1. The projectile in a gas gun is subjected to a propulsive force

\[ F = F_0 \left(1 - \frac{v}{3c}\right)^5 \]

where \( F_0 \), c are constants and \( v \) is the projectile's speed. The projectile has mass \( m \). It starts at rest at time \( t=0 \) at position \( x=0 \).

1.1 Use Newton’s law to determine the acceleration of the projectile and hence determine an expression for its speed as a function of time and other parameters. Air resistance and friction may be neglected.

\[ F = ma \quad \Rightarrow \quad a = \frac{dv}{dt} = \frac{F_0}{m} \left(1 - \frac{v}{3c}\right)^5 \]

Separate, vary & integrate

\[ \int_0^v \frac{dv}{(1 - \frac{v}{3c})^5} = \int_0^t \frac{F_0}{m} \, dt \]

\[ \frac{3c}{4} \left[ \frac{1}{(1 - \frac{v}{3c})^4} \right]_0^v = \frac{F_0}{m} \cdot t \]
$v = 3c \left\{ 1 - \frac{1}{\left(1 + \frac{4F_0}{3mc} t\right)^{1/4}} \right\}$

1.2 Find a formula for the distance traveled by the projectile as a function of time.

$$v = \frac{dx}{dt}$$

$$\int_0^x dx = \int_0^t 3c \left\{ 1 - \frac{1}{\left(1 + \frac{4F_0}{3mc} t\right)^{1/4}} \right\} dt$$

$$\Rightarrow x = 3c t - 3c \left[ \frac{mc}{F_0} \left(1 + \frac{4F_0}{3mc} t\right)^{3/4} \right]_0^t$$
\[ x = 3ct - \frac{3mc^2}{F_0} \left[ (1 + \frac{4F_0t}{3mc})^{3/4} - 1 \right] \]

Velocity a function of position

\[ \frac{dx}{dt} = V = V_0 \left( 1 - \frac{x}{x_0} \right) \]

Separate variables

\[ \int_0^x \frac{dx}{(1 - x/x_0)} = \int_0^t V_0 \, dt \]

\[ -x_0 \ln \left( 1 - \frac{x}{x_0} \right) \bigg|_0^x = V_0 t \]

\[ -x_0 \ln \left( 1 - \frac{x}{x_0} \right) = V_0 t \Rightarrow x = x_0 \left( 1 - e^{-\frac{V_0 t}{x_0}} \right) \]
An airport “People Mover” travels at constant speed $V$ around a circular path with radius $R$.

(a) Write down the acceleration of the vehicle in the $\{n, t\}$ basis

$$\alpha = \frac{V^2}{R} \hat{n}$$

(b) The figure shows a passenger in the vehicle. Draw a FBD showing the forces acting on the person.

(c) Find formulas for the reaction forces acting on the passenger in terms of $m, g, V, R$. Not all the forces can be determined uniquely.
\[ F = ma \Rightarrow -(T_A + T_B) \hat{n} + (N_A + N_B - mg) \hat{k} = m \frac{V^2}{R} \hat{n} \]

\[ \sum M_c = 0 \quad (T_A + T_B) h + (N_B - N_A) \frac{d}{2} = 0 \]

\[ T_A + T_B = -m \frac{V^2}{R} \]

\[ N_A + N_B = mg \]

\[ -N_A + N_B = -(T_A + T_B) \frac{h^2}{d} = \frac{m V^2}{R} \frac{2h}{d} \]

Add \( 2N_B = mg + m \frac{V^2}{R} \frac{2h}{d} \)

\[ 2N_A = mg - m \frac{V^2}{R} \frac{2h}{d} \]
(d) Find a formula for the minimum allowable radius of the path that allows the passenger to remain standing, in terms of $g, V, h, d$

For no tipping $N_a > 0$, $N_b > 0$

Here $N_a$ is concern

$N_a > 0 \Rightarrow R > \frac{V^2}{2gh} \frac{2h}{gd}$
3. Two spheres with identical mass and restitution coefficient $e=0$ have initial positions shown in the figure below. Before impact sphere B is stationary and sphere A has velocity $V_i$. The collision is frictionless. By answering the true/false questions below, identify which of the figures (a-d) shows the correct position of the spheres after collision.

(a) Total Momentum is conserved in the $j$ direction.
Momentum of B is conserved in the $t$ direction.
The restitution formula is satisfied in the $n$ direction.

\[ \dot{p} = m \dot{v} \]
\[ \text{Initial momentum} \neq 0 \]

(B): Initial momentum $\neq 0$
in $t$ dir
Final nonzero

\[
\text{Restitution } (V_n^B - V_n^A) = -e (V_n^A - V_n^B)
\]

Hence distance between particles in $n$ dir is fixed.
(b) Total Momentum is conserved in the \( \mathbf{j} \) direction  
Momentum of B is conserved in the \( \mathbf{t} \) direction  
The restitution formula is satisfied in the \( \mathbf{n} \) direction  

\[ T \quad F \quad T \quad F \]
(c) Total Momentum is conserved in the \( j \) direction
Momentum of B is conserved in the \( t \) direction
The restitution formula is satisfied in the \( n \) direction