HW6 2015 ENGN 0040

SOLUTIONS

1pt 1.1. $\underline{r}_{B/A} = (.200\hat{i} + .120\hat{j})\ m$

1pt 1.2. $\underline{r}_{C/B} = (.100\hat{i} - .100\hat{j})\ m$

1pt 1.3. $\underline{r}_{C/A} = \underline{r}_{C/B} + \underline{r}_{B/A} = (.3\hat{i} + .02\hat{j})\ m$

4pt 1.4. $\underline{V}_B = \underline{V}_A + \underline{\omega}_{AB} \times \underline{r}_{B/A}$ BUT $\underline{V}_A = 0$ **

$$\underline{V}_C = \underline{V}_B + \underline{\omega}_{BC} \times \underline{r}_{C/B}$$

2 pts → $\boxed{\underline{V}_C = \underline{\omega}_{AB} \times \underline{r}_{B/A} + \underline{\omega}_{BC} \times \underline{r}_{C/B}}$

show this vector eqn

split into components, substitute $\underline{V}_C = (0\hat{i} - 1\hat{j})\ m$

$$\underline{V}_C = \omega_{AB}\hat{k} \times (.2\hat{i} + .12\hat{j}) + \omega_{BC}\hat{k} \times (.1\hat{i} - .1\hat{j})$$

$$\underline{V}_C = -.12\,\omega_{AB}\hat{i} + .2\,\omega_{AB}\hat{j} + .1\,\omega_{BC}\hat{i} + .1\,\omega_{BC}\hat{j}$$

$$\left.\begin{aligned} \hat{i}:&\quad 0 = -.12\,\omega_{AB} + .1\,\omega_{BC} \\ \hat{j}:&\quad -1 = .2\,\omega_{AB} + .1\,\omega_{BC} \end{aligned}\right\}$$ solve 2 eqns & 2 unknowns

$$\boxed{\begin{aligned} \underline{\omega}_{AB} &= -3.12\,\hat{k}\ \tfrac{rad}{s} \\ \underline{\omega}_{BC} &= -3.75\,\hat{k}\ \tfrac{rad}{s} \end{aligned}}$$ ← 2 pts

5pts 1.5. $\underline{a}_B = \underline{a}_A + (\underline{\alpha}_{AB} \times \underline{r}_{B/A}) + (\underline{\omega}_{AB} \times \underline{\omega}_{AB} \times \underline{r}_{B/A})$

$\underline{a}_C = \underline{a}_B + (\underline{\alpha}_{BC} + \underline{r}_{C/B}) + (\underline{\omega}_{BC} \times \underline{\omega}_{BC} \times \underline{r}_{C/B})$

$* \underline{a}_A = 0$

2 pts

$$\boxed{\begin{aligned}\underline{a}_C = (\underline{\alpha}_{AB} \times \underline{r}_{B/A}) + (\underline{\omega}_{AB} \times \underline{\omega}_{AB} \times \underline{r}_{B/A}) \\ + (\underline{\alpha}_{BC} \times \underline{r}_{C/B}) + (\underline{\omega}_{BC} \times \underline{\omega}_{BC} \times \underline{r}_{C/B})\end{aligned}}$$

$\underline{a}_C = (0\hat{\imath} - 3\hat{\jmath})\,m/s^2$

evaluating cross products: (matlab/mupad OK)

$\underline{a}_C = -.12\alpha_{AB}\hat{\imath} + .2\alpha_{AB}\hat{\jmath} - .2\omega_{AB}^2\hat{\imath} - .12\omega_{AB}^2\hat{\jmath}$
$+ .1\alpha_{BC}\hat{\imath} + .1\alpha_{BC}\hat{\jmath} - .1\omega_{BC}^2\hat{\imath} + .1\omega_{BC}^2\hat{\jmath}$

$\hat{\imath}: \quad 0 = -.12\alpha_{AB} - .2\omega_{AB}^2 + .1\alpha_{BC} - .1\omega_{BC}^2$

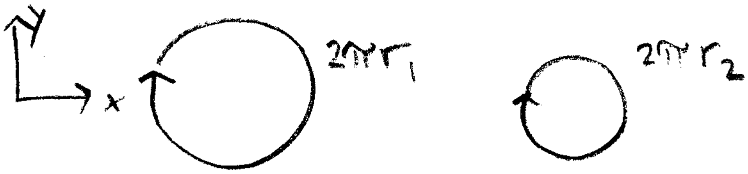
$-3 = .2\alpha_{AB} - .12\omega_{AB}^2 + .1\alpha_{BC} + .1\omega_{BC}^2$

3pts

$$\boxed{\begin{aligned}\underline{\alpha}_{AB} &= -20.6\,\hat{k}\ rad/s^2 \\ \underline{\alpha}_{BC} &= 8.87\,\hat{k}\ rad/s^2\end{aligned}}$$

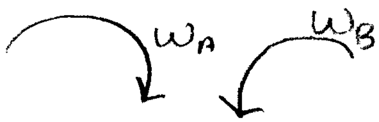
(10pts)
2.1

"Teeth" ratio same as gear ratio



teeth same as circumference, gives r_1/r_2

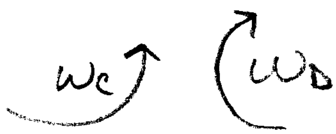
$$\omega_A = -30 \frac{\text{rot.}}{\text{min.}} \cdot \frac{1 \text{ min}}{60 \text{ sec.}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rot.}} = -\pi \text{ rad/s}$$



$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{120}{40}\right)\omega_A$$

$$\omega_B = 3\pi \text{ rad/s}$$ $\Rightarrow$ opposite direction!

$\omega_B = \omega_c$ they are rigidly attached - move at same ω (1 rigid body)



$$\omega_D = \omega_c\left(\frac{r_c}{r_D}\right) = \left(\frac{80}{20}\right)\omega_c$$

3pts $\rightarrow$ $\boxed{\omega_D = -12\pi \text{ rad/s} \quad \underline{\text{clockwise}}}$

* must have sign or direction!

(2pts)
2.2. 10 min = 600 seconds = T

$$\theta_A = \int_0^T \omega_A \, dt = -600\pi \text{ radians}$$ $\Rightarrow$ $\boxed{-300 \text{ revolutions}}$

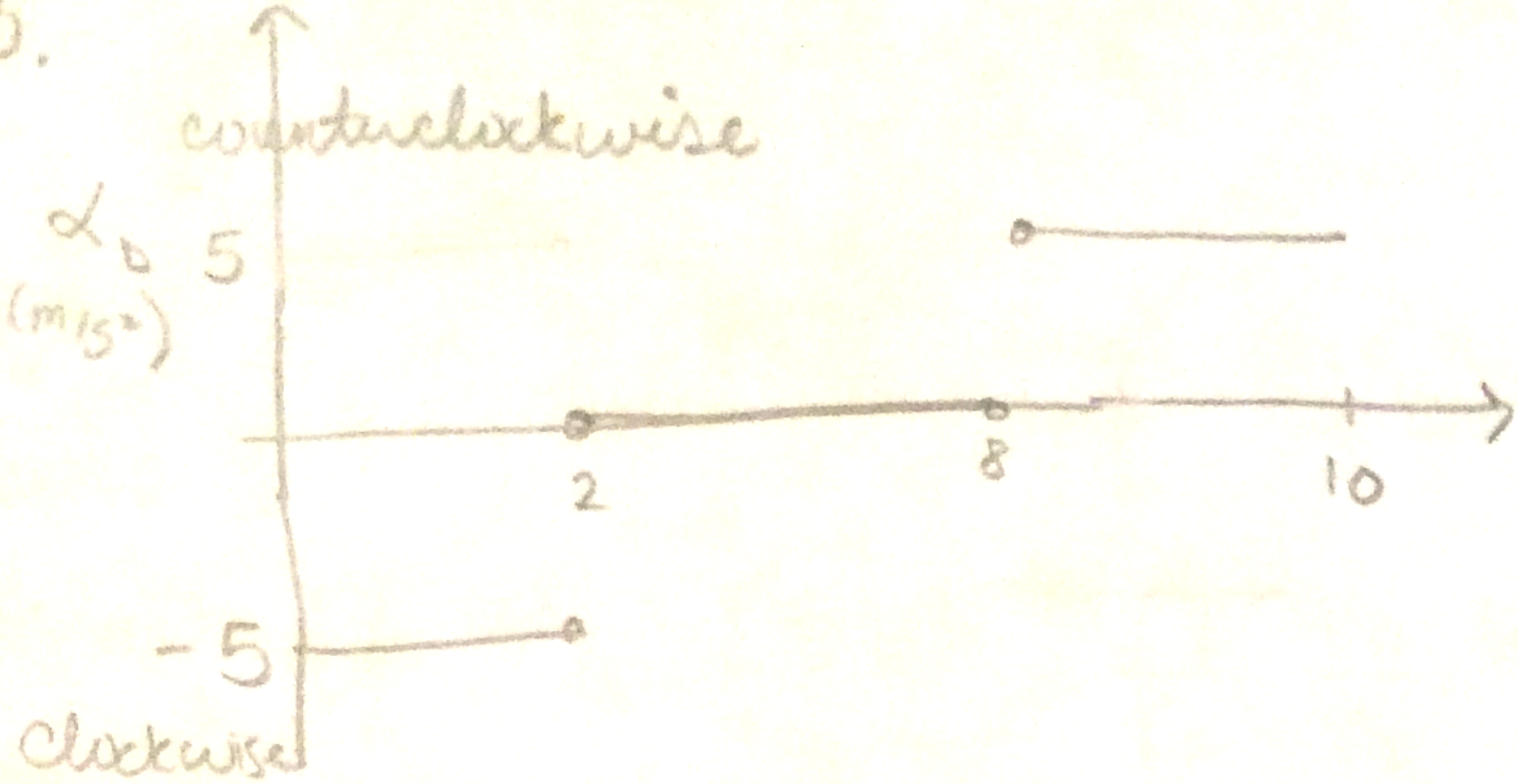
$\uparrow$ denotes clockwise

$$\theta_B = \omega_B T / 2\pi = \frac{3\pi(600)}{2\pi} \Rightarrow \boxed{900 \text{ revolutions}}$$

$$\boxed{\theta_c = \theta_B}$$

$$\theta_D = \frac{-12\pi(600)}{2\pi} \Rightarrow \boxed{-3600 \text{ revolutions}}$$

(5pts) 2.3.



$$\omega_D = \int \alpha_D \, dt = \underbrace{\int_0^2 \alpha_D \, dt}_{\omega_{D1}} + \underbrace{\int_2^8 \alpha_D \, dt}_{\omega_{D2}} + \underbrace{\int_8^{10} \alpha_D \, dt}_{\omega_{D3}}$$

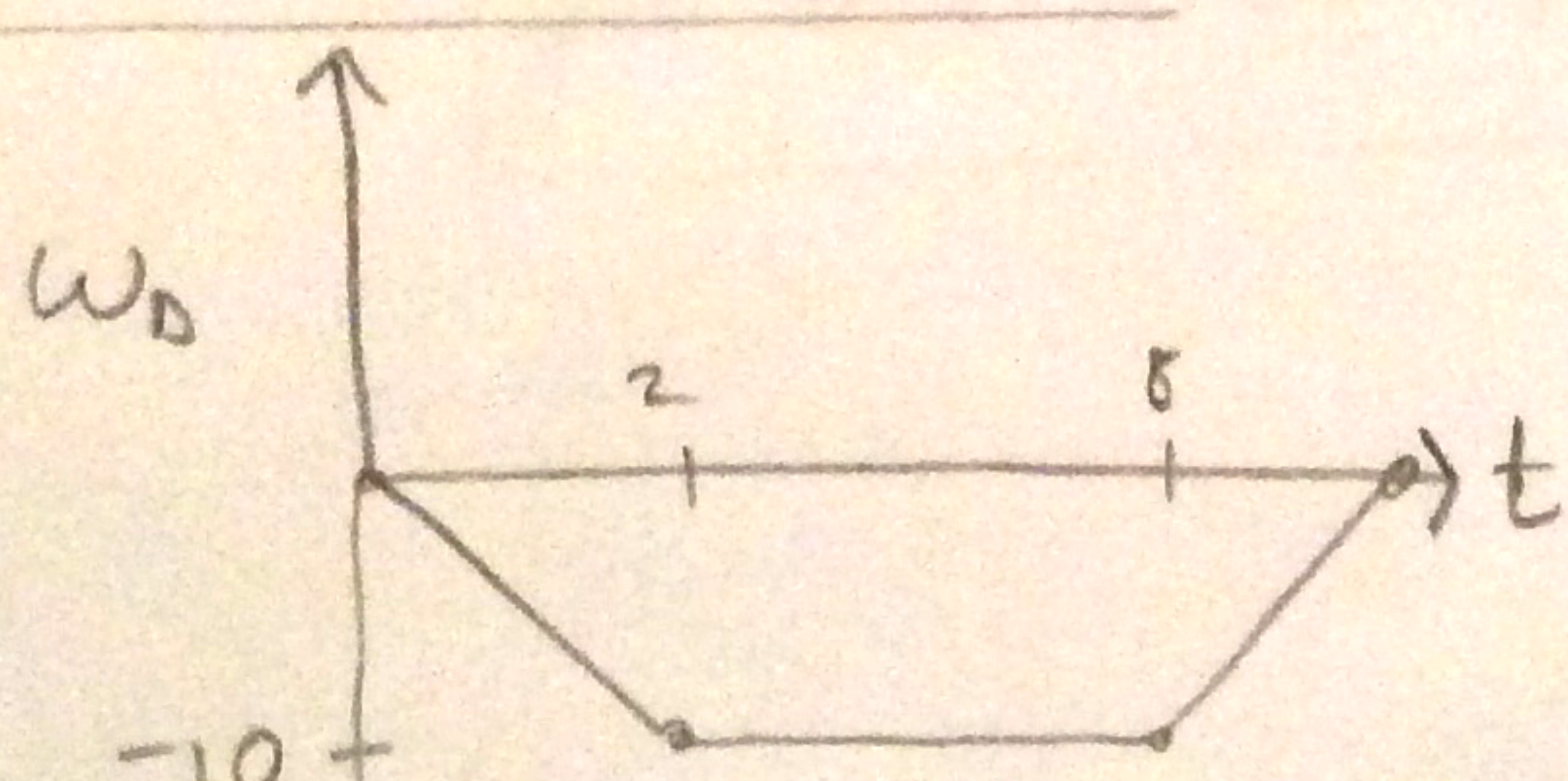
$$\boxed{\begin{aligned} \omega_{D1} &= -5t \text{ rad/s} \\ \omega_{D2} &= -10 \text{ rad/s} \end{aligned}}$$

$$\omega_{D3} = 5t + \text{const.} \Rightarrow t = 8 \text{ sec. } \omega = -10$$

$$-10 = 5 \cdot 8 + \text{const}$$

$$\text{const} = -50$$

$$\boxed{\omega_{D3} = 5t - 50 \text{ rad/s}}$$



negative means clockwise rotation

$$\theta_{D1} = \int_0^2 \omega_{D1} \, dt = -\frac{5t^2}{2} \Big|_0^2 = -10 \text{ radians}$$

$$\theta_{D2} = \int_2^8 \omega_{D2} \, dt = -10t \Big|_2^8 = -10 \cdot 6 = -60 \text{ radians}$$

$$\theta_{D3} = \int_8^{10} \omega_{D3} \, dt = \frac{5t^2}{2} - 50t \Big|_8^{10} = -10 \text{ radians}$$

80 radians ⇒ 12.74 revolutions clockwise

$\boxed{12.74 \text{ total revolutions}}$ 3pts

gear A:

$$\omega_A = \frac{r_B}{r_A}\omega_B = \frac{r_B}{r_A}\omega_C = \frac{r_B}{r_A}\frac{r_D}{r_C}\omega_D$$

$$\omega_A = \left(\frac{40}{120}\right)\left(\frac{20}{80}\right)\omega_D = \frac{1}{3}\frac{1}{4}\omega_D = \frac{1}{12}\omega_D$$

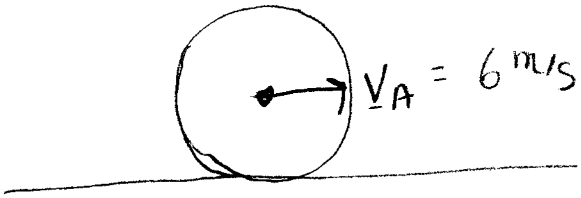
$$\omega_{A1} = \frac{1}{12}\omega_{D1}, \quad \omega_{A2} = \frac{1}{12}\omega_{D2}, \quad \omega_{A3} = \frac{1}{12}\omega_{D3}$$

$$\theta_A = \frac{1}{12}\int \omega_D \, dt = \frac{1}{12}\theta_D$$

All answers the same except multiplied by 1/12.

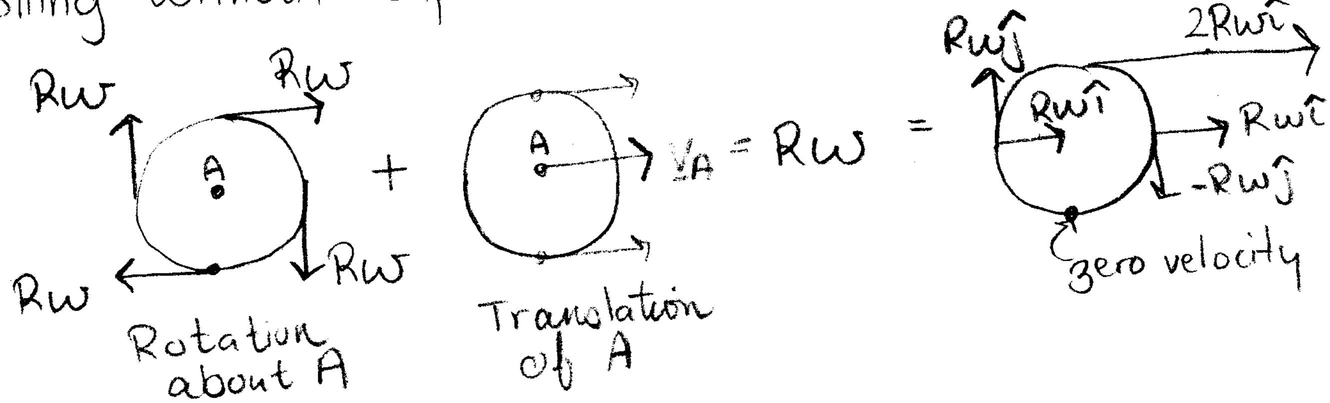
$\boxed{\text{gear A makes } 1.06 \text{ revolutions}}$ 2pts

3. (8 pts) ω^+, α^+ $R = 0.5\ m$



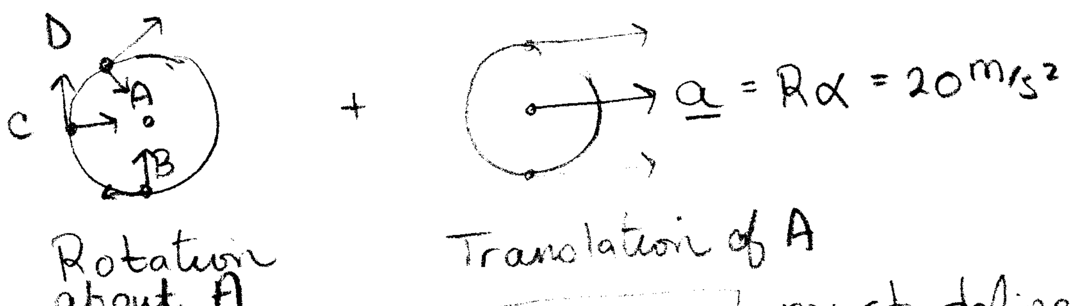
$\underline{V}_A = 6\ m/s,\ a_A = 20\ m/s^2$

Rolling without slip:



$$\omega = \frac{6\ m/s}{.5\ m} = 12\ rad/s$$

Acceleration:



$$\alpha = 20/.5 = \boxed{40\ rad/s^2\quad \overset{+}{\alpha}}$$

2pts

must define direction or define coordinate axis ($\uparrow\hat{j}\ \rightarrow\hat{i}$ $\underline{\alpha} = -40\hat{k}$)

point B:

$$\underline{a}_B = \underline{a}_A + \underline{\alpha} \times \underline{r}_{B/A} + \underline{\omega} \times \underline{\omega} \times \underline{r}_{B/A}$$

$\uparrow\hat{j}\ \rightarrow\hat{i}$

$$\left.\begin{aligned}\underline{\alpha} &= -40\hat{k}\\ \underline{\omega} &= -12\hat{k}\\ \underline{r}_{B/A} &= -.5\hat{j}\end{aligned}\right\}\quad \underline{a}_B = 20\hat{i} - .5\alpha\hat{i} + .5\omega^2\hat{j}$$

$$\boxed{\underline{a}_B = (0\hat{i} + 72\hat{j})\ m/s^2}\ 2pts$$

$$\underline{a}_C = \underline{a}_A + \underline{\alpha} \times \underline{r}_{C/A} + \underline{\omega} \times \omega \times \underline{r}_{C/A}$$

same as but $\underline{r}_{C/A} = -.5\hat{\imath}$

$$\underline{a}_C = 20\hat{\imath} + .5(40)\hat{\jmath} + .5(12)^2\hat{\imath}$$

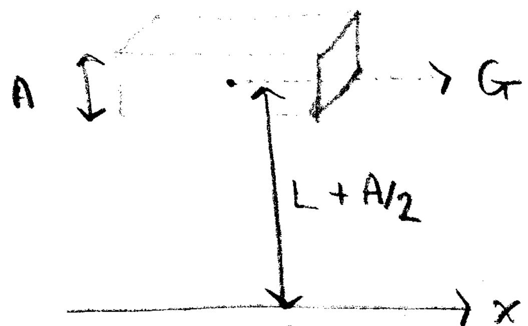
$$\boxed{\underline{a}_C = 92\hat{\imath} + 20\hat{\jmath} \ m/s^2}$$ 2pts

$\underline{a}_D$, repeat $\underline{r}_{D/A} = -.5\frac{\sqrt{2}}{2}\hat{\imath} + .5\frac{\sqrt{2}}{2}\hat{\jmath}$

$$\underline{a}_D = 20\hat{\imath} + \frac{\sqrt{2}}{4}(40)\hat{\imath} + \frac{\sqrt{2}}{4}(40)\hat{\jmath} + \frac{\sqrt{2}}{4}(12^2)\hat{\imath} - \frac{\sqrt{2}}{4}(12^2)\hat{\jmath}$$

$$\boxed{\underline{a}_D = 85.0\hat{\imath} - 36.8\hat{\jmath} \ m/s^2}$$ 2pts

4.1 (5 pts) use of tables OK

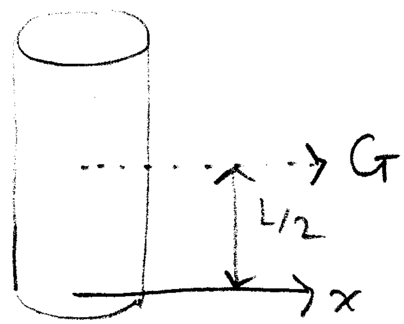


$m_1 = \rho B A^2$

$I_G = \frac{1}{12} m_1 (2A^2)$

$I_{x1} = \frac{1}{12} m_1 2A^2 + m_1 (L + A/2)^2$

$I_{x1} = m_1 \left[6A^2 + (L + A/2)^2 \right]$



$m_2 = \rho \pi R^2 L$

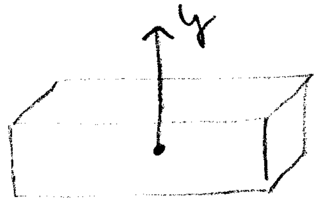
$I_G = \frac{1}{12} m_2 (3R^2 + L^2)$

$I_{x2} = I_G + m_2 (L/2)^2$

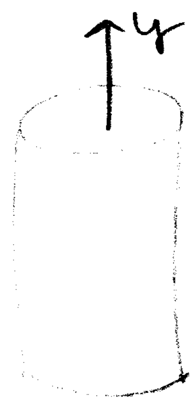
$I_{x2} = m_2 \left[\frac{1}{12}(3R^2 + L^2) + \left(\frac{L}{2}\right)^2 \right]$

$I_{x2} = m_2 \left[\frac{1}{12}(3R^2 + L^2) + \frac{L^2}{4} \right]$

3 pts $\boxed{I_x = I_{x1} + I_{x2}}$



$I_{y1} = \frac{1}{12} m_1 (B^2 + A^2)$

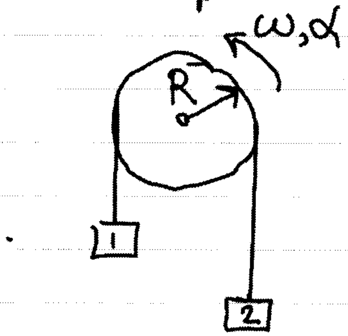


$I_{y2} = \frac{1}{2} m_2 R^2$

2 pts $I_y = I_{y1} + I_{y2}$

HW 6, 5. (8 pts)

[redacted] pulley with mass



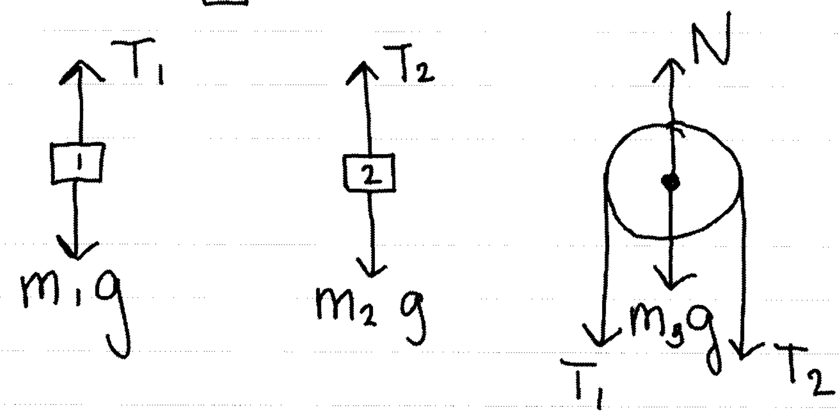
$m_1 = 10\ kg$

$m_2 = 8\ kg$

$m_{pulley} = m_3 = 4\ kg$

what is the acceleration of m_2?

FBD



EOM

$$\boxed{\begin{aligned} T_1 - m_1 g &= m_1 a_1 \\ T_2 - m_2 g &= m_2 a_2 \end{aligned}}$$

pulley: $\Sigma M_G = I_G \alpha$

$$\boxed{R T_1 - R T_2 = I_G \alpha}$$

Mass Moment of Inertia (I_G)

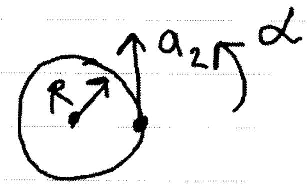
thin disk radius R $\Rightarrow$ look up in table $\Rightarrow$ $\boxed{I_G = \frac{1}{2} m_3 R^2}$

<u>constraints?</u> $a_1 = -a_2$ on same cable, inextensible

unknowns: a_2, α, T_1, T_2 (4 unknowns, 3 eqns)

<u>kinematics</u> (4th eqn)

rotational motion of the pulley



$$\boxed{a_2 = R\alpha}$$

<u>solve</u>:

$$T_1 = m_1 g - m_1 a_2$$
$$T_2 = m_2 g + m_2 a_2$$
$$R(T_1 - T_2) = \frac{1}{2} m_3 R^2 \alpha$$

plug T_1, T_2 into 3rd eqn

use $\alpha = a_2/R$

solve for a_2

$$\boxed{a_2 = \frac{g(m_1 - m_2)}{\frac{1}{2} m_3 + m_1 + m_2}}$$

sub. values $\boxed{a_2 = \frac{g}{10}}$ 6 pts

What if $m_3 = 0$ (massless pulley)

1pt → $\boxed{a_2 = \frac{g}{\cancel{g}}\ g}$ a_2 is faster! (if pulley is mass-less)

Mass of pulley slows acceleration.

I_G, mass moment of inertia, adds resistance to rotational motion

Explanation (1pt)