Lecture 4: rigid body dynamics examples

rigid body dynamics problems: 2D planar motion

- *Free Body Diagram!*
- *3 equations of motion:*

$$F_x = ma_x$$

$$F_y = ma_y$$

- *problem constraints*

$$M_z = I\alpha$$

- *mass moment of inertia calculation*
- *can we solve? if not, need more eqns:*
- *kinematics equations: connection between*

$$\alpha,\ \omega\ \text{AND}\ v,\ a$$

Rigid Body Kinematics

$$v_A = v_B + (\omega \times r_{A/B})$$

$$a_A = a_B + (\alpha \times r_{A/B}) + (\omega \times (\omega \times r_{A/B}))$$

- Useful Shortcuts for 2D planar motion

$$\alpha = \alpha \mathbf{k}$$
$$\omega = \omega \mathbf{k}$$
$$\mathbf{r} = r_x \mathbf{i} + r_y \mathbf{j}$$

$$\omega \times r = -r_y \omega \mathbf{i} + r_x \omega \mathbf{j}$$

$$\alpha \times r = -r_y \alpha \mathbf{i} + r_x \alpha \mathbf{j}$$

$$\omega \times (\omega \times r) = -r_x \omega^2 \mathbf{i} - r_y \omega^2 \mathbf{j}$$

Types of Constraints

- *Fixed Point A:*

  $V_A=0$

- *Sliding:*

  one velocity component will be zero

- *Gears:*

  equal linear velocities at point of intersection

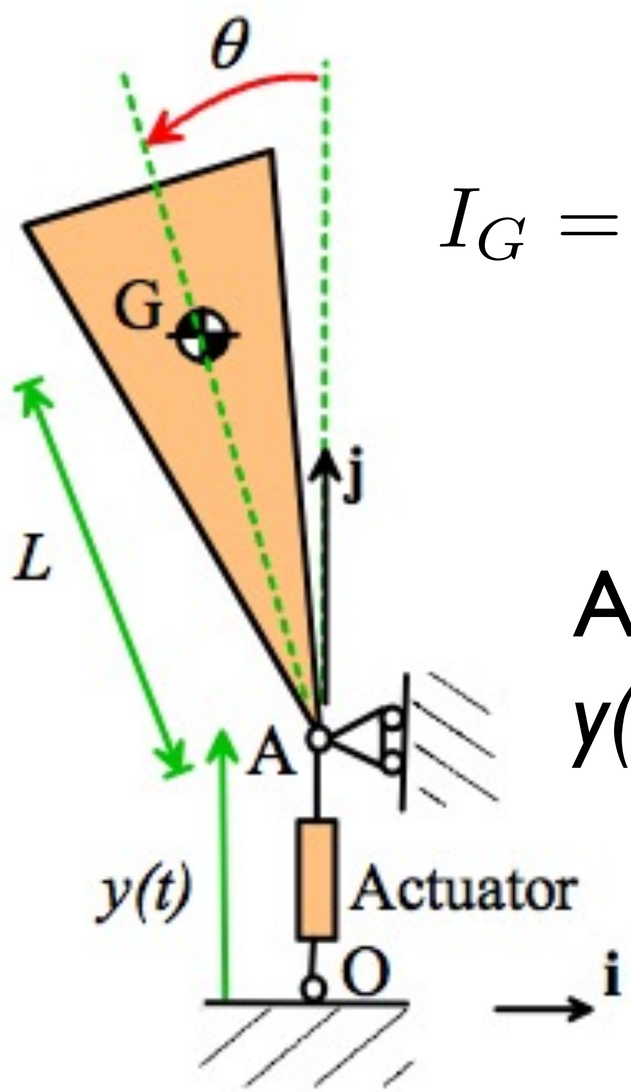
- *Inextensible Rope:*

  equal acceleration magnitudes

- *Pure Translation:*

  sum Moments = 0

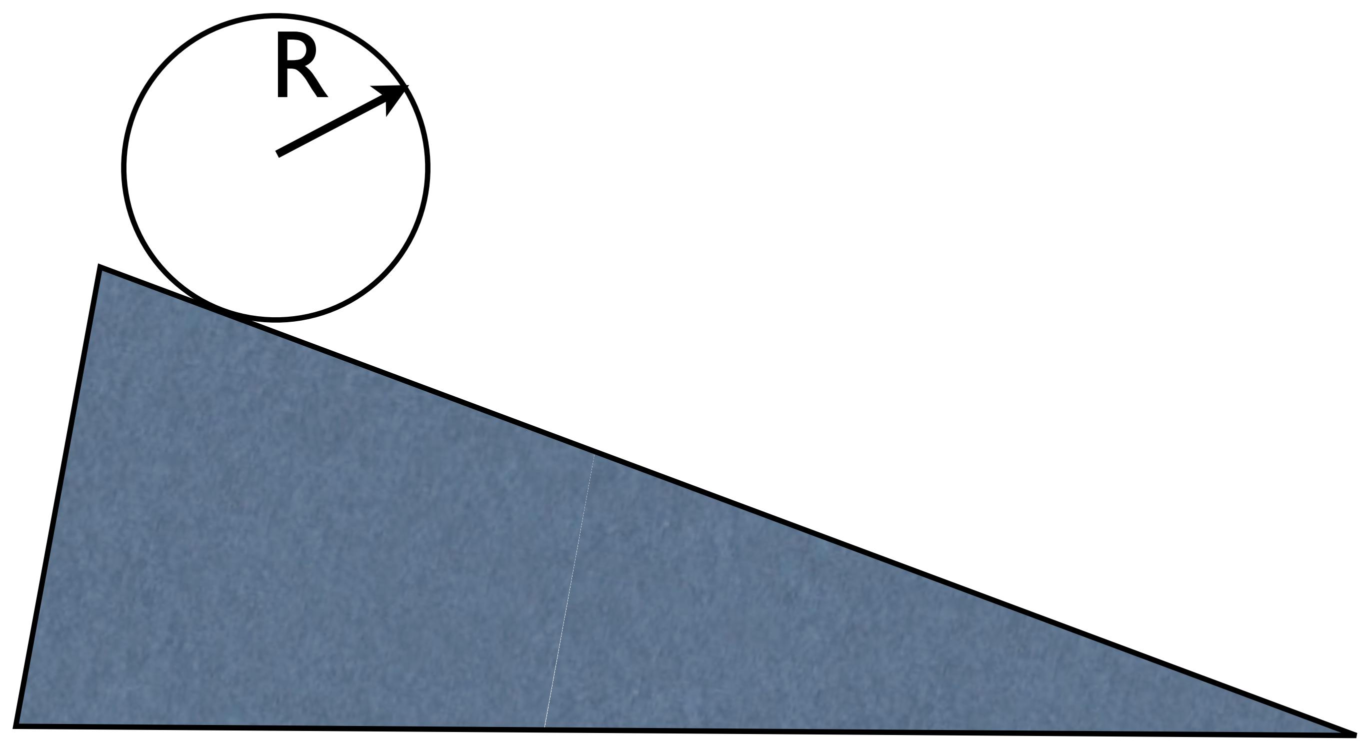
Inverted Pendulum Example



$$I_G = \frac{1}{10} m L^2$$

Actuator has displacement
y(t)=-sin(wt)

Rolling Cylinder: No Slip



who will win?

A: hollow cylinder

Ro = 0.037 m

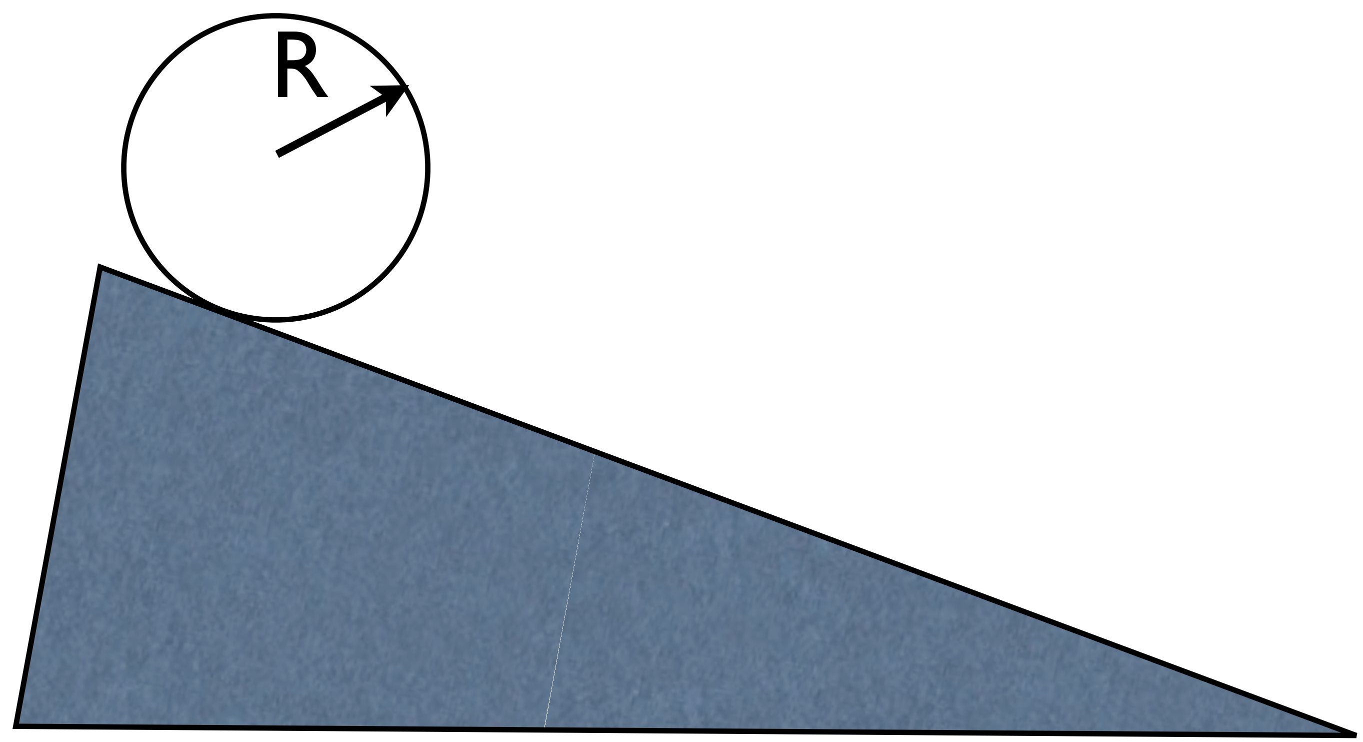
m = 0.258 kg

B: solid cylinder

R = 0.037 m

m = 0.494 kg

Rolling Cylinder: No Slip



who will win?

B: solid cylinder
$R = 0.037\ m$
$m = 0.494\ kg$

C: solid cylinder
$R = 0.050\ m$
$m = 2.42\ kg$

Boeing 747 Landing

landing speed = 260 km/hr
aircraft mass = 400 tonnes
16 rear wheels (assume all contact the ground at the same time)
The lift on the aircraft when the aircraft just lands is 95% of the aircraft weight.
coefficient of kinematic friction between the wheel and the runway is 0.5
mass of a tire = 125 kg
tire radius of gyration = 75 cm [moment of inertia = mass * (radius of gyration)^2]
Radius of the tire = 1 m

(i) Upon landing, how far would the tires skid before they begin to roll without slipping (this is also the length of the skid marks that the aircraft leaves on the runway)?
(ii) What's the skidding time?
(iii) What is the deceleration of the aircraft while it's skidding?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