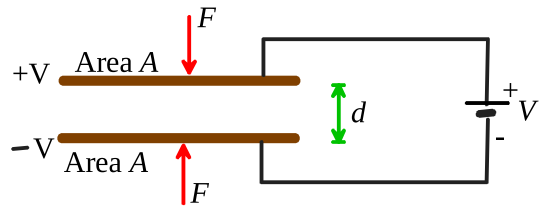
Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 4: Work and Energy Methods
Due Friday February 27th

1. Two parallel plates in an electrostatic actuator experience an attractive force $F = AV^2\varepsilon/(2d^2)$ where ε is the permittivity of the medium between them.

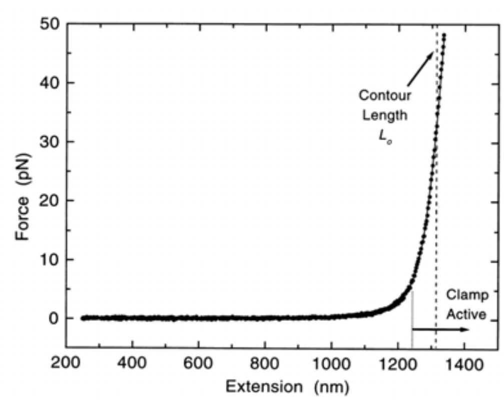


a. Suppose that the upper plate of the actuator is displaced to reduce the separation from d_0 to d_1. Find a formula for the work done by the electrostatic force that acts on the upper plate.

b. A typical actuator has $A = 10^{-7}\,m^2$, $d_1 = 10^{-7}\,m$ and is operated at 5V. Estimate the maximum work that can be done by closing the plates of the actuator from a large (infinite) separation to d_1

c. Suppose that the actuator operates at 1 kHz (1000 cycles per second) with $d_1 = 10^{-6}$ m. Estimate (roughly) the mechanical power that could be generated by the device.

2. The figure shows the measured force-extension relation for a DNA molecule (the figure is from Wang *et al*, (1997), Biophysical Journal, **72**, 1335-1346). For low stretches, (less than about 1250nm), the forces exerted by the molecule are generated by its thermal vibration – much as pressure in a gas is generated by thermal motion of the atoms. In this regime the force-extension relation for the molecule can be approximated as



$$F = \left(\frac{kT}{L_p}\right)\left(\frac{1}{4(1-x/L_0)} - \frac{1}{4} + \frac{x}{L_0}\right)$$

where, k is the Boltzmann constant, T is absolute temperature (the product kT quantifies energy of thermal vibration in a the system), L_p is the 'persistence length' of the molecule (the correlation length of the thermal vibrations), and L_0 is the total length of the molecule.

a. Find a formula for the work done to stretch the molecule from length $x=0$ to length a (use MAPLE to do the integral).

b. Using data $L_p = 43$ nm, $L_0 = 1317$ nm, calculate the work required to stretch a molecule from zero length to half its total length, at room temperature (T=288 Kelvin).

3. The table below shows specifications for an electric motor (available for purchase from http://www.motortech.com/dcmotorCIR_MD.htm.)

Voltage	No-load Speed (rpm)	Stall torque (oz-in)	Stall current (A)	No load current (A)
27	16400	5.31	3.6	0.17

a. Estimate the maximum power that can be generated by the motor. Please give the answer in SI units!

b. Calculate values for the parameters R, β, T_0 and τ_0 in the equations describing the behavior of an electric motor.

c. Determine the speed that maximizes the efficiency of the motor.

4. Estimate the power developed by the motor that lifts one of the elevators in the sciences library. Assume that you are the sole occupant. Explain briefly how you estimated values for relevant variables in the calculation.

5. Bungee jumps are subject to extensive legal restrictions – see for example the Ohio standards at http://codes.ohio.gov/oac/901:9-1, and in particular http://codes.ohio.gov/oac/901%3A9-1-29.

The goal of this problem is to establish a design procedure that will ensure that the standards are met. To provide some perspective, you may be interested in reading Kockelman, J.W. and Hubbard, M. (2004) "Bungee jumping cord design using a simple model," Sports Engineering, **7**, 89.

Assume that the jumper has mass m, and the bungee cord has cross-sectional area A, unstretched length L_0, and is made from a material with Young's modulus E and tensile strength σ_{max}. (If you have not done EN3, you may need to read Section 10 of the EN3 lecture notes at http://www.engin.brown.edu/courses/en3/notesframe.htm for definitions of E and σ_{max})

a. **The physics:** Using (i) energy conservation, (ii) the equation of motion for the jumper at the instant that the cord is stretched to its maximum extent, and (iii) the force-extension equations for an elastic two force member, show that

b. The maximum acceleration during the jump is $a_{max} = g\sqrt{1+\frac{2AE}{mg}}$. Note that this means that a light person will experience higher accelerations than a heavy one, and also, surprisingly, the acceleration does not depend on the height of the jump or the length of the bungee cord.

- The jumper falls through a total distance $d = L_0\left(1+\frac{mg}{AE}+\frac{mg}{AE}\sqrt{1+\frac{2AE}{mg}}\right)$
- The maximum tensile stress (force/area) in the cord is $\frac{F}{A} = \left(\frac{mg}{A}+\frac{mg}{A}\sqrt{1+\frac{2AE}{mg}}\right)$

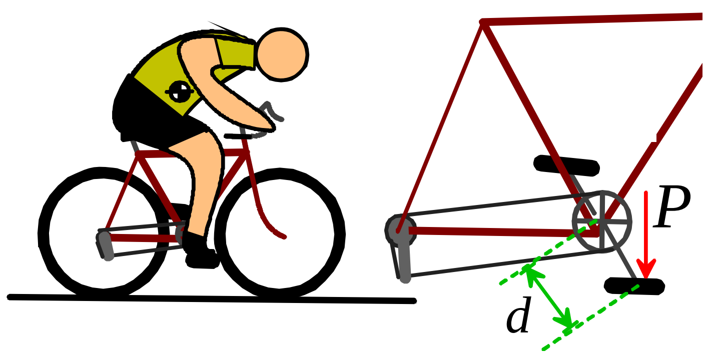
MAPLE can simplify the rather tedious algebra involved in solving the equations from (i) and (ii) for a_{max} and d.

c. **Design constraints:** The standards require that the maximum acceleration of the jumper must not exceed αg, where $\alpha = 3.5$ for a waist and chest harness, and $\alpha = 2.5$ for an ankle harness (the codes actually prescribe 'g forces' but this is what they really mean). They also require the maximum stress in the cable to be at least a factor of five lower than the tensile strength of the cable. Assume that the mass of the jumper will lie in the range $m_0 \le m \le m_1$. Show that the design constraints require that:

- For the acceleration to be within limits for all jumpers $\dfrac{gm_0}{AE} \ge \dfrac{2}{(\alpha^2 - 1)}$
- For the force to be within limits for all jumpers $\dfrac{gm_1}{AE} \le \left(\dfrac{\sigma_{\max}}{5E}\right)^2 \dfrac{1}{2\left[1 + \sigma_{\max}/(5E)\right]}$
- For the jump to be feasible for any jumper, the material in the cord must have strength/stiffness ratio $\sigma_{\max}/E > 10/(\alpha - 1)$

d. **Design Decision:** Suppose that the cord is to be made from rubber with E=0.7MPa and $\sigma_{\max} = 30MPa$. Choose suitable values for the maximum and minimum allowable weight for the jumper, and recommend a value for the cross-sectional area of the cord that will ensure that the design limits are not exceeded (the solution is not unique – use your judgment)..

7. A bicycle travels at constant speed v. Assume that
(i) The rider exerts a vertical force P on the downward moving pedal;
(ii) The pedal cranks have length d,
(iii) The pedal cranks rotate at angular speed n revolutions per minute.
(iv) The bicyclist has frontal area A and drag coefficient C_D, and rides through air with density ρ.
(v) The air resistance of the bicycle can be neglected.



a. Draw a free body diagram showing all the forces acting on the bicycle (without the rider) (assume that the front wheel rolls freely)

b. Draw a free body diagram showing all the forces acting on the rider

c. Deduce that the total horizontal force exerted by the rider on the bicycle is equal to $\rho C_D A v^2 / 2$

d. Find a formula for the average rate of work done on the bicycle by the force P, in terms of d, P and n.

e. Calculate the rate of work done on the bicycle by all the other (external) forces acting on the bicycle.

f. Hence, show that the speed of the bicycle is related to the force exerted on the pedal and the rotational speed by

$$v = \left(\frac{2Pdn}{15\rho A C_D} \right)^{1/3}$$

g. Calculate the speed of the bicycle for A=1.5m^2, d=20cm, P=100N, $C_D = 0.5$, $\rho = 1.02 kgm^{-3}$, n=30

h. Calculate the required transmission ratio v/n for these operating conditions (the transmission ratio can be designed by selecting the bicycle's gears and the radius of the bike wheels appropriately – this will be discussed in more detail when we consider motion of rigid bodies.)