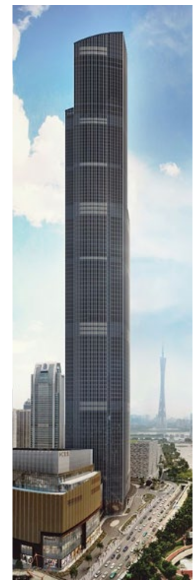
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Kinematics and Dynamics of Particles
Due Friday June 4, 2021

1. Straight Line Motion. The CTF Finance Center in Guanzhou has the world's fastest elevator, with the following specifications:

- Height: 400m
- Maximum speed 20 m/s
- Maximum acceleration (and deceleration) 2 m/s^2



Assume that the elevator (i) starts at rest on the ground floor; (ii) accelerates with constant acceleration up to the maximum speed, then (iii) travels at constant speed; then (iv) decelerates to rest with constant acceleration and comes to rest on the top floor.

1.1 How long does the elevator take to reach the maximum speed?

1.2 What distance does the elevator travel while accelerating?

1.3 What distance does the elevator travel at constant speed?

1.4 What is the total time required to reach the top floor at 400m ?

2. Calculus review (apologies for inflicting this on you but hopefully it will be helpful. We suggest doing the problems by hand rather than MATLAB; the problems are meant to help you brush up on calculus that sometimes appears on exams. The calculus in this problem appears again in later parts of the course, so try to remember it!): A particle starts at position $x = 0$ with speed $v = 0$ at time $t=0$. For each case below, please find formulas for the speed v and position x of the particle as functions of time.

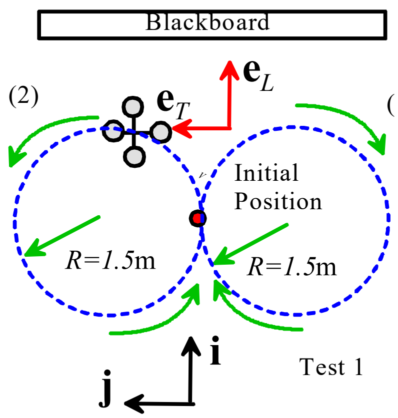
2.1 The acceleration of the particle depends on time $a(t) = A_0 \exp(-t) - 2A_0 \exp(-2t)$ where A_0 is a constant and $\exp(x) \equiv e^x$

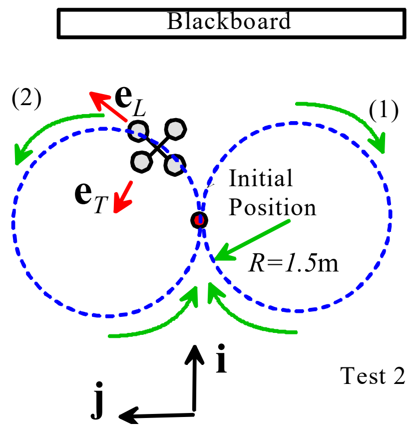
2.2 The acceleration of the particle depends on its speed as $a(v) = 1 - c^2 v^2$, where c is a constant

2.3 The acceleration of the particle depends on its position as

$$a(x) = 2\left(1 - 3\sqrt{x} + 2x\right)$$

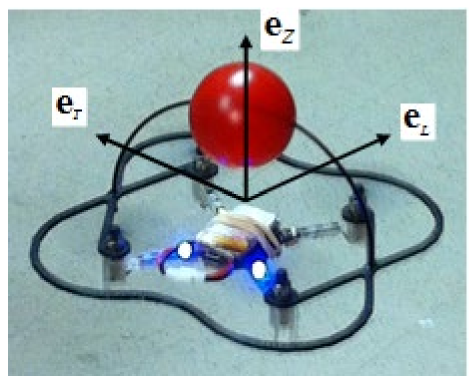
(You can solve this problem by finding v as a function of x first, then find x as a function of t, and finally use your answer to find v as a function of t. There are other ways too, if you know how to solve differential equations).





3. Using MATLAB to process position / acceleration / velocity measurements: [This video demonstration] shows a small quadcopter flying around a 'figure of eight' path, while recording its position, velocity and acceleration.

1. Position was measured using a radio system located in a fixed position in the room. The coordinate system is shown in the figure (the origin is just some random position determined by the location of the radio anchors; **i** is towards the blackboard, and **j** is to the left of the screen in the video).
2. Velocity, acceleration, and the yaw of the quadcopter were measured using on-board sensors. The sensors measure data in the $\{\mathbf{e}_L, \mathbf{e}_T, \mathbf{e}_N\}$ basis that rotates with the quadcopter (to keep things simple you can assume that the quadcopter remains approximately level).



The following flight was programmed into the controller:

(0) The quadcopter started on its flight stand, with the $\{\mathbf{e}_L, \mathbf{e}_T, \mathbf{e}_N\}$ quadcopter axes parallel to $\{\mathbf{i,j,k}\}$

(1) Ascend to the target altitude with constant vertical speed in 5 seconds

(2) Fly two circles with 1.5m radius at constant speed, in 15 seconds, i.e. the controller was programmed to fly the path

$$\mathbf{r} = \begin{cases} \mathbf{r}_0 + R\sin\omega(t-5)\mathbf{i} - R\{1-\cos\omega(t-5)\}\mathbf{j} & 5 < t < 12.5 \\ \mathbf{r}_0 + R\sin\omega(t-5)\mathbf{i} + R\{1-\cos\omega(t-5)\}\mathbf{j} & 12.5 < t < 20 \end{cases}$$

$$\omega = 4\pi/15\, rad/s \qquad R = 1.5m$$

(3) Descend to land on the ground in front of the lecture bench in 5 seconds

Two tests were run. In the first test, the quadcopter was programmed to keep a fixed orientation, with the $\{\mathbf{e}_L, \mathbf{e}_T, \mathbf{e}_N\}$ quadcopter axes parallel to $\{\mathbf{i,j,k}\}$ throughout the flight. Data from this test are stored in the file called Figure-eight-fixed.csv In the second test, the quadcopter turned so that the $\mathbf{e}_L$ direction was parallel to the (desired) direction of motion around the circle. Data from this test are stored in the file called Figure-eight-with-turn.csv. In both files, the first column of data is the time (s); the second-4th colums are (x,y,z) position, in m; the 5th-7th columns are velocity (v_L, v_T, v_N), in m/s ; the 8th-10th columns are acceleration (a_L, a_T, a_N), in m/s^2, and the last column is the yaw (in degrees).

3.1 Before trying to process the data, sketch graphs (you can do this by hand, or use MATLAB, or some other software is fine too if you are averse to MATLAB) showing:

(a) The expected variation of the velocity components (v_L, v_T) for the two tests during the time interval $5<t<20$ (assuming the path is a perfect circle, which of course it is not)….

(b) The expected variation of the acceleration components (a_L, a_T) for the two tests during the time interval $5<t<20$ (again, assuming perfectly circular motion)

3.2 Write a MATLAB script that accomplishes the following tasks (Your MATLAB code should be uploaded to CANVAS as a submission to this problem): For each data file

(a) Read the data into a matrix using the MATLAB 'csvread' command
(b) Plot the trajectory (on a 3D plot).
(c) Plot the measured (v_L, v_T) as a function of time
(d) Plot the measured (a_L, a_T) as a function of time

3.3 Add MATLAB code to check the consistency of the data. For the data recorded with fixed quadcopter orientation, calculate the velocity by integrating the accelerometer readings (a_L, a_T) using the 'cumtrapz' function, and (on the same graph) plot the measured (v_L, v_T).

Can this procedure be used to calculate the velocity from the acceleration with the data for the rotating quadcopter? If not, how would you determine the velocity components from the measured acceleration components? (you don't actually have to do the calculation, unless you are curious to see what happens)

4. Simple circular motion problem The figure shows the dimensions of the rotor from an 'ultracentrifuge' sold by Thermo-Fisher Scientific.

4.1 During operation, the rotor spins about its axis at a maximum speed of 100,000 rpm. Calculate the magnitude of the acceleration of a point at the 'maximum radius' of the rotor. Give your answer both in m/s^2 and also as the 'relative centrifugal force (rcf)' typically quoted by centrifuge manufacturers – which is the acceleration divided by the gravitational acceleration.

4.2 In one mode of operation the centrifuge spins up to a speed of 500rpm in one minute (with constant acceleration). Calculate the angular acceleration of the rotor, and hence calculate the magnitude of the acceleration of a point at the 'maximum radius' on the rotor at the instant the rotor reaches 500rpm.

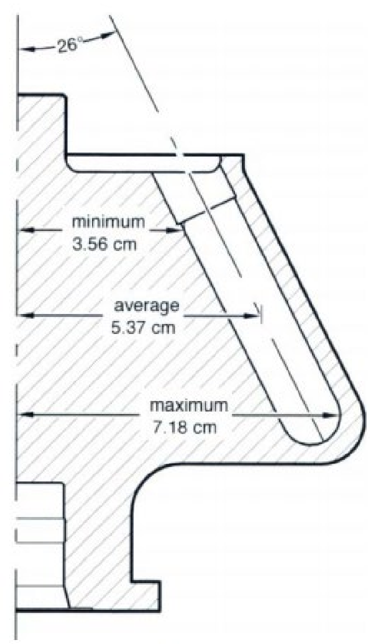


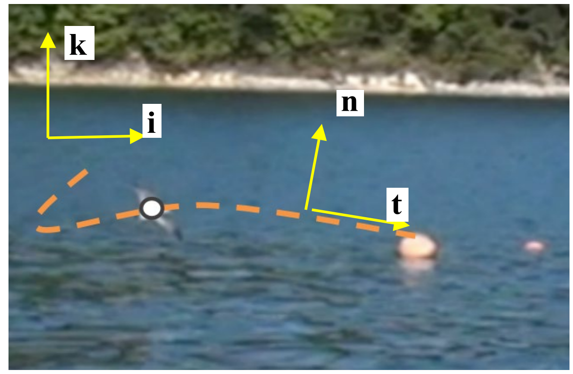
Figure 2-1. Rotor Radii

5. Normal-Tangential Coordinates. The *x,z* coordinates (*x* is horizontal, *z* is vertical) as a function of time of a seagull landing on a buoy (from this paper) can be fit by polynomials

$$x = -0.6471t^3 - 1.0593t^2 + 6.7421t - 0.0394$$

$$z = -0.8033t^3 + 2.0538t^2 - 1.5713t + 0.0039$$

with *x* and *z* in meters, and *t* in seconds. Use a MATLAB live script to:



5.1 Plot the trajectory (*z* –*v*- *x*) for a time interval $0<t<1.5$s

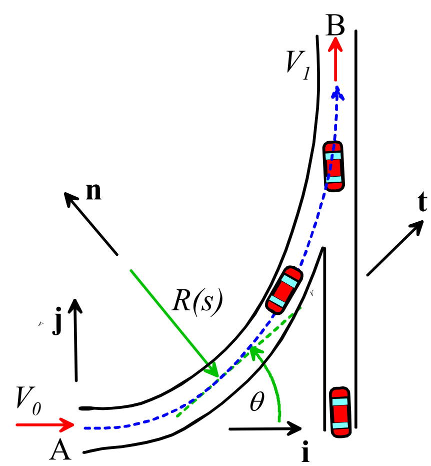
5.2 Plot the speed as a function of time, for a time interval $0<t<1.5$s

5.3 Let **n**,**t** be unit vectors normal and tangent to the path (take **n** to be the vector with a positive **k** component, as shown in the figure). Plot the normal and tangential components of acceleration (on the same plot), for a time interval $0<t<1.5$s.

5.4 The lift force on the seagull acts in the **n** direction, the drag force acts in the **-t** direction (note the negative sign!). The seagull has a mass of 0.37kg. Plot the lift and drag forces as a function of time for $0<t<1.5$s. (You will need to (i) draw a FBD for the gull; and (ii) use Newton's law and the calculations in 5.3 to find the lift and drag forces).

6. The goal of this problem is to calculate the optimum shape of a highway entry ramp. Vehicles enter the ramp with speed V_0 at *A* and merge with traffic moving with higher speed V_1 at *B*. They will travel with constant tangential acceleration a_t between A and B.

The magnitude of the vehicle's acceleration between A and B cannot not exceed $a_{\max}$ (a constant), otherwise the car will skid.



6.1 Find a formula for the car's speed V in terms of the distance (arc length) s travelled from A, the tangential acceleration a_t and V_0

6.2 The highway ramp is to be designed so that cars can travel with
(i) a constant tangential acceleration, and (ii) with acceleration magnitude equal to the maximum possible acceleration all the way from A to B. Show that the radius of curvature of the car's path must satisfy

$$\frac{d\theta}{ds} = \frac{1}{R(s)} = \frac{\sqrt{a_{\max}^2 - a_t^2}}{\left(V_0^2 + 2a_t s\right)}$$

(you can assume that $1/R = d\theta/ds$)

6.3 Hence (by integrating 6.2, noting that $\theta = 0$ at $s=0$), find a formula for the angle θ between the tangent to the curve and the **i** direction in terms of distance travelled *s,* (as well as $a_{\max}, a_t, V_0$)

6.4 Find formulas for (a) the velocity vector and (b) the acceleration vector of a car traveling along the ramp as a function of time, $V_0, a_t, a_{\max}$ in {**n,t**} coordinates (don't think too hard about this problem – you can just write down the answer. The velocity vector depends on time, but the acceleration vector is constant).

6.5 Assume that the entry ramp must turn through an angle of 90 degrees. Find formulas for the tangential acceleration a_t and the total arc-length of the ramp *L*, in terms of $V_1, V_0, a_{\max}$ (use the conditions that $V = V_1, \quad s = L$ at $\theta = \pi / 2$ to get equations you can solve for a_t and *L*) . Calculate values for these quantities for $V_1 = 55mph, \; V_0 = 25mph, \; a_{\max} = 0.2g$.

6.6 Note that the (*x,y*) coordinates of the point at a distance *s* along the ramp are related to the angle θ by

$$\frac{dx}{ds} = \cos\theta \qquad \frac{dy}{ds} = \sin\theta$$

By finding formulas for the parametric equations *x(s)*, *y(s)* of the curve of the ramp, plot the shape of the curve for $V_1 = 55mph, \; V_0 = 25mph, \; a_{\max} = 0.2g$. You only need to submit your plot, there is no need to submit MATLAB code. There is also no need to write down the formulas for *x(s),y(s)* – they are very messy. If you do the plot in a 'Live Script' using the 'fplot' function, you may (depending on your version of MATLAB) have to tell MATLAB to convert x and y to real numbers (the formulas may have a very small imaginary part because of rounding errors). You can do this with, eg
fplot(real(x),real(y),[0,L])