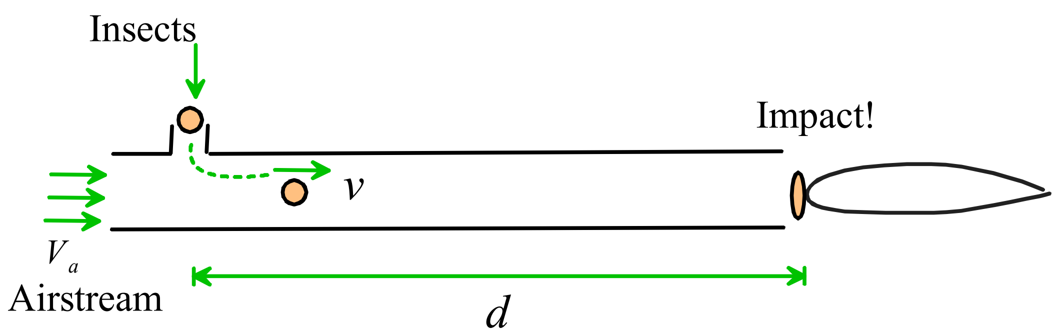
1 (8 points) _____________

2. (11 points) _____________

3. (9 points) _____________

4. (7 points) _____________

TOTAL (35 points) ___________



1. In an experiment to study the effects of insect debris on lift generated by aircraft wings, insects are injected into an air-stream that flows with constant speed V_a . The insects are at rest at time $t=0$ and have a speed dependent acceleration

$$a = \omega(V_a - v)$$

where ω is a constant (with dimensions of 1/time), and v is the insects speed. The insects collide with a stationary airfoil a distance d downstream.

1.1 Find an expression for the speed of an insect as a function of time

$$\frac{dv}{dt} = \omega(V_a - v)$$

$$\Rightarrow \int_0^v \frac{dv}{(V_a - v)} = \int_0^t \omega dt \Rightarrow -\log\left[V_a - v\right]\Big|_0^v = \omega t$$

$$\Rightarrow -\log\left[V_a - v\right] + \log(V_a) = -\log\frac{V_a - v}{V_a} = \omega t$$

$$\Rightarrow V_a - v = V_a \exp(-\omega t) \Rightarrow v = V_a\left(1 - \exp(-\omega t)\right)$$

[3 POINTS]

1.2 Find an expression for the distance traveled by an insect as a function of time

$$\frac{dx}{dt} = v = V_a\left[1-\exp(-\omega t)\right]$$
$$\Rightarrow \int_0^x dx = \int_0^t V_a\left[1-\exp(-\omega t)\right]dt$$
$$\Rightarrow x = V_a\left[t+\frac{1}{\omega}\exp(-\omega t)\right]_0^t = V_a\left[t+\frac{1}{\omega}\left\{\exp(-\omega t)-1\right\}\right]$$

[2 POINTS]

1.3. The insects must reach the airfoil with speed exceeding $V_a/2$. Find a formula for the minimum value of the distance *d* necessary to reach the reach this speed, in terms of V_a and ω .

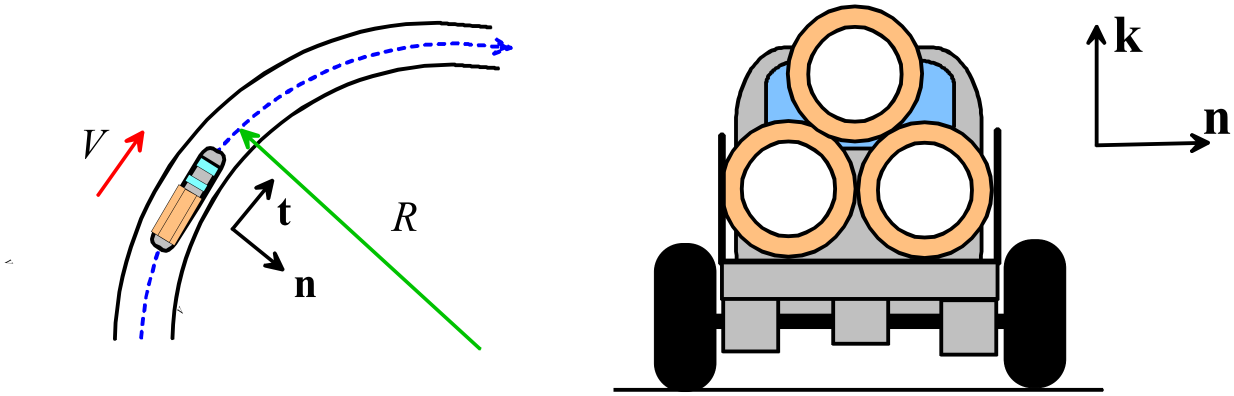
We can calculate the time required to reach the necessary speed from 1.1

$$V_a/2 = V_a(1-\exp\left(-\omega t\right)) \Rightarrow 1-\exp(-\omega t) = \frac{1}{2} \Rightarrow t = -\frac{1}{\omega}\log(1/2) = \frac{1}{\omega}\log(2)$$

Substitute back into 1.2 to see that

$$d = \frac{V_a}{\omega}\left[\log(2)-1/2\right] = 0.1931\frac{V_a}{\omega}$$

[3 POINTS]



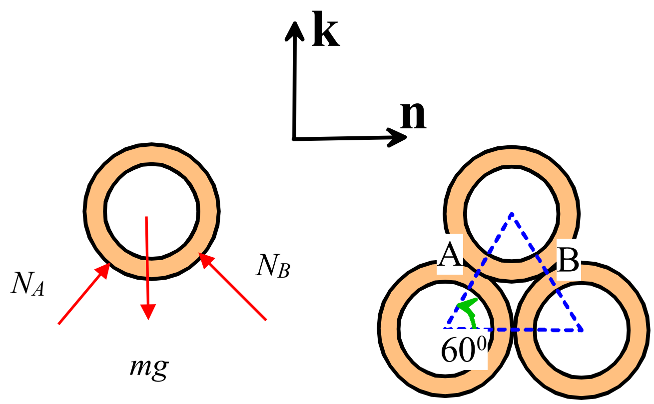
2. A flatbed truck carrying three long cylindrical water-pipes travels around a curved road with radius R at constant speed V. The pipes are arranged on the truckbed as shown in the figure. The goal of this problem is to determine the maximum allowable speed that will ensure that the **topmost** cylinder does not roll off the truck as it negotiates the bend.

2.1 Write down the acceleration of the vehicle in terms of V and R, using the normal-tangential coordinate system shown in the figure.

Use the circular motion formula $\mathbf{a} = \frac{V^2}{R}\mathbf{n}$

[1 POINT]

2.2 Draw the forces acting on the topmost cylinder on the diagram below (the figure on the right is provided for information). **Neglect friction** (but do include gravity!)



[2 POINTS]

2.3 Write down Newton's law in the normal-tangential-vertical coordinate system, and hence find formulas for the (normal) reaction forces acting at the two contact points A and B

$$\mathbf{F} = m\mathbf{a} \Rightarrow \left(N_A - N_B\right)\cos 60\mathbf{n} + \left[\left(N_A + N_B\right)\sin 60 - mg\right]\mathbf{k} = m\frac{V^2}{R}\mathbf{n}$$

The two components of this equation give

$$\left(N_A - N_B\right)\frac{1}{2} = m\frac{V^2}{R}$$

$$\left(N_A + N_B\right)\frac{\sqrt{3}}{2} = mg$$

$$\Rightarrow \sqrt{3}N_A = \sqrt{3}m\frac{V^2}{R} + mg \Rightarrow N_A = m\frac{V^2}{R} + \frac{mg}{\sqrt{3}}$$

$$\sqrt{3}N_B = mg - \sqrt{3}m\frac{V^2}{R} \Rightarrow N_B = \frac{mg}{\sqrt{3}} - m\frac{V^2}{R}$$

[4 POINTS]

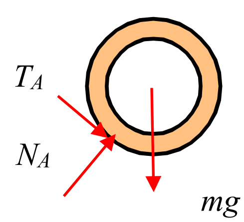
2.4 Hence, find the critical speed at which the topmost cylinder will just start to roll off the truck

The pipe will roll if either reaction force is negative (the negative reaction force is impossible). At the critical speed the reaction force is zero. Clearly the reaction at A will never be negative, so the condition is

$$N_B = \frac{mg}{\sqrt{3}} - m\frac{V^2}{R} = 0 \Rightarrow V = \sqrt{\frac{gR}{\sqrt{3}}}$$

[2 POINTS]

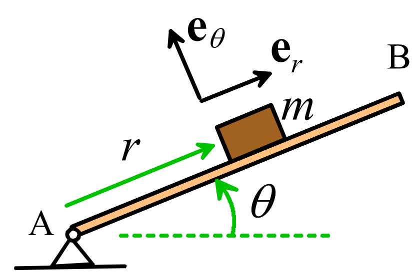
2.5 If friction acts at the contact points, will this change the critical speed? Please explain your reasoning with the aid of relevant calculations (and possibly a new free body diagram).



The figure shows a FBD at the critical speed (when the normal force at B is zero), but including a friction force at A. Taking moments about the COM of the cylinder shows that $T_A = 0$. This means that friction makes no difference.

[2 POINTS]

3. The figure shows an idealized model of the centrifugal pump discussed in class. The bar AB has negligible mass, and rotates in the horizontal plane about A. The rotation of the bar is driven by a motor that supplies a constant power P to the system (**NB: the bar does not rotate at constant angular speed)**. A particle with mass m slides up the bar. **Friction between the bar and mass can be neglected.**



3.1 Write down formulas for the velocity and acceleration vectors of the mass, in terms of $\theta, \frac{d\theta}{dt}, \frac{d^2\theta}{dt^2}, r, \frac{dr}{dt}, \frac{d^2r}{dt^2}$, using the polar coordinate system shown.

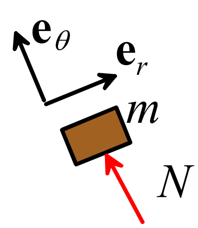
Formulas from the notes

$$\mathbf{v} = r\mathbf{e}_r + r\frac{d\theta}{dt}\mathbf{e}_\theta$$

$$\mathbf{a} = \left(\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)\mathbf{e}_r + \left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\mathbf{e}_\theta$$

[2 POINTS]

3.2 Draw a free body diagram showing the forces acting on the mass *m*. (Neglect friction and gravity).



[2 POINTS]

3.3 Write down Newton's law **F** = m**a** for the mass using the polar coordinate system.

$$N\mathbf{e}_\theta = m\left(\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)\mathbf{e}_r + m\left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\mathbf{e}_\theta$$

[1 POINT]

3.4 Hence, show that the rate of work done by the force acting on the mass is related to r, θ by

$$P = m\left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)r\frac{d\theta}{dt}$$

The rate of work is

$$\begin{aligned} P &= \mathbf{F}\cdot\mathbf{v} \\ &= m\left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)r\frac{d\theta}{dt} \end{aligned}$$

3.5 The work done by the forces on the mass must equal the constant power P supplied by the motor. Use this (and 3.3) to show that θ and r must satisfy

$$\frac{d^2\theta}{dt^2} = \frac{P}{mr^2\left(\frac{d\theta}{dt}\right)} - \frac{2}{r}\frac{dr}{dt}\frac{d\theta}{dt}$$

$$\frac{d^2r}{dt^2} = r\left(\frac{d\theta}{dt}\right)^2$$

[2 POINTS]

The first equation comes from rearranging (3.4). The second equation comes from the $\mathbf{e}_r$ component of 3.3.

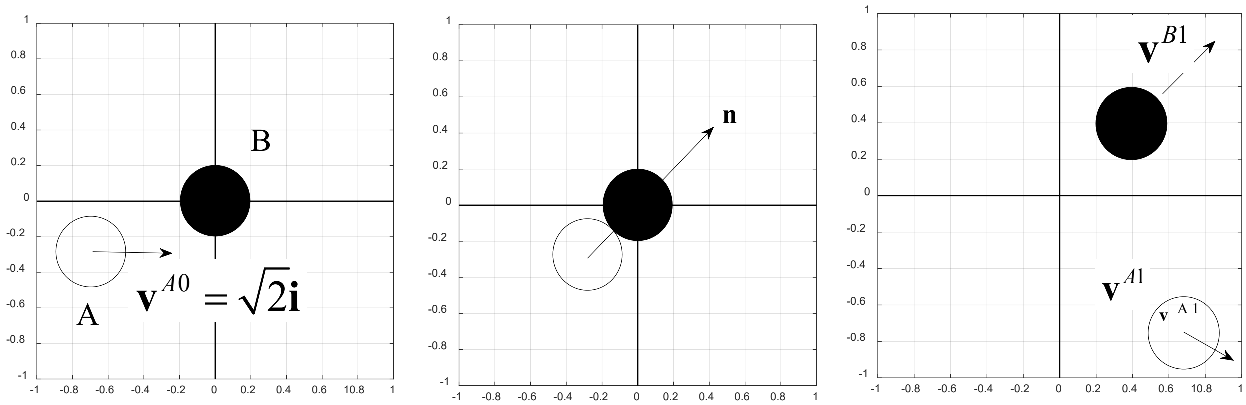
3.6 Re-write the equations in part (3.5) into a form that can be solved for r and θ using the MATLAB ode45 function.

Following the usual procedure we introduce $\omega = \frac{d\theta}{dt}$ $\quad v = \frac{dr}{dt}$ as additional variables, and solve for $[r, v, \theta, \omega]$

Re-writing the equations in terms of these variables gives

$$\frac{d}{dt}\begin{bmatrix} r \\ v \\ \theta \\ \omega \end{bmatrix} = \begin{bmatrix} v \\ r\omega^2 \\ \omega \\ P/(mr^2\omega) - 2v\omega/r \end{bmatrix}$$

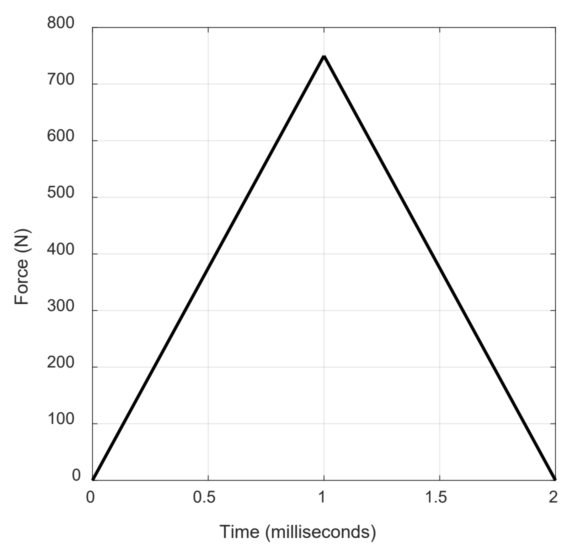
[2 POINTS]



Before impact At impact After impact

4. The figure shows a frictionless oblique impact between two spheres. The spheres both have mass 1 kg. At time *t=0* sphere B is at rest, and sphere *A* has velocity vector $\mathbf{v}^{A0} = \sqrt{2}\mathbf{i}$ m/s. The collision direction **n** is at 45 degrees to the **i** and **j** directions.

4.1 The figure shows the magnitude of the force acting on the particles at the point of contact during the collision. Calculate the magnitude of the impulse.



The impulse is the area under the curve, i.e. 0.75 Ns.

[1 POINT]

4.2 Explain why the direction of the impulse on each individual particle during the collision must be parallel to $(\mathbf{i}+\mathbf{j})$

The contact force during the impact acts parallel to the line connecting the two centers (because the impact is frictionless).

[1 POINT]

4.3 Hence, calculate the velocity vectors for particles A and B after the collision

We can use the impulse-momentum relations for a single particle.

For A $\mathbf{I}_A = m(\mathbf{v}^{A1} - \mathbf{v}^{A0}) \Rightarrow -\frac{3}{4}\frac{(\mathbf{i}+\mathbf{j})}{\sqrt{2}} = 1\times\left(\mathbf{v}^{A1} - \sqrt{2}\mathbf{i}\right) \Rightarrow \mathbf{v}^{A1} = -\frac{3}{4}\frac{(\mathbf{i}+\mathbf{j})}{\sqrt{2}} + \frac{2\mathbf{i}}{\sqrt{2}} = \frac{5\mathbf{i}-3\mathbf{j}}{4\sqrt{2}}$

For B $\mathbf{I}_B = m\left(\mathbf{v}^{B1} - \mathbf{v}^{B0}\right) \Rightarrow \frac{3}{4}\frac{(\mathbf{i}+\mathbf{j})}{\sqrt{2}} = 1\times\mathbf{v}^{B1}$

[3 POINTS]

4.4 Calculate the restitution coefficient for the collision.

We can calculate the restitution coefficient from the normal components of velocity

$$e = -\frac{\left(\mathbf{v}^{B1} - \mathbf{v}^{A1}\right)\cdot\mathbf{n}}{\left(\mathbf{v}^{B0} - \mathbf{v}^{A0}\right)\cdot\mathbf{n}} = -\frac{\left(-2\mathbf{i}+6\mathbf{j}\right)/(4\sqrt{2})\cdot(\mathbf{i}+\mathbf{j})/\sqrt{2}}{(-\mathbf{i}\sqrt{2})\cdot(\mathbf{i}+\mathbf{j})/\sqrt{2}} = \frac{1}{2}$$

[2 POINTS]