School of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Fri May 10 2019: 9am-12:00pm

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1-20 [40 points] _____________

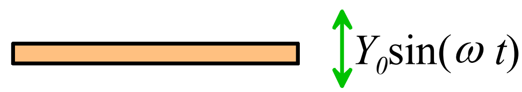
21 [10 POINTS] _____________

22 [10 POINTS] _____________

TOTAL [60 POINTS] _____________

FOR PROBLEMS 1-20 WRITE YOUR ANSWER IN THE SPACE PROVIDED. ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. The platform shown in the figure vibrates in the vertical direction with a harmonic displacement $y(t) = Y_0 \sin \omega t$. The amplitude of its vertical acceleration is



(a) $Y_0 \omega^2$

(b) Y_0 / ω^2

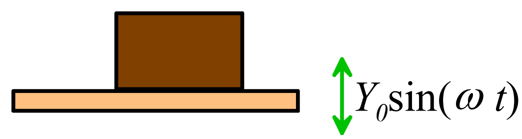
(c) $Y_0 \omega$

(d) Y_0 / ω

(e) None of the above

ANSWER________A______ **(2 POINTS)**

2. The platform shown in the figure vibrates in the vertical direction with a harmonic displacement $y(t) = Y_0 \sin \omega t$, with $\omega = 100$ rad/s. For each value of the amplitude listed below, indicate (by circling the appropriate response) whether the mass will remain in contact with the platform

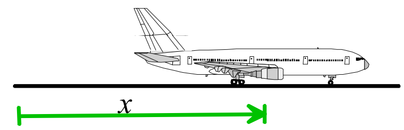


(a) 0.01mm	Remains in contact	Loses contact
(b) 0. 1mm	Remains in contact	Loses contact
(c) 1mm	Remains in contact	Loses contact
(d) 10mm	Remains in contact	Loses contact

(2 POINTS)

For the mass to remain in contact the normal force at the contact must remain positive. Newtons law shows that $N - mg = mY_0\omega^2 \sin \omega t \Rightarrow N \geq m(g - Y_0\omega^2)$ so $Y_0 < g / \omega^2 = 0.00098m = 0.98mm$

3. The aircraft shown in the figure starts at rest and has and acceleration that depends on its speed v as $a = A_0\left\{1-(v^2/v_0^2)\right\}$, where A_0 and v_0 are constants. After the aircraft has traveled a distance x, its speed is



(a) $v=\sqrt{2A_0 x}$

(b) $v=\sqrt{2A_0 x\left\{1-(v^2/v_0^2)\right\}}$

(c) $v=v_0\sqrt{1-\exp(-2A_0 x/v_0^2)}$

(d) $v=v_0\sqrt{1-\exp(-A_0 x/v_0^2)}$

(e) None of the above

ANSWER________C______ **(2 POINTS)**

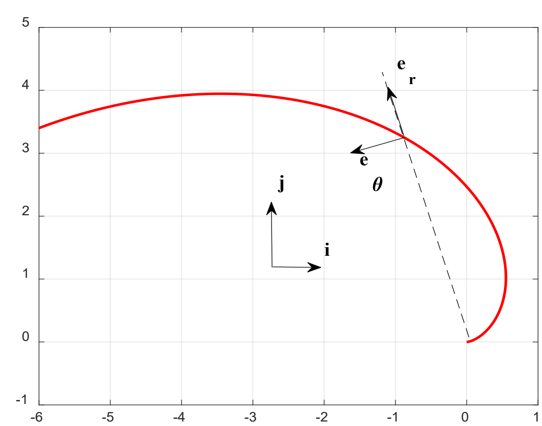
The acceleration-distance relation gives

$$v\frac{dv}{dx}=A_0(1-v^2/v_0^2)\Rightarrow\int_0^v\frac{vdv}{(1-v^2/v_0^2)}=\int_0^x A_0 dx\Rightarrow\left[\frac{-v_0^2}{2}\log(1-v^2/v_0^2)\right]_0^v=A_0 x\Rightarrow v=v_0\sqrt{1-\exp(-2A_0 x/v_0^2)}$$

4. The trajectory of a particle is specified in polar coordinates as a function of time as

$$r=t^2 \qquad \theta=t$$

At time $t=1$s, the particle's velocity vector is



(a) $\mathbf{v}=\mathbf{e}_r+\mathbf{e}_\theta$

(b) $\mathbf{v}=\frac{1}{2}\mathbf{e}_r+\mathbf{e}_\theta$

(c) $\mathbf{v}=2\mathbf{i}+\mathbf{j}$

(d) $\mathbf{v}=2\mathbf{e}_r+\mathbf{e}_\theta$

(e) None of the above

ANSWER________D______ **(2 POINTS)**

The polar coordinate formula for velocity $\mathbf{v}=(dr/dt)\mathbf{e}_r+r(d\theta/dt)\mathbf{e}_\theta$ gives (d)

5. The trajectory of a particle is specified in polar coordinates as a function of time as

$$r = t^2 \qquad \theta = t$$

At time $t=1$s, the particle's acceleration vector is

(a) $\mathbf{a} = 2\mathbf{e}_r + 4\mathbf{e}_\theta$

(b) $\mathbf{a} = \mathbf{e}_r + 2\mathbf{e}_\theta$

(c) $\mathbf{a} = \mathbf{e}_r + 4\mathbf{e}_\theta$

(d) $\mathbf{a} = -\mathbf{e}_r$

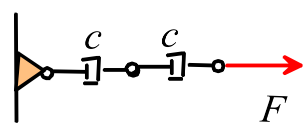
(e) None of the above

ANSWER________C______ **(2 POINTS)**

The polar coordinate formula for acceleration

$\mathbf{a} = \left\{(d^2r/dt^2) - r(d\theta/dt)^2\right\}\mathbf{e}_r + \left\{rd^2\theta/dt^2 + 2(dr/dt)(d\theta/dt)\right\}\mathbf{e}_\theta$ gives (c)

6. Two dashpots connected in series are stretched by a constant force F as shown in the figure. The rate of work done by the force is



(a) cF

(b) $2F^2/c$

(c) F/c

(d) $F^2/(2c)$

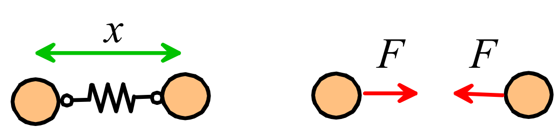
(e) None of the above

ANSWER______B________ **(2 POINTS)**

The two dashpots are in series so their effective coefficient is $c_{eff} = c/2$. The stretch rate is $dL/dt = F/c_{eff}$ and the rate of work done by the force is $F(dL/dt) = 2F^2/c$

7. The bond between atoms in a diatomic molecule has a potential energy given by

$$U = -F_0 d\left\{\left(\frac{x}{d}\right)^{-2} - \left(\frac{x}{d}\right)^{-4}\right\}$$



where x is the separation between the atoms and F_0 and d are constants. When $x = 2d$ the attractive force between the atoms is

(a) $F = F_0/2$

(b) $F = F_0/4$

(c) $F = F_0/8$

(d) $F = F_0/16$

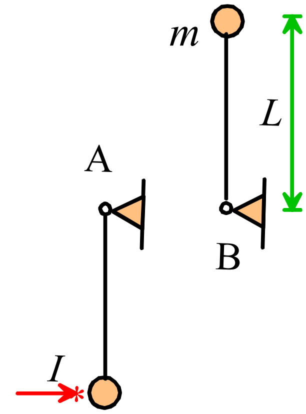
(e) None of the above

ANSWER______C________ **(2 POINTS)**

$$F = -\frac{dU}{dx} = -F_0\frac{d}{x}\left\{2\left(\frac{x}{d}\right)^{-2} - 4\left(\frac{x}{d}\right)^{-4}\right\} = -F_0\frac{1}{2}\left\{\frac{2}{4} - \frac{4}{16}\right\} = -\frac{F_0}{8}$$

. Negative forces are attractive.

8. At time $t=0$ the pendulum shown in the figure is at rest. It is then subjected to a horizontal impulse with magnitude I. During its subsequent motion it just comes to rest in the inverted configuration shown. The magnitude of the impulse is

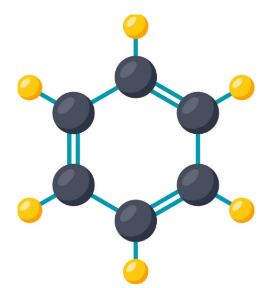


(a) $I = m\sqrt{2gL}$
(b) $I = 2mgL$
(c) $I = mgL$
(d) $I = 2m\sqrt{gL}$
(e) None of the above

ANSWER______D________ **(2 POINTS)**

Energy conservation after the impact gives $\frac{1}{2}mV^2 = 2mgL \Rightarrow V = 2\sqrt{gL}$. Impulse-momentum gives $I = mV = 2m\sqrt{gL}$

9. How many degrees of freedom and vibration modes has the molecule shown in the figure?



(a) 18 DOF and 24 vibration modes
(b) 24 DOF and 18 vibration modes
(c) 36 DOF and 30 vibration modes
(d) 30 DOF and 36 vibration modes
(e) None of the above

ANSWER______C________ **(2 POINTS)**

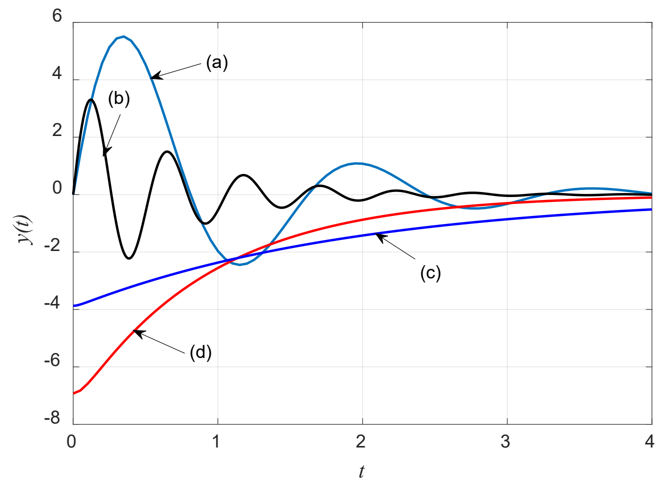
There are 12 atoms; the formula gives $3N = 36DOF$ and 30 vibration modes

10. Which curve in the figure shows a solution to the differential equation

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+16y=0$$

Select (a)-(d) or

(e) none of the curves are a correct solution



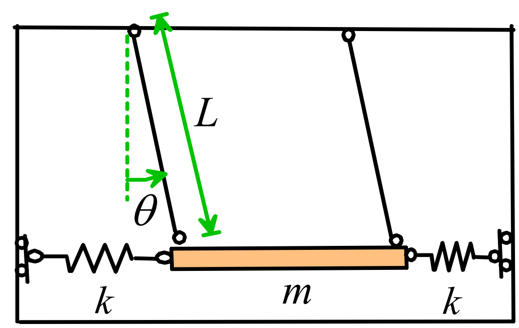
ANSWER______A________ **(2 POINTS)**

Rearrange the equation in standard form $\frac{1}{4^2}\frac{d^2y}{dt^2}+\frac{2\times 0.25}{4}\frac{dy}{dt}+y=0$ showing that $\omega_n=4$ and $\zeta=0.25$. This is an underdamped solution with period approximately $2\pi/4$; i.e. (a)

11. The spring-pendulum vibration isolation system shown in the figure has an equation of motion

$$mL^2\frac{d^2\theta}{dt^2}+2kL^2\sin\theta\cos\theta+mgL\sin\theta=0$$

The natural frequency of small amplitude oscillations is



(a) $\sqrt{\frac{2k}{m}+\frac{g}{L}}$

(b) $\sqrt{\frac{2k}{m}}$

(c) $\sqrt{\frac{2k}{m}+\frac{2g}{L}}$

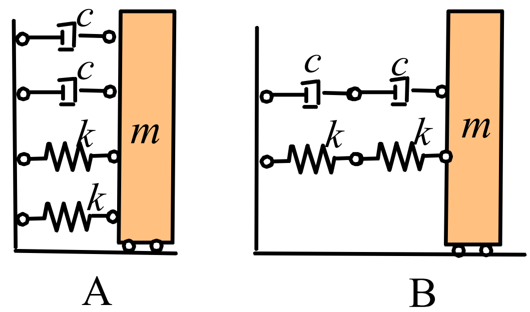
(d) $\sqrt{\frac{g}{L}}$

(e) None of the above

ANSWER______A________ **(2 POINTS)**

Linearize the EOM and rearrange $\frac{1}{(2k/m)+(g/L)}\frac{d^2\theta}{dt^2}+\theta=0$ and compare to Case I to see (a)

12. System A has natural frequency and damping coefficient ω_n, ζ. System B has natural frequency and damping coefficient

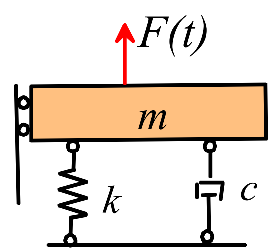


(a) $\omega_{nB} = \omega_n / \sqrt{2}, \quad \zeta_B = \zeta / \sqrt{2}$
(b) $\omega_{nB} = \omega_n / 2, \quad \zeta_B = \zeta / 2$
(c) $\omega_{nB} = 2\omega_n, \quad \zeta_B = 2\zeta$
(d) $\omega_{nB} = \omega_n / 2, \quad \zeta_B = \zeta / \sqrt{2}$
(e) None of the above

ANSWER______B________ **(2 POINTS)**

The springs/dashpots in A are in parallel; in B they are in series. Therefore $\omega_{nA} = \sqrt{2k/m} \quad \zeta_A = 2c/(2\sqrt{2km})$ and $\omega_{nB} = \sqrt{k/2m} \quad \zeta_B = c/4\sqrt{km/2}$ giving (b)

13. The spring-mass system shown in the figure has undamped natural frequency ω_n and damping coefficient ζ . It is subjected to a harmonic force with amplitude F_0 and frequency equal to its undamped natural frequency ($\omega = \omega_n$). The amplitude of vibration of the mass is



(a) $Y_0 = \dfrac{F_0}{2\zeta}$

(b) $Y_0 = \dfrac{F_0}{c}\sqrt{\dfrac{m}{k}}$

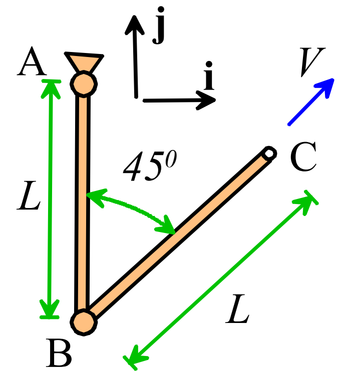
(c) $Y_0 = \dfrac{F_0}{c}\sqrt{\dfrac{k}{m}}$

(d) $Y_0 = \dfrac{F_0}{k}$

ANSWER______B________ **(2 POINTS)**

Using the standard formulas for the magnification of an externally forced spring mass system, the amplitude is $Y_0 = KF_0/(2\zeta) = \dfrac{F_0/k}{2(c/2\sqrt{km})} = \dfrac{F_0}{c}\sqrt{\dfrac{m}{k}}$

14. Point A on the actuator shown in the figure is stationary, and point C moves at constant speed V along a straight line parallel to the vector $\mathbf{i}+\mathbf{j}$. The angular velocities of arms AB and BC are



(a) $\omega_{BC}=V/(L\sqrt{2}) \quad \omega_{AB}=V/L$

(b) $\omega_{BC}=V/L \quad \omega_{AB}=-V/(L\sqrt{2})$

(c) $\omega_{BC}=V/L \quad \omega_{AB}=\sqrt{2}V/L$

(d) $\omega_{BC}=\sqrt{2}V/L \quad \omega_{AB}=V/L$

(e) None of the above

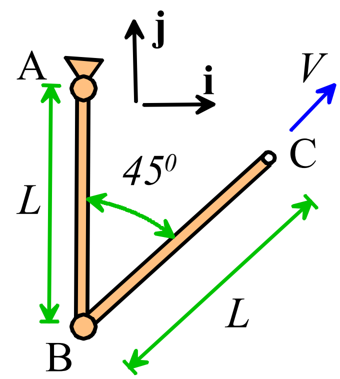
ANSWER________C______ **(2 POINTS)**

The rigid body formula gives $\mathbf{v}_B-\mathbf{v}_A=\omega_{AB}\mathbf{k}\times(-L\mathbf{j})=\omega_{AB}\mathbf{i}$

$\mathbf{v}_C-\mathbf{v}_B=\omega_{BC}\mathbf{k}\times(L\mathbf{i}+L\mathbf{j})/\sqrt{2}=\omega_{BC}(-L\mathbf{i}+L\mathbf{j})/\sqrt{2}$. Adding and using the given velocity of C

$\mathbf{v}_C=(\omega_{AB}-\omega_{BC}/\sqrt{2})L\mathbf{i}+\omega_{BC}L/\sqrt{2}\mathbf{j}=V(\mathbf{i}+\mathbf{j})/\sqrt{2}$ and hence $\omega_{BC}=V/L \quad \omega_{AB}=\sqrt{2}V/L$

15. Point A on the actuator shown in the figure is stationary, and point C moves at constant speed V along a straight line parallel to the vector $\mathbf{i}+\mathbf{j}$. The angular accelerations of arms AB and BC are



(a) $\alpha_{BC}=(1-\sqrt{2})V^2/L^2 \quad \alpha_{AB}=(\sqrt{2}-1)V^2/L^2$

(b) $\alpha_{BC}=V^2/L^2 \quad \alpha_{AB}=\sqrt{2}V^2/L^2$

(c) $\alpha_{BC}=(1-2\sqrt{2})V^2/L^2 \quad \alpha_{AB}=(\sqrt{2}-2)V^2/L^2$

(d) $\alpha_{BC}=-2\sqrt{2}V^2/L^2 \quad \alpha_{AB}=V^2/(\sqrt{2}L^2)$

(e) None of the above

ANSWER________C______ **(2 POINTS)**

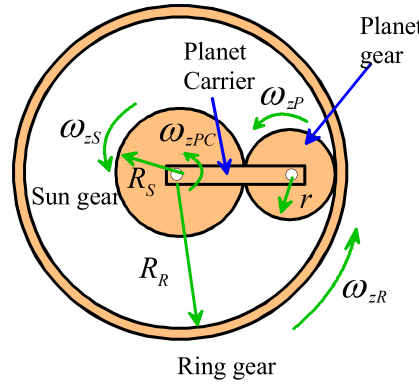
The rigid body formula gives $\mathbf{a}_B-\mathbf{a}_A=\alpha_{AB}\mathbf{k}\times(-L\mathbf{j})+\omega_{AB}^2L\mathbf{j}=\alpha_{AB}L\mathbf{i}+\omega_{AB}^2L\mathbf{j}$

$\mathbf{a}_C-\mathbf{a}_B=\alpha_{BC}\mathbf{k}\times(L\mathbf{i}+L\mathbf{j})/\sqrt{2}-\omega_{BC}^2L(\mathbf{i}+\mathbf{j})/\sqrt{2}=\alpha_{BC}(-L\mathbf{i}+L\mathbf{j})/\sqrt{2}-\omega_{BC}^2L(\mathbf{i}+\mathbf{j})/\sqrt{2}$. Adding

$\mathbf{a}_C=(\alpha_{AB}-\alpha_{BC}/\sqrt{2}-\omega_{BC}^2/\sqrt{2})L\mathbf{i}+(\alpha_{BC}/\sqrt{2}+\omega_{AB}^2-\omega_{BC}^2/\sqrt{2})L\mathbf{j}=\mathbf{0}$ using the given (zero) acceleration of C

and hence $\alpha_{BC}=\omega_{BC}^2-\omega_{AB}^2\sqrt{2}=(1-2\sqrt{2})V^2/L^2 \quad \alpha_{AB}=(\alpha_{BC}+\omega_{BC}^2)/\sqrt{2}=(\sqrt{2}-2)V^2/L^2$

16. In the figure shown, the planet carrier (the bar) rotates counterclockwise with angular speed $\omega_{zPC} = \omega_0$. The planet gear has zero angular speed. The angular speed of the sun gear is

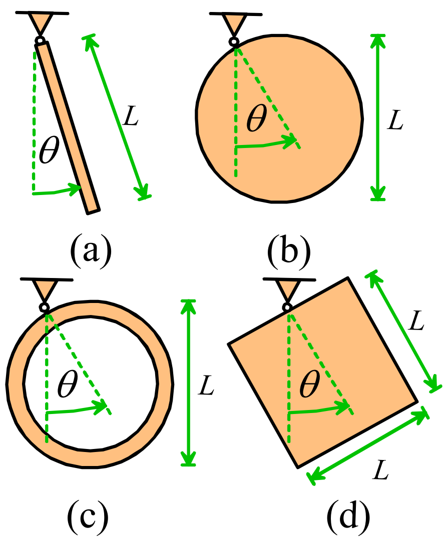


(a) $\omega_0(R_S - r)/R_S$ clockwise
(b) $\omega_0(R_S - r)/R_s$ counterclockwise
(c) $\omega_0(R_S + r)/R_S$ clockwise
(d) $\omega_0(R_S + r)/R_S$ counterclockwise
(e) None of the above

ANSWER________D______ **(2 POINTS)**

Use method from class – consider motion of system in a reference frame rotating with planet carrier $(\omega_{zP} - \omega_{zPC})/(\omega_{zS} - \omega_{zPC}) = -R_S/r \Rightarrow \omega_{zPC}(1 + R_S/r) = \omega_{zS} R_S/r$ and so (d)

17. The four objects shown in the figure are suspended from frictionless pivots, and set swinging (as pendula) with the same (small) amplitude. Which pendulum has the highest natural frequency?



(a) The rod
(b) The disk
(c) The ring
(d) The square
(e) All four have the same frequency.

ANSWER________A______ **(2 POINTS)**

Derive the EOM for the rigid body pendulum using energy (or remember formula from baseball bat example from class) $d(KE+PE)/dt = (1/2)(d/dt)\left[I_{Ozz}(d\theta/dt)^2 - mgL\cos\theta\right] = 0$ and hence

$I_{Ozz}(d^2\theta/dt^2) + (mgL/2)\sin\theta = 0 \Rightarrow (2I_{Ozz}/mgL)(d^2\theta/dt^2) + \theta \approx 0$ and $\omega_n = \sqrt{mgL/(2I_{0zz})}$.

Since $I_{0zz} = I_{Gzz} + m(L/2)^2$ for each shape the one with the lowest I_{Gzz} will have the highest natural frequency, i.e. the rod. Or more directly the shape with the lowest KE for a given $d\theta/dt$ at the instant $\theta = 0$ must be swinging fastest and must have the highest frequency.

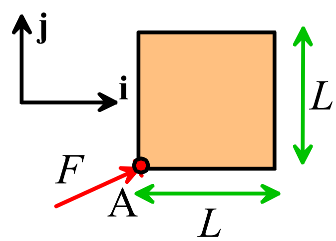
18 A two bladed wind-turbine with total mass 120kg (60kg per blade) and rotor diameter 20m is spun up from rest to an angular speed of 10 radians per second in 100 sec. The (constant) torque exerted by the wind on the turbine is

(a) 200 Nm
(b) 400 Nm
(c) 800 Nm
(d) 1000 Nm
(e) None of the above

ANSWER________B______ **(2 POINTS)**

The mass moment of inertia of the blades about the COM at the axle is $I_G = mD^2/12 = 4000kgm^2$ and the moment-d**h**/dt formula gives $Q = I_G\alpha = 4000\times 10/100 = 400Nm$

19. A square plate with uniform mass density, mass *m,* and side length *L* is subjected to a force *F* at the corner located at point A in the figure. The direction of the force is such that the angular acceleration of the plate is zero. The linear acceleration of its center of mass is

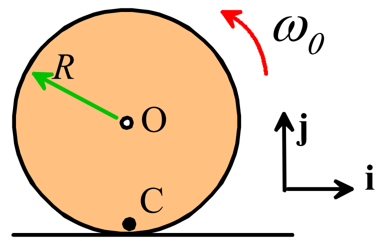


(a) $(F/m)(\mathbf{i}+\mathbf{j})/\sqrt{2}$
(b) $(2F/(3m)(\mathbf{i}+\mathbf{j})/\sqrt{2}$
(c) $(F/(4m)(\mathbf{i}+\mathbf{j})/\sqrt{2}$
(d) $(F/m)\mathbf{i}$
(e) None of the above

ANSWER________A______ **(2 POINTS)**

For zero angular acceleration the moment of the force about the COM must be zero, so the direction must act through the COM at the center of the plate. **F**=m**a** gives (A)

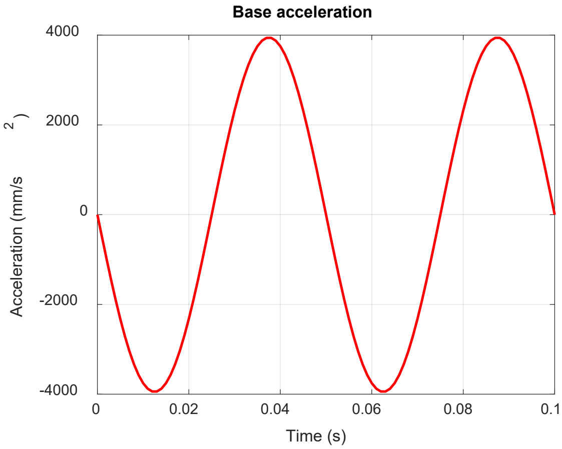
20. At time $t=0$ the cylinder shown in the figure has angular velocity $\boldsymbol{\omega} = \omega_0 \mathbf{k}$ with $\omega_0 > 0$ and its center is stationary. It is in contact with a stationary surface. The contact between the cylinder and surface has friction coefficient μ. Identify whether the statements below are true or false

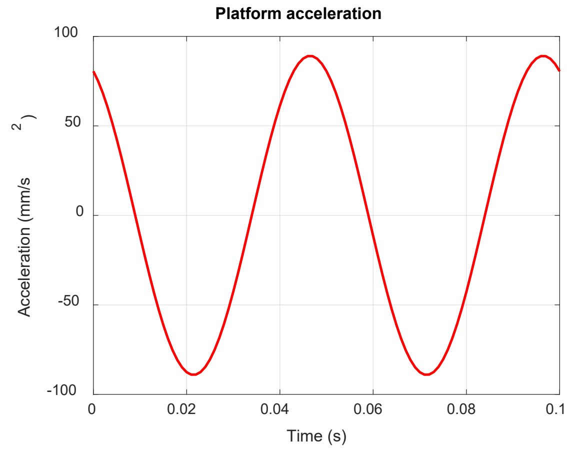


At time $t=0$

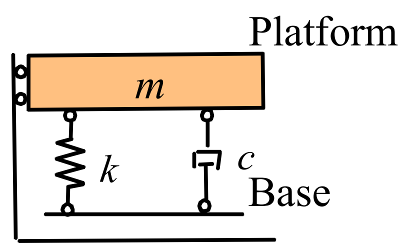
(a) No slip occurs at the contact	T	F
(b) The friction force on the cylinder has magnitude μmg	T	F
(c) The friction force on the cylinder acts to the right	T	F
(d) The cylinder has acceleration $\mathbf{a}_G = -\mu g \mathbf{i}$	T	F

At time $t=0$ the rigid body kinematics formula gives $\mathbf{v}_C = \omega_0 R \mathbf{i}$ so C moves to the right over the surface. There is slip, and friction acts to the left. The normal force is equal to mg, so the friction force has magnitude μmg and the acceleration is $\mathbf{a}_G = -\mu g \mathbf{i}$





21 The figure shows readings from accelerometers attached to the base and platform of a vibration isolation table.



21.1 Find

(1) The period of the vibration;

There are 2 cycles in 0.1s so the period is 0.05s

[1 POINT]

(2) The frequency of the vibration (in both Hz and radians per second);

The frequency is 20Hz, or 126 rad/s

[1 POINTS]

(3) The amplitude of the displacement (in mm) of the base

The acceleration amplitude of the base is 4 m/s^2. The amplitude of the displacement is $Y_0 = A_0 / \omega^2 = 0.25mm$

[1 POINT]

(4) The amplitude of the displacement (in mm) of the platform

The acceleration amplitude of the platform is 85 mm/s^2. The amplitude of the displacement is $X_0 = A_0 / \omega^2 = 0.0054mm$

[1 POINT]

(5) The magnification.

The magnification is $X_0 / Y_0 = 0.0054 / 0.25 = 0.022$

[1 POINT]

21.2 In a separate experiment, the natural frequency of vibration of the table is measured to be 2Hz. Use this information and your solution to 21.1 to calculate the damping coefficient ζ for the platform

We know that the magnification is $M = \dfrac{\sqrt{1+(2\zeta\omega/\omega_n)^2}}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2+(2\zeta\omega/\omega_n)^2}}$

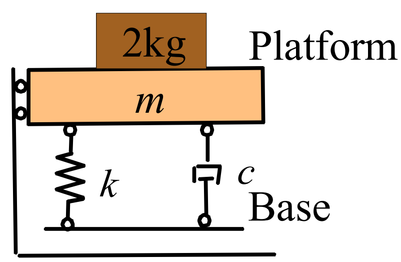
$$M^2\left[\left(1-\omega^2/\omega_n^2\right)^2+(2\zeta\omega/\omega_n)^2\right]=1+(2\zeta\omega/\omega_n)^2$$

Hence $\Rightarrow (2\zeta\omega/\omega_n)^2\left[1-M^2\right]=M^2\left(1-\omega^2/\omega_n^2\right)^2-1$

$$\Rightarrow \zeta = \frac{\omega_n}{2\omega}\sqrt{\frac{M^2\left(1-\omega^2/\omega_n^2\right)^2-1}{\left[1-M^2\right]}} \approx \frac{M\omega}{2\omega_n} = 0.1$$

[3 POINTS]

21.3 A 2kg mass is placed on the table. It is found that the natural frequency of the table with the mass on its surface decreases to 1Hz. Calculate the value of the table mass *m*, the spring constant *k* and the dashpot coefficient *c*.



Using the formulas for natural frequency

$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow 2\times 2\pi = \sqrt{\frac{k}{m}} \qquad 2\pi = \sqrt{\frac{k}{m+2}}$$

$$\Rightarrow 4 = \frac{m+2}{m} \Rightarrow m = \frac{2}{3}kg$$

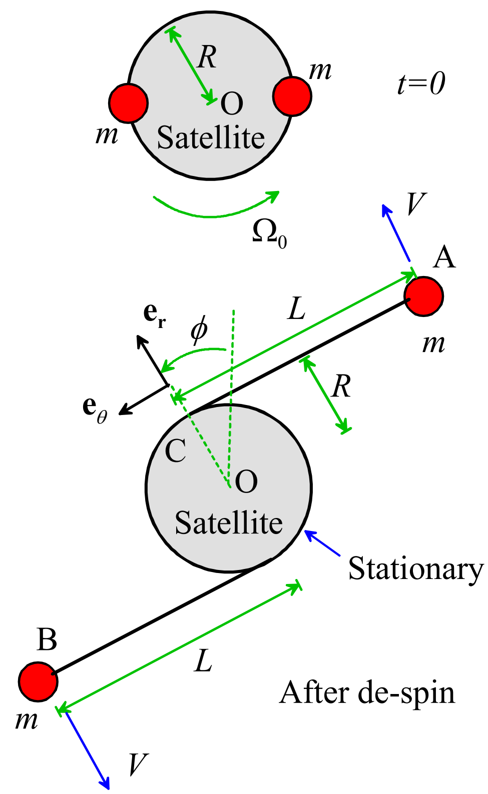
$$k = 16\pi^2 m = 105 N/m$$

The formula for damping coefficient gives

$$\zeta = \frac{c}{2\sqrt{km}} \Rightarrow c = 0.1\times 2\sqrt{105\times 2/3} = 1.7 Ns/m$$

[2 POINTS]

22 The figure shows a so called 'yoyo de-spin' of a satellite. The satellite is cylindrical, with radius R and mass moment of inertia about its center I_{Gzz} . Two masses m (with negligible mass moment of inertia about their COM) are attached to the cylinder by inextensible, massless tethers.



At time $t=0$ the tethers are wound tightly around the cylinder and the assembly spins with angular speed Ω_0 . To de-spin the satellite, the masses are released, and the rotational motion of the assembly causes the tethers to unwind from the cylinder. This slows the rotation of the cylinder. When the rotation of the cylinder stops, the tethers are cut. The goal of this problem is to determine the length of the tethers necessary to stop the rotation of the satellite.

22.1 Consider the system at time $t=0$. Assume that the center of the satellite at O is stationary and the satellite (with masses attached) spins at angular speed Ω_0 . Find the total kinetic energy of the system (small masses + cylinder together) at time $t=0$, in terms of Ω_0, m, R, I_{Gzz} . Treat the small masses m as particles.

The masses are in circular motion about O and so move at speed $V = \Omega_0 R$ and have KE $\frac{1}{2}mV^2$. The total KE is thus

$$T = \frac{1}{2} I_{Gzz}\Omega_0^2 + mR^2\Omega_0^2$$

[2 POINTS]

22.2 Find a formula the total angular momentum of the system (masses + cylinder) about the center of the cylinder at time $t=0$ in terms of Ω_0, m, R, I_{Gzz}.

The masses have angular momentum $\mathbf{h} = RmV\mathbf{k} = mR^2\Omega_0\mathbf{k}$. The total angular momentum is therefore

$$\mathbf{h} = \left[I_{Gzz}\Omega_0 + 2mR^2\Omega_0 \right]\mathbf{k}$$

[1 POINT]

22.3 Consider the assembly at the instant that the cylinder just comes to rest (after de-spin). Write down the position vector of the mass at A at this instant, (taking the origin to be at O) in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis in terms of R, L.

$$\mathbf{r} = R\mathbf{e}_r - L\mathbf{e}_\theta$$

[1 POINT]

22.4 Suppose that at the instant the satellite stops rotating, the tether CA has angular speed $\omega = d\phi / dt$ and length L. Note that the cylinder (i.e. the satellite) and tether must have the same velocity where they touch at point C. Use the rigid body kinematics formula to find the velocity vector of the mass at A in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis in terms of ω, L .

Since the satellite is not rotating, point C is stationary. The rigid body kinematics formula for CA gives

$$\mathbf{v} = \mathbf{v}_C + \omega\mathbf{k} \times (-L\mathbf{e}_\theta) = \omega L \mathbf{e}_r$$

[1 POINT]

22.5 Hence, find formulas for the total kinetic energy and angular momentum about O of the system (i.e. the satellite and both masses combined) at the instant that the cylindrical satellite comes to rest, in terms of ω, L, m.

By symmetry both A and B must have the same KE and angular momentum so the total is

$$T = 2 \times \frac{1}{2} m(\omega L)^2$$

$$\mathbf{h} = \mathbf{r} \times m\mathbf{v} = 2 \times \left(R\mathbf{e}_r - L\mathbf{e}_\theta\right) \times m\omega L \mathbf{e}_r = 2m\omega L^2 \mathbf{k}$$

[2 POINTS]

22**.**6 Finally, by considering the energy and angular momentum of the system show that the cylinder comes to rest when the tether length reaches

$$L = \sqrt{\frac{\left(I_{Gzz} + 2mR^2\right)}{2m}}$$

No external forces act on the system consisting of the two masses and cylinder, so its total energy and angular momentum must be conserved. The cylinder is stationary and so has neither KE nor angular momentum. Therefore (since there are two masses)

$$m(\omega L)^2 = \frac{1}{2}\left(I_{Gzz} + 2mR^2\right)\Omega_0^2$$

$$2m\omega L^2 \mathbf{k} = \left[I_{Gzz} + 2mR^2\right]\Omega_0 \mathbf{k}$$

We can solve these equations for ω, L to see that

$$\omega L = \sqrt{\frac{\left(I_{Gzz} + 2mR^2\right)}{2m}}\Omega_0$$

$$\omega L^2 = \frac{\left[I_{Gzz} + 2mR^2\right]}{2m}\Omega_0 \Rightarrow L = \sqrt{\frac{\left(I_{Gzz} + 2mR^2\right)}{2m}}$$

[3 POINTS]