Special Edition

Design Project 4
Solar Car

ENGN0040: Dynamics and Vibrations
Allan Bower, Yue Qi

School of Engineering
Brown University

Topics for todays class

Design Project 3: Solar Car

1. Introduction to project

2. Background Theory
   Equations for DC motors
   Equations for solar panel I-V curves
   Friction losses

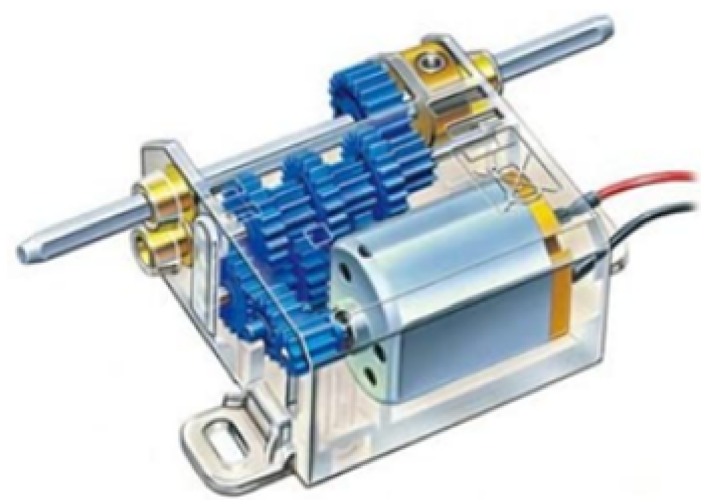
3. Design Calculations – predicting optimal gear ratio & wheel diameter

Design Project 3: Solar Car

Goal: Design a solar car to climb a ramp at maximum speed

Components provided:

1. 4x 3V 200mA Solar Modules
2. DC Motor
3. Gearbox
4. Wheels
5. Materials to construct chassis



Design Project 3: Solar Car

Goal: Design a solar car to climb a ramp at maximum speed

Ramp Details:

1. Length: 48", Width: 12"
2. Inclination: 0 to 17.5 degrees (2.5 deg increments)

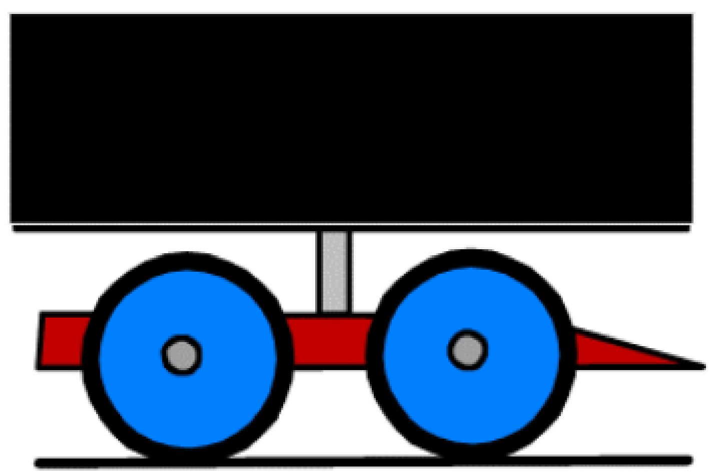
Approach:

(1) Select transmission ratio (gears, wheel radius) that will extract max average power from solar panels and motor.

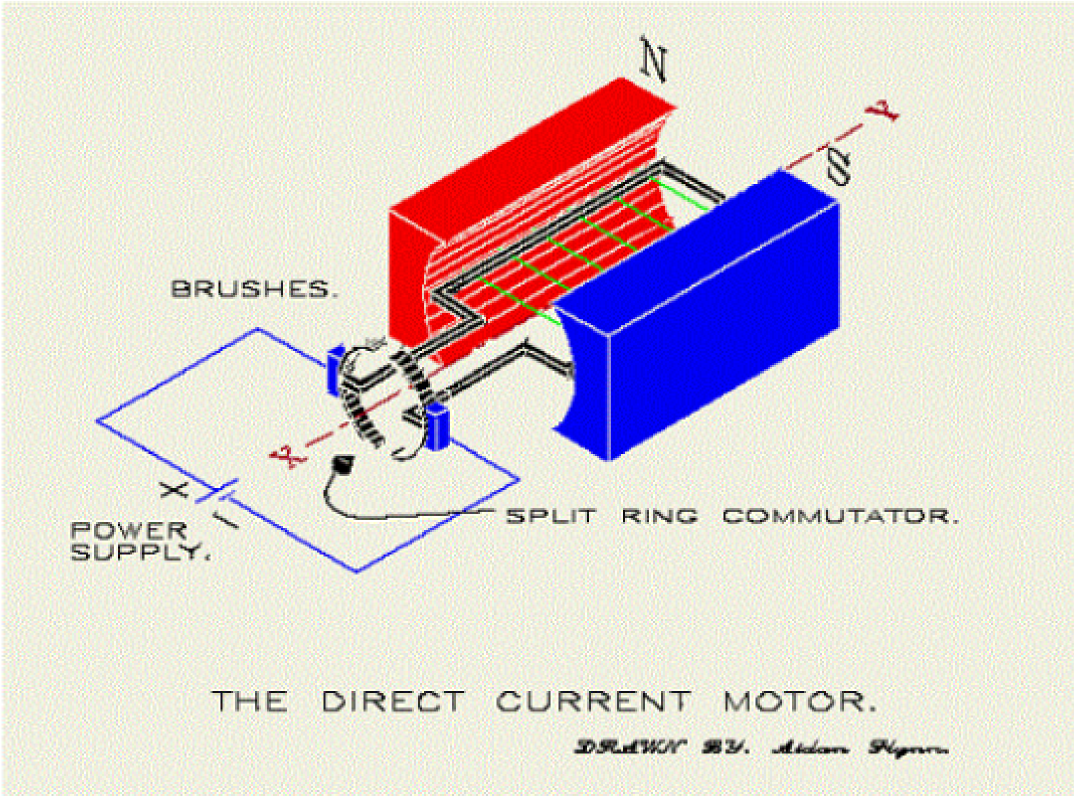
(2) Construct lightest possible vehicle with lowest friction.

Suggested design/test schedule

1. Design chassis
   - Needs to support panels and motor and have smallest possible mass.
   - Helpful to be able to changes gears/wheels quickly
   - You will probably want to be able to adjust angle of panels
   - You can (optionally) 3D print parts – but there is an early deadline to submit STL files for printing
2. Develop MATLAB code to predict optimal gear ratio & wheel diameter, given slope, mass, friction losses & panel characteristics
3. Construct vehicle
4. Run tests to characterize solar panels; measure friction losses
5. Predict optimal gear ratio & wheel diameter as function of slope
6. Compete!



Background Theory: DC Motors

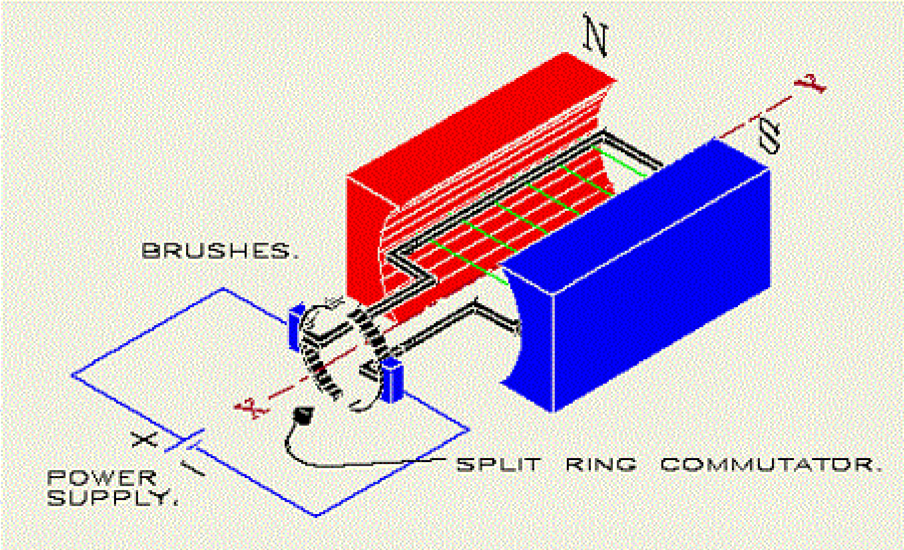


Power supply (e.g. Battery, or solar cell) applies voltage *V* to motor
Voltage induces current flow *I* through winding
Current interacts with magnetic field to generate torque *T* and motor speed ω

We need:

(1) Equations relating *V*,*I*,*T*, ω
(2) A way to calibrate the equations for the motor provided

Equations for DC motors



Goal: Relate V, I, T, ω

Physics:

(1) Winding has resistance R

(2) If motor rotates, motion of coil through magnetic field induces a "back EMF" $\beta\omega$

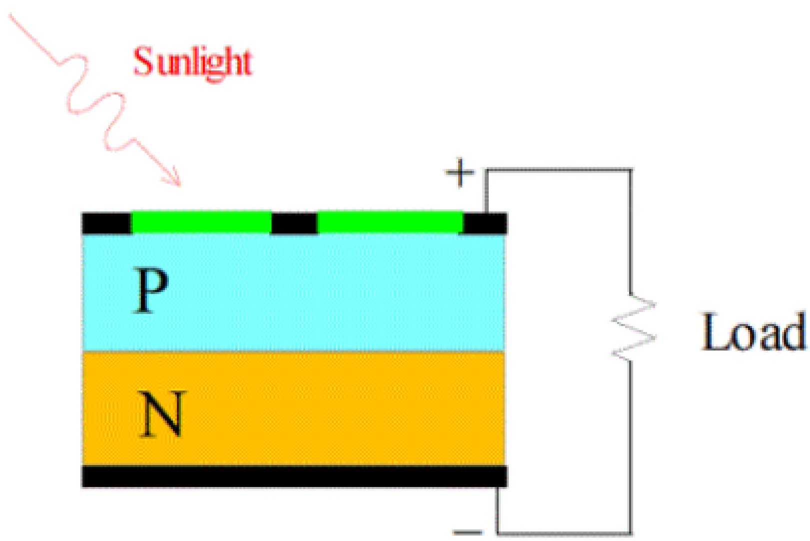
(3) Current flow through wire generates torque $T = \beta I$

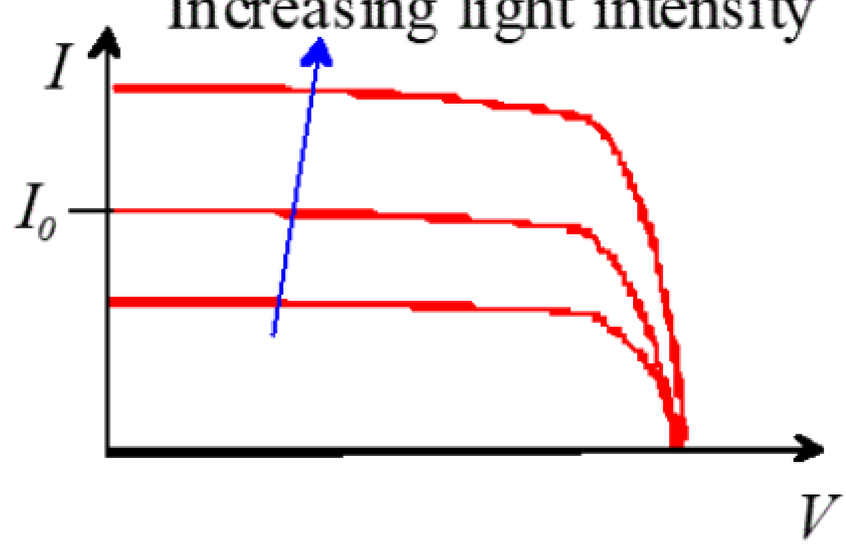
(4) Friction / eddy currents reduce torque by $T_0 + \tau_0 \omega$

$$I = (V - \beta\omega)/R \qquad T = \begin{cases} \beta I - T_0 - \tau_0 \omega & \beta I > T_0 \\ 0 & \beta I < T_0 \end{cases}$$

Background Theory: Solar Panels

- A photoelectric cell is a junction between P/N semiconductors – (it behaves like a diode)
- Illumination provides energy to promote electrons (or holes) into conduction band. The P/N junction forces the charge carriers to flow through the external circuit



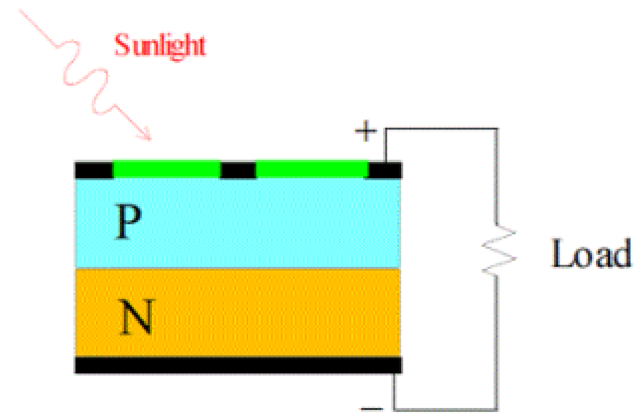


We need:

(1) Equations relating *V,I*

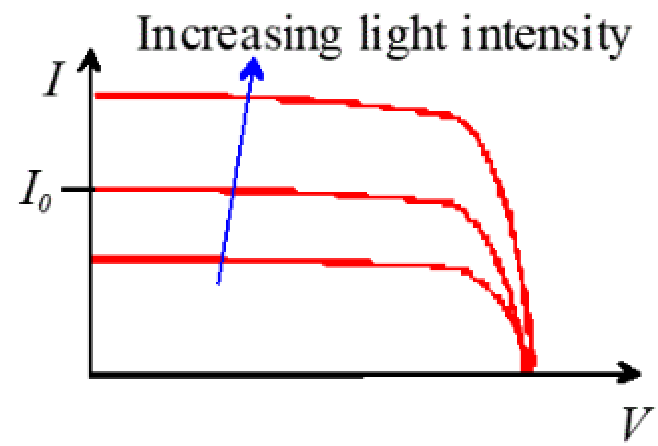
(2) A way to calibrate the equations for the panels provided

Solar panel equation



(1) Charge separation @ P-N junction generates voltage jump across junction

(2) Photoelectric effect generates e^-/hole pairs : e^- can flow through external circuit



$$I = I_0 - I_1\left[\exp\left(\frac{qV}{nkT}\right) - 1\right]$$

I_0 : Forward current (depends on light intensity)

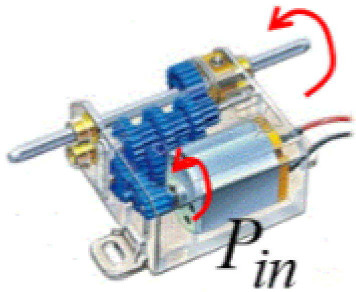
I_1 : "Reverse saturation current" (constant)

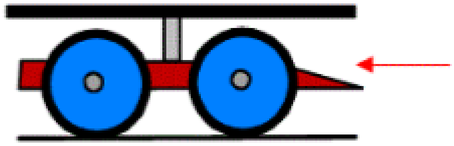
q : electron charge magnitude

k : Boltzmann constant

n : # cells in panel (used as a constant here)

Energy Losses





2 main causes:

① Friction in gearbox
(proportional to transmitted power)

② Axle / rolling friction / air resistance

Gearbox: $P_{out} = \eta P_{in}$

P_{out}: useful power; P_{in}: Power from motor

Use $\eta \approx 0.2$ as initial approx ; measure later

Axle / Rolling friction

$$P_{loss} = C_R v + C_D v^2$$

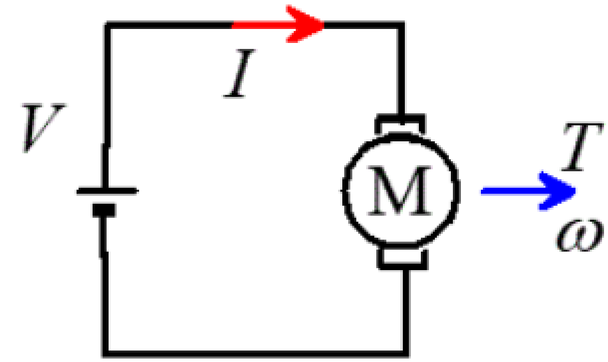
C_R, C_D constants (Measure)

Design Problem #1: Given the following experimental data (in project description):

At 3 Volts

- No load speed ω_{nl}
- No load current I_{nl}
- Stall torque T_s
- Stall current I_s

Find $\{R, \beta, T_0, \tau_0\}$



Approach: substitute test vals into motor eqs

① No load experiment $T=0$ $\omega=\omega_{nl}$ $I=I_{nl}$

$\Rightarrow \beta I_{nl} - T_0 - \tau_0 \omega_{nl} = 0$ $\quad (V - \beta\omega_{nl})/R = I_{nl}$

② Stall experiment $\omega = 0$ $T = T_s$ $I = I_s$

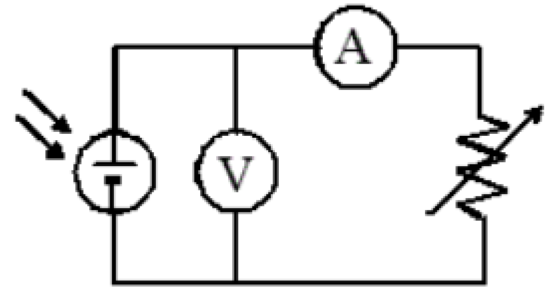
$\Rightarrow \beta I_s - T_0 = T_s$ $\quad V/R = I_s$

4 equations for 4 unknowns — solve!

Design Problem #2: Given:

(1) Measured I-V curve for a solar panel (your data)

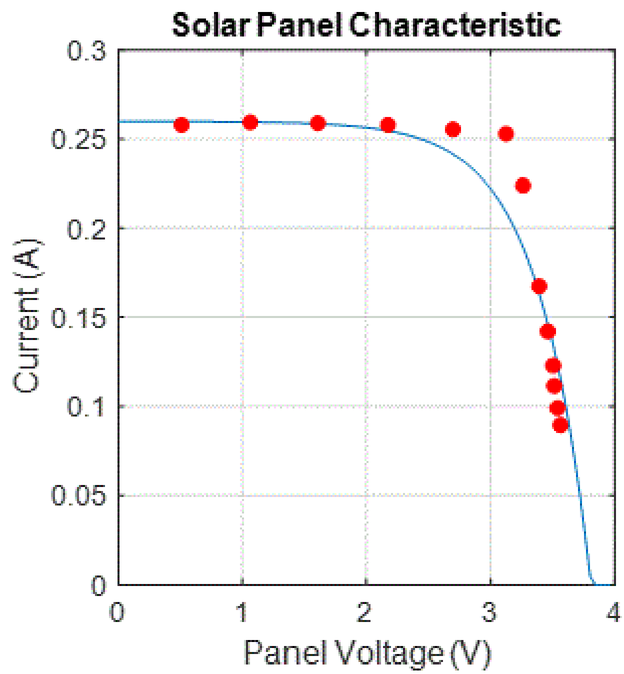
(2) Values for electronic charge *q*, Boltzmann constant *k*, reverse saturation current I_1



Find I_0, n for your panel(s)

Procedure: ① Measure (I, V) curve for panel (see project description)

② Plot experimental data and adjust I_0, n to get an approx. fit

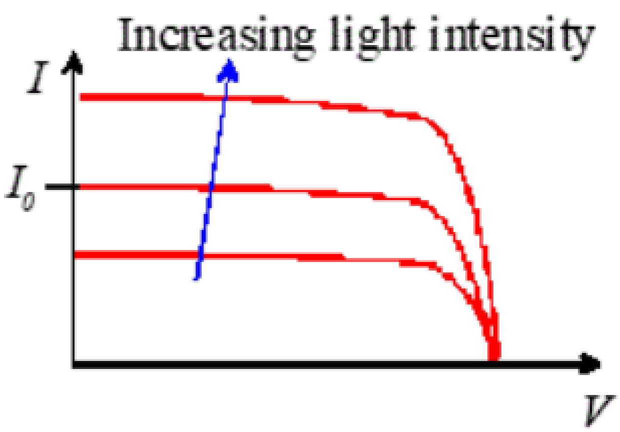


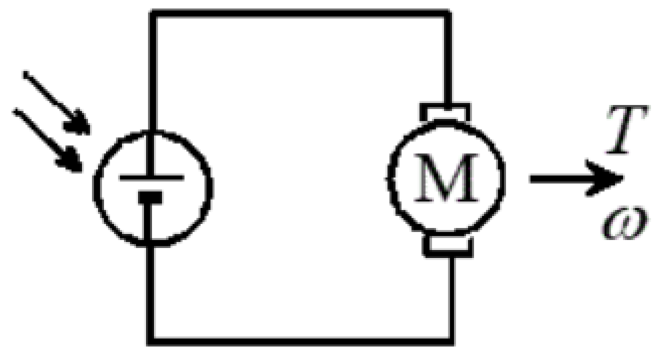
For preliminary design calcs can use nominal values

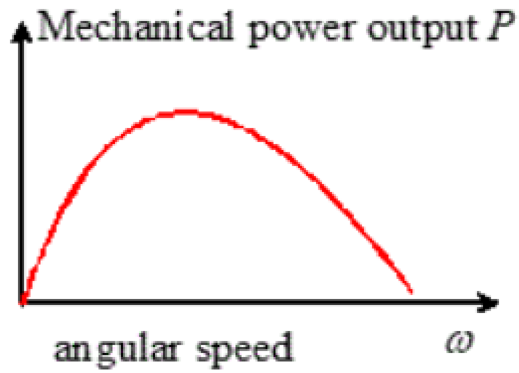
$I_0 = 200$ mA

$n = 16$

Design Problem #3: Plot the power and torque curves for panel/motor







① Panel equation: $I = I_0 - I_1 [\exp(qV/nkT) - 1]$

[Note: you will use 4 panels in parallel
⇒ multiply I for 1 panel by 4 in motor eq.]

② Motor $\omega = (V - IR)/\beta$ $\quad T = \beta I - T_0 - \tau_0 \omega$

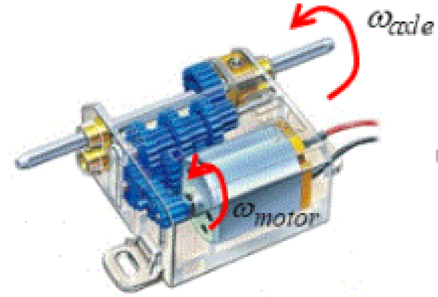
③ Power $P = T\omega$

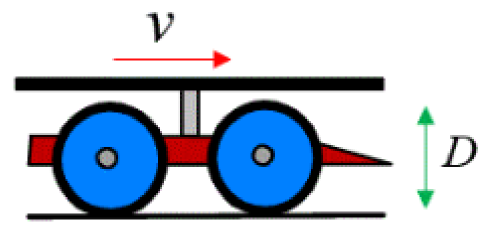
Plot T-v-ω & P-V-ω as a parametric plot (voltage V is the parameter $0 < V < 3.6$)

Design Problem #4: Given:

(1) Diameter of car's wheels D

(2) Gear ratio in transmission $\rho = \omega_{motor} / \omega_{axle}$





Find a formula relating car speed v and ω_{motor}

Rigid body kinematics

(1) Rolling wheel formula

(2) Gear formulas

See Episode 17 of lecture recordings

Design Problem #5: Given:

(1) Diameter of car's wheels *D*

(2) Gear ratio in transmission $\rho = \omega_{motor} / \omega_{axle}$

(3) Motor equations $I = (V - \beta\omega) / R$ $\quad T = \begin{cases} \beta I - T_0 - \tau_0 \omega & I > T_0 / \beta \\ 0 & I < T_0 / \beta \end{cases}$

(4) Solar panel equation $I = I_0 - I_1 (e^{qV/nkT} - 1)$

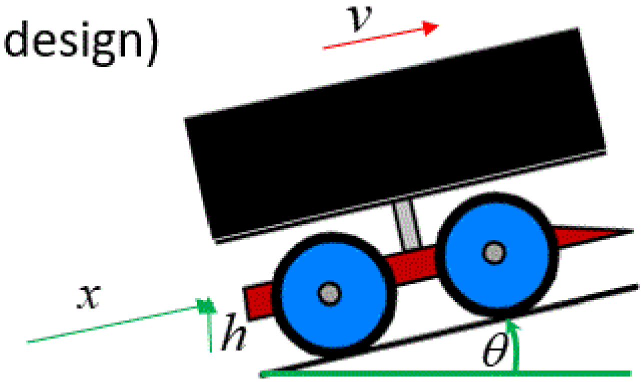
(5) Gearbox efficiency η (use 0.2 for preliminary design)

(6) Slope (between 0 and 17.5 degrees)

(6) Rolling friction & air drag loss coefficients
(use zero for preliminary design)

(7) Car speed *v*

Find a way to calculate the car's acceleration



Procedure:

① Find ω_{motor} (Use DP#4)

② Substitute for V in panel eq from first motor eq
Solve for I (need to use "fsolve" in MATLAB – see tutorial

③ Find torque T from 2nd motor eq

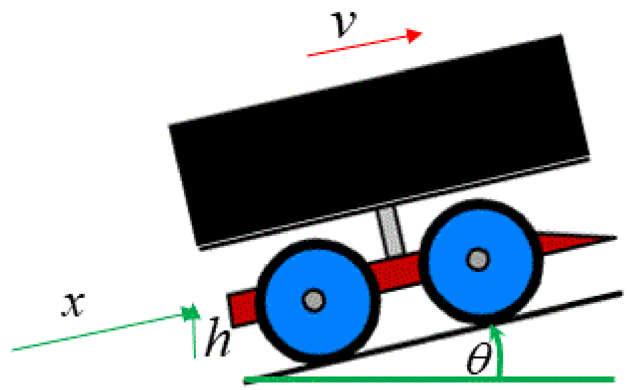
④ Motor power = $T\omega_{motor}$

Power-energy relation for vehicle

Total useful power $P = \eta T \omega_{motor} - C_R v - C_D v^2$

$$\Rightarrow \frac{d}{dt}\left(T + U \right) = P$$

$$\Rightarrow \frac{d}{dt}\left\{ \frac{1}{2} m v^2 + mgh \right\} = P$$



$$h = x \sin\theta$$

$$\Rightarrow m v \frac{dv}{dt} + mg \frac{dx}{dt} \sin\theta = P$$

$$\Rightarrow \frac{dv}{dt} = \frac{1}{m}\left\{ \underbrace{\eta T \frac{\omega_{motor}}{v}} - mg \frac{v}{v} \sin\theta - C_R - C_D v \right\}$$

Depends on wheel diameter D and gear ratio e

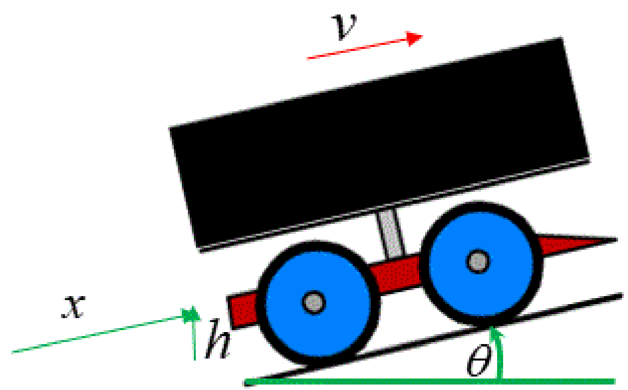
Design Problem #6: Given

(1) Solution to Design Problem #5

(2) Car starts from rest

(3) Slope and Length of test track

Find the time to reach the end of the test track



Re-write differential eq from DP#5 in MATLAB form

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} v \\ \frac{1}{m}\left\{ \eta T \frac{W_{motor}}{v} - mg\sin\theta - C_R - C_D v \right\} \end{bmatrix}$$

Solve with ODE45

Use "event" function to detect $x = L$

& hence find time

Test code with sensible guess for (D, ρ)