In 1934, French entomologist Antoine Magnan (1881-1938) included the following passage in the introduction to his book *Le Vol des Insectes*: “ *Tout d'abord poussé par ce qui se fait en aviation, j'ai appliqué aux insectes les lois de la résistance de l'air, et je suis arrivé avec M. Sainte-Laguë à cette conclusion que leur vol est impossible.* ” This translates to: “ First prompted by what is done in aviation, I applied the laws of air resistance to insects, and I arrived, with Mr. Sainte-Laguë, at this conclusion that their flight is impossible. ” Magnan refers to his assistant André Sainte-Laguë, a mathematician..

http://www.youtube.com/watch?v=-yZPrrboTkY

EN4 : Vibrations March, 2013

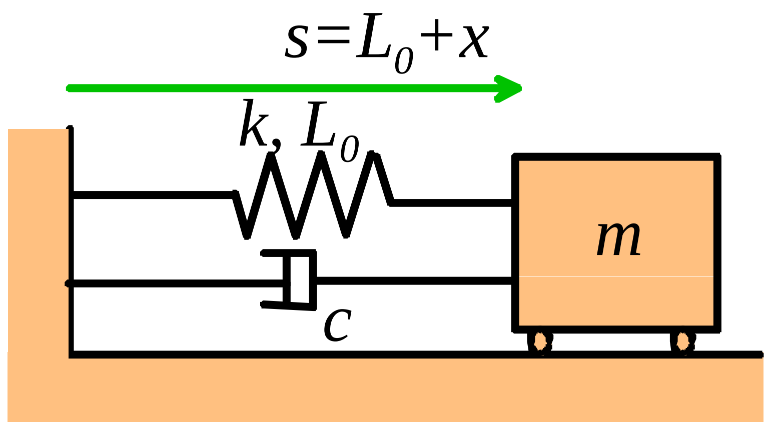
Lectured by K.-S. Kim

3/7 Lecture 11: Free Vibrations:
Natural frequency & Graphical representation
Atomic Clock / GPS and Friction Oscillator

3/12 Lecture 12: Free Vibrations:
Multiple component systems, Linearization, etc.
Dvorak / Cherry Tree Shaker and **LASER**

3/14 Lecture 13: Free Vibrations:
Degree of freedom and Modes
Violin / Everest and **IR Spectrophotometer**

3/19 Lecture 14: Damped Free Vibrations:
Transient response and Damping criticality
Car Suspensions and **Atomic Force Microscope**



$$\mathbf{F} = m\mathbf{a} \Rightarrow \frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = 0 \qquad \omega_n = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{c}{2\sqrt{km}}$$

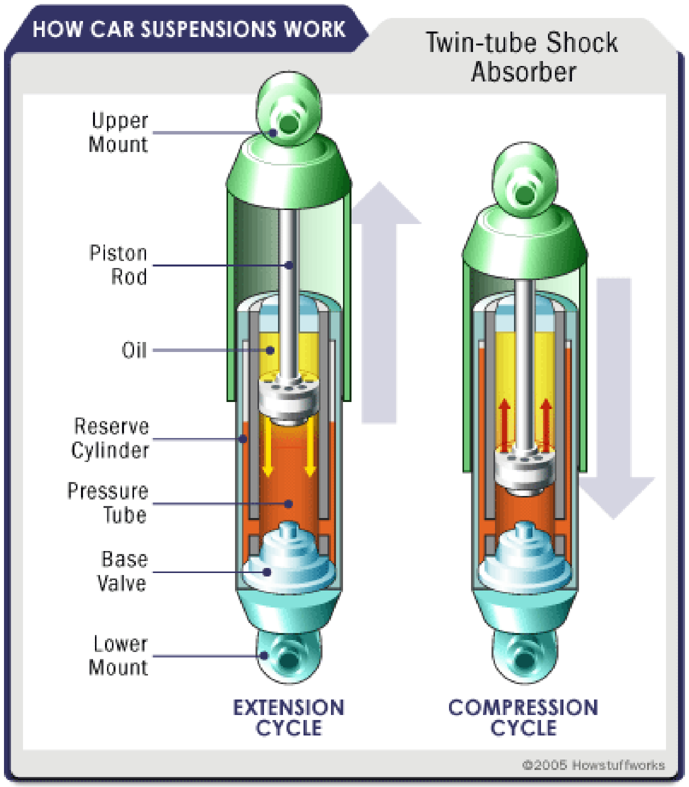
Lecture 14: Damped Free Vibrations:
Transient response and Damping criticality

Properties of Newtonian Fluid

$$F = \mu A \frac{\dot{x}}{d}$$

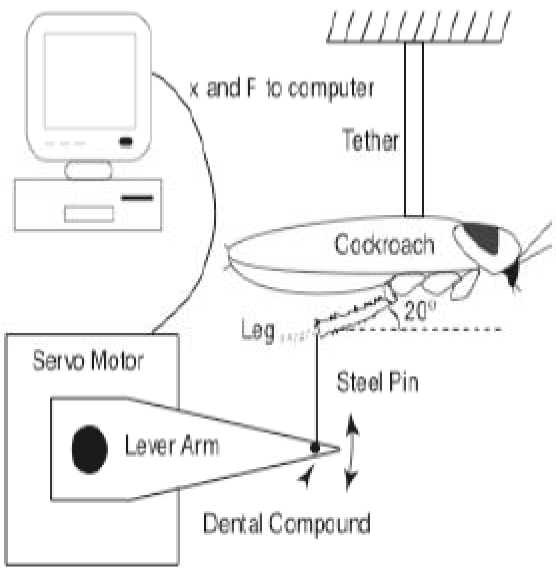
μ : viscosity

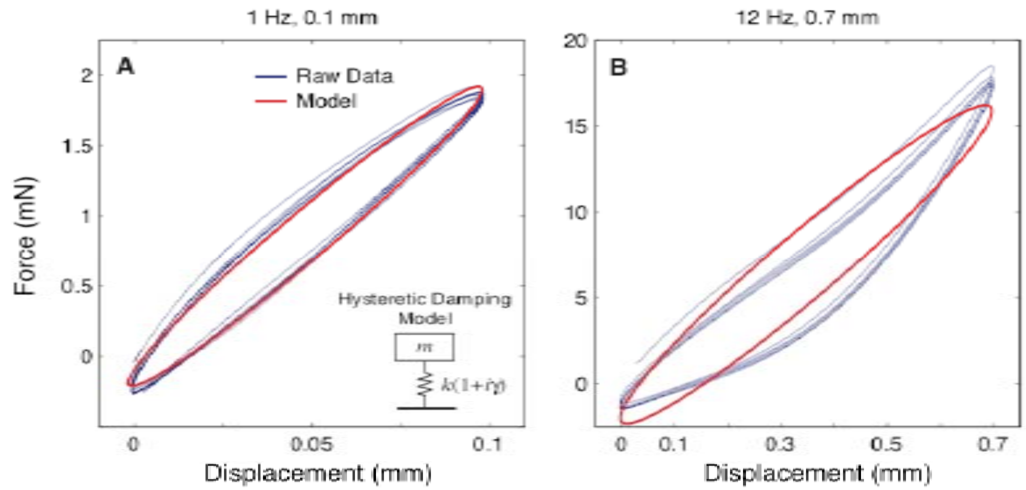
Car suspension & shock absorber

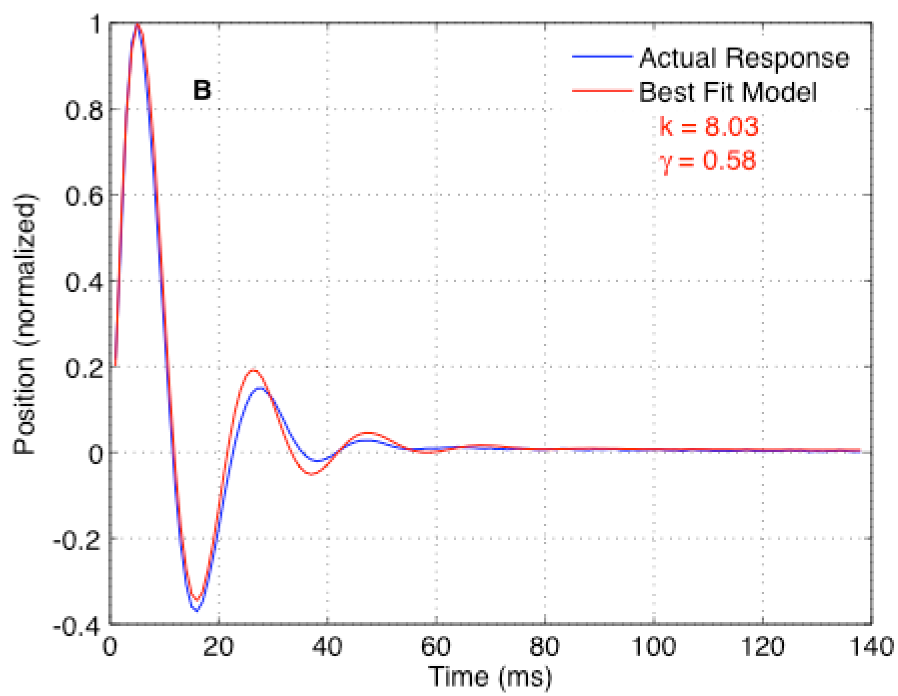


http://www.youtube.com/watch?v=mHj4vnSEdtg

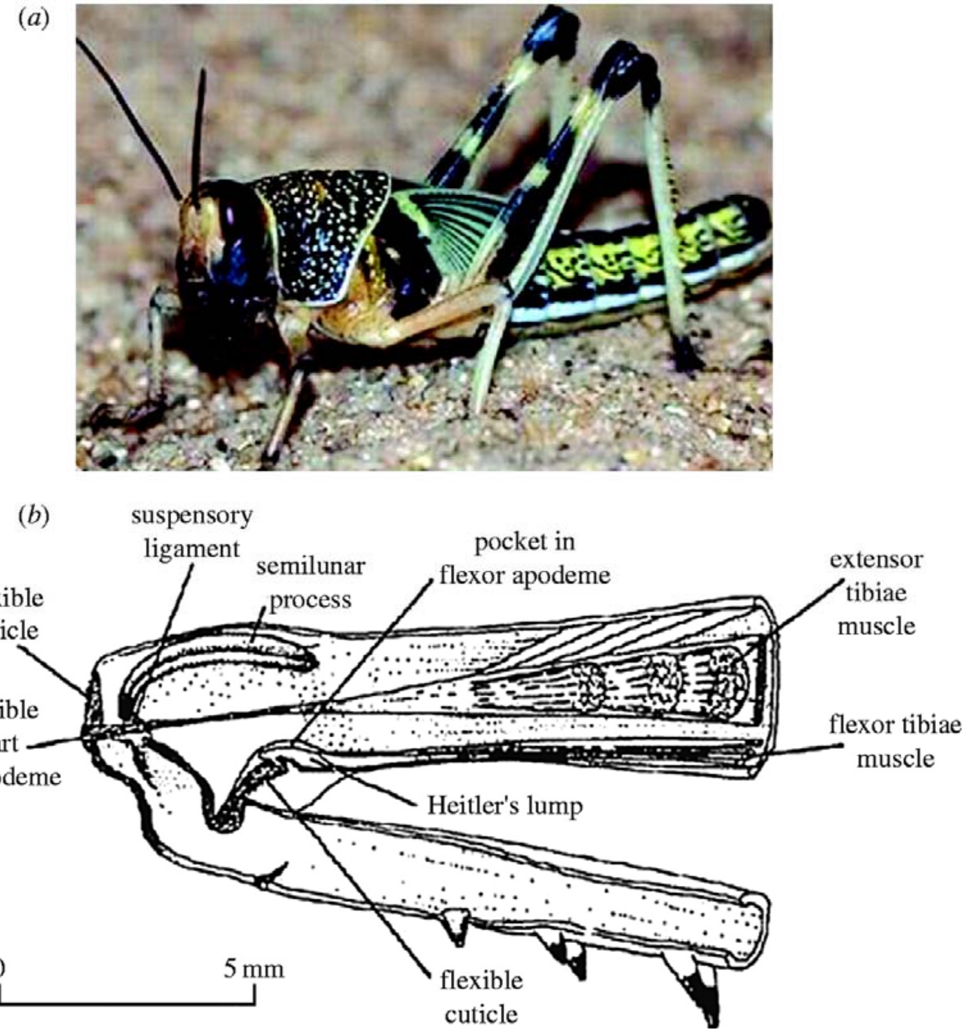
D Free Coxa Preparation







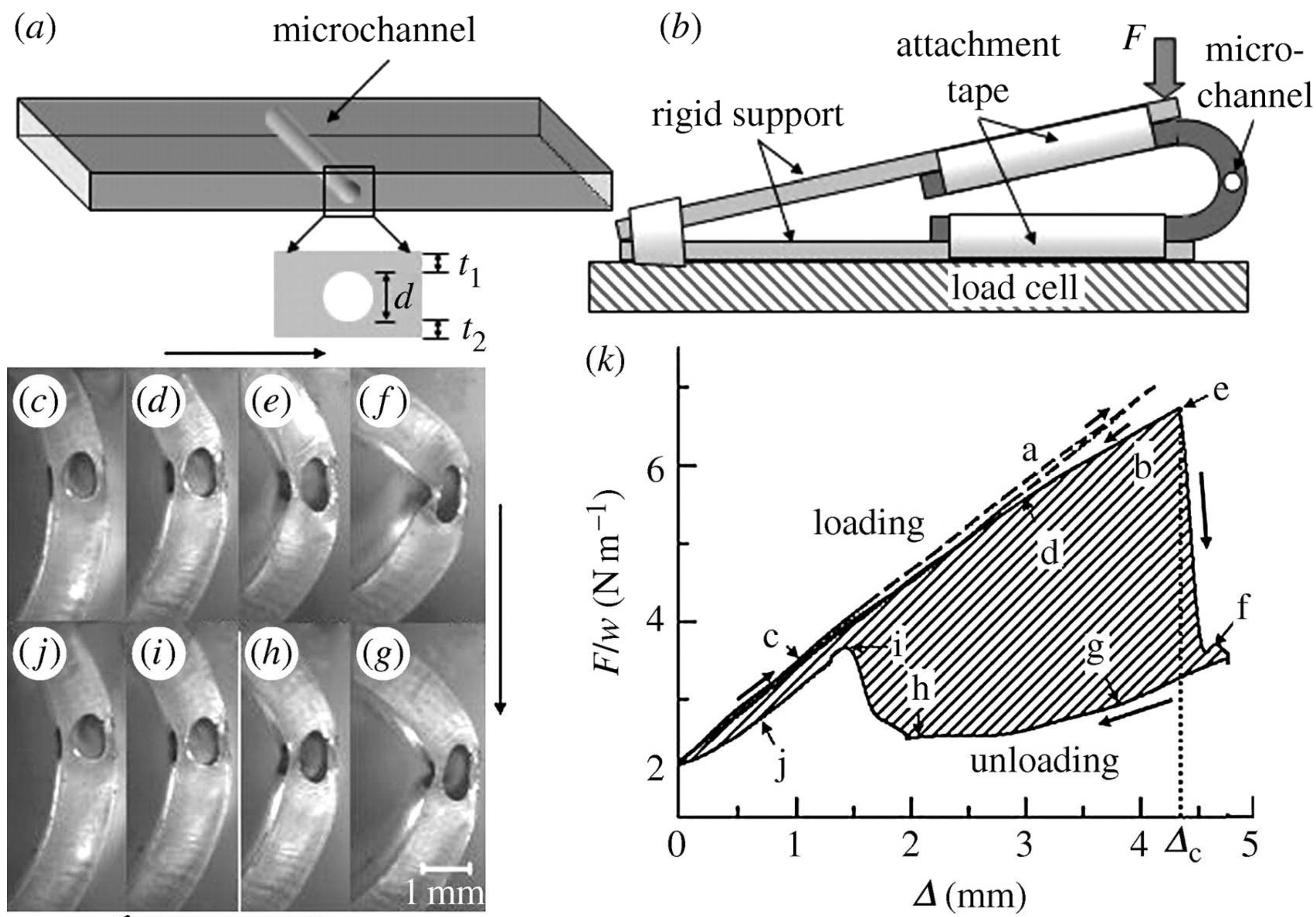
(a) Locust and (b) the simplified internal anatomy of its leg (adapted from fig. 1 on p. 6 of Bennet-Clark (1975)) showing the fluid-filled chamber in the leg including at the joint between the femur and tibia.



Ghatak A et al. J. R. Soc. Interface 2009;6:203-208

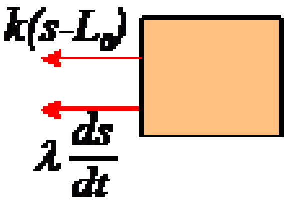
Interface

Schematic of the experiment.



Ghatak A et al. J. R. Soc. Interface 2009;6:203-208

To proceed, we draw a free body diagram, showing the forces exerted by the spring and damper on the mass.



Newton's law then states that

$$k(s-L_0)+\lambda\frac{ds}{dt}=ma=m\frac{d^2s}{dt^2}$$

$$\Rightarrow\frac{m}{k}\frac{d^2s}{dt^2}+\frac{\lambda}{k}\frac{ds}{dt}+s-L_0=0$$

This is our equation of motion for s.

Now, we check our list of solutions to differential equations, and see that we have a solution to:

$$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2}+\frac{2\varsigma}{\omega_n}\frac{dx}{dt}+x=0$$

We can get our equation into this form by setting

$$s=L_0+x \qquad \omega_n=\sqrt{\frac{k}{m}} \qquad \varsigma=\frac{\lambda}{2\sqrt{km}}$$

As before, ω_n is known as the natural frequency of the system. We have discovered a new parameter, ς, which is called the **damping coefficient**. It plays a very important role, as we shall see below.

Overdamped System $\varsigma > 1$

$$x(t) = \exp(-\varsigma\omega_n t)\left\{\frac{v_0 + (\varsigma\omega_n + \omega_d)x_0}{2\omega_d}\exp(\omega_d t) - \frac{v_0 + (\varsigma\omega_n - \omega_d)x_0}{2\omega_d}\exp(-\omega_d t)\right\}$$

where $\omega_d = \omega_n\sqrt{\varsigma^2 - 1}$

Critically Damped System $\varsigma = 1$

$$x(t) = \left\{x_0 + \left[v_0 + \omega_n x_0\right]t\right\}\exp(-\omega_n t)$$

Underdamped System $\varsigma < 1$

$$x(t) = \exp(-\varsigma\omega_n t)\left\{x_0\cos\omega_d t + \frac{v_0 + \varsigma\omega_n x_0}{\omega_d}\sin\omega_d t\right\}$$

where $\omega_d = \omega_n\sqrt{1 - \varsigma^2}$ is known as the damped natural frequency of the system.

In all the preceding equations,

$$x_0 = s_0 - L_0 \qquad v_0 = u_0$$

are the values of x and its time derivative at time $t=0$.

Diamagnetic Levitation Lateral Force Calibrator (D-LFC)

Q. Li, K.-S. Kim and A. Rydberg, Rev. Sci. Inst., 77(6), 065105, 2006.

$$k = m\omega^2$$

$$k \approx 10 \text{ pN/nm}$$