

# EN4 Dynamics and Vibrations

## Design Project

### Design and construction of a solar powered vehicle

#### Synopsis

In this project you will design and construct a small solar powered vehicle to climb a ramp at the maximum possible speed.

#### 1. Organization

- This project may be done individually or in groups of up to four students.
- The project 'deliverables'; are (i) An assembled and tested vehicle; (ii) A short (2-3 page) report (one report per group) describing your design, and describing the calculations you used to design it; (iii) a short oral presentation describing your design and design procedures; and (iv) an evaluation of the performance of your team-mates on the project.
- Materials for building your vehicle can be found in and near the Peterutti Design lab within the Prince Laboratory (the usual place...).
- Workshop facilities for building your vehicle will be open between Mon April 30 and Mon May 7. You may also take your vehicle away and work on it in your dorm room, if you prefer – but please don't hoard vast quantities of supplies in your dorm room. **DO NOT REMOVE THE SOLAR PANELS FROM PRINCE LAB** – they are too expensive to provide one per group, so you will have to share panels. Make sure you design your vehicle so that the panels can quickly be attached and removed from the car.
- You will demonstrate your design to faculty or TAs on Tuesday May 8th. Just prior to testing, you should give a short (10 min) presentation describing your design procedure. Your written report is due at the presentation.
- Note that solar panels produce very little power – far less than a battery – so even if your car works under battery power it may not function with the panels. Constructing your vehicle to minimize friction and mass will be very important. There are clever ways to use the materials provided to do this, so be creative! Also expect to be doing a good deal of trouble-shooting.
- **Warning:** We only have 5 ramps with light bars to test vehicles. Unlike the pendulum project, we will not be able to accommodate large numbers of groups testing their designs at the same time. If a large number of teams leave construction and testing to the day before the test date it may be impossible for everyone to get access to the test ramps. Your team will also be able to work much more efficiently if you come to the workshop at less crowded times – the lab is usually nearly empty early in the first week, and during the 9am and lunchtime timeslots. It will also make construction much easier for everyone if you can help by returning unused tools and materials once you are done with them – this will help minimize the time people need to spend hunting around for tools and supplies. We experienced much more pressure on facilities, supplies, and tools in the pendulum project this year than usual (possibly because your class did not go through the dreaded lunar project, which traumatized people so badly they always started future projects really early...). Although we've run the solar car project for several years without difficulty, there may be unanticipated pressures this year.

## 2. Overview of design requirements

The goal of this project is to design and construct a small solar-powered vehicle that will climb a ramp at the maximum possible speed, using materials and supplies supplied for this purpose. A detailed list of materials and supplies can be found in Appendix B. Supplies include

1. A set of six solar modules;
2. A brushed DC permanent magnet motor; with a built-in gearbox that can be modified to provide 6 different transmission ratios;
3. Perforated Al sheet, which can be cut and bent to build a chassis;
4. Poster-board and rubber O rings, which can be cut to build wheels with a wide range of different radii;
5. 8-32 threaded rod, which can be used as axles;
6. A set of miter gears, which are helpful to drive the axle; and
7. Assorted hardware – spacers, nuts, collars, etc that can be used to assemble the vehicle.

The solar panels must be carried by your vehicle – you can't save weight by attaching the panels to the vehicles using wires. The solar panels and motor must also be the sole power source for your vehicle.

To complete the design, you will need to:

- (i) Design a power transmission system that will ensure that the car climbs the slope at maximum speed;
- (ii) Assemble and test the vehicle.
- (iii) Document your design calculations and design specification in a report (one per group).

There are four particularly important design considerations:

1. The transmission and wheels must be designed to extract the maximum power from the solar panels and electric motor;
2. The total weight of the vehicle must be minimized;
3. Friction losses must be avoided as far as possible (very difficult!)
4. You need to place the solar panels as close to the light source as possible – **but not so close that they will melt** (and since the spots illuminate a relatively small area, try to design the system so that a few panels remain close to the spotlight as the vehicle moves past the light-bank)

This project description provides all the information you need to design the transmission system.

## 3. Design calculations

The goal of your calculation is (i) To determine the motor speed that yields the maximum power output from the transmission, and the corresponding maximum power; (ii) to use energy methods to compute the resulting car speed as it moves up the ramp; (iii) knowing the car speed and motor speed, to determine the transmission ratio; and (iv) use your knowledge of gear ratios and rigid body kinematics to design a transmission that will achieve the desired ratio. Note that the calculations are only a rough design guide – because a large (and unknown) fraction of the power output from the motor is lost in friction, so you can't predict the optimal design very accurately. If you have time, you can attempt to measure the friction loss after you've constructed your vehicle, which will allow you to refine the calculations. Your design calculations should involve the following steps:

1. Start by characterizing the solar panels. Using EXCEL, plot the experimental data provided in Appendix A1 to show a graph of current provided by the solar cell as a function of voltage. You can plot each dataset if you wish, but it is likely you will want the panels to be as close to the light sources as you are allowed. On the same graph, plot the predictions of equation (1) with

some suitable guess for  $I_0$  and  $n$ . Then adjust the values of  $I_0$  and  $n$  to give a reasonable fit to the experiment (there is no need to be very precise).

2. Next, determine values for the parameters characterizing the electric motor. You will need to determine values for the parameters  $\beta, R, T_0, \tau_0$  in the motor equations (2) and (3), from the values provide for the stall current (measured by us), and the stall torque, no-load speed and no-load current provided on the manufacturers data sheet. The procedure for doing this is discussed in Section 4.1.11 of the online notes. Be very careful with units...
3. Now you are ready to analyze the behavior of the power transmission system. Note that the behavior of the system you are designing is very unlike a conventional electric motor that is powered by a battery, so the formulas in the online notes for the optimal motor speed don't work. You need to analyze the behavior of the entire system – panels and motor together. We suggest doing this using an EXCEL spreadsheet organized as shown below:

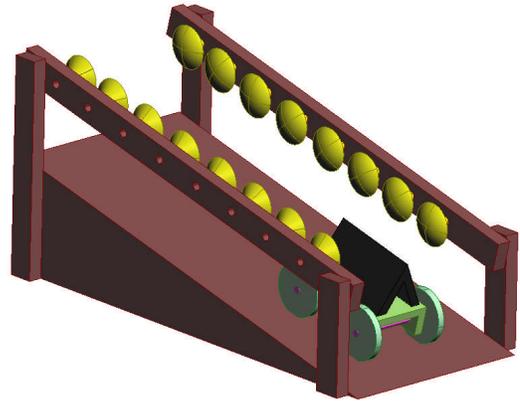
Panel Voltage	Current	Motor Speed	Motor Torque	Motor output power
V	A	rad/s	Nm	Watts
0				
0.1				
0.2				
0.3				
0.4				
<i>Etc</i>				

Here, the first column is a possible operating point on the solar panel characteristic (somewhere between 0 and about 3.5V. The second column is the current output by the solar panels at the appropriate voltage, computed using the solar panel equations (don't forget you are using 6 panels, connected in parallel). The third and fourth columns are the motor speed and torque, computed using the motor equations. Finally, the last column is the power output from the motor. Once you have completed the table, plot the motor power output as a function of the motor speed. This will tell you the motor speed corresponding to the maximum power output.

4. Finally, assume that some fraction of the motor output power will be lost to friction (you have to guess this – it will depend on how good a job you do of building your car. Losses can be as much as 80%). You will then know how much useful power is available to drive the car up the ramp. Using energy methods (available power = rate of change of potential energy) determine the vehicle speed as a function of the slope. Note your calculation will predict an infinite vehicle speed for a zero slope – this is clearly nonsense – but for higher slopes the calculation is more accurate.
5. Now you know the desired car speed and motor speed – you can proceed to design a system of gears and wheels to achieve the necessary ratio. The manufacturer's datasheet for the motor includes a table of all the possible gear ratios that can be accomplished using the gears that come with the motor. You can add other more complicated gears if you wish. Of course, the more gears you use, the greater the friction losses. Note that if you accidentally run your motor with speed below the max power output, it may stall.

#### 4. Testing your vehicle

The figure shows a rendering of the test-track for your vehicle. It consists of a ramp, which has banks of incandescent light-bulbs attached to each side to illuminate the solar panels.



The ramp is 48” long and 12” wide, and its inclination can be adjusted.

You will test your vehicle as follows:

1. Measure the speed for two different ramp angles between 0 degrees and 15 degrees to be specified on the test day (so you should be ready to adjust your vehicle’s gear ratios/wheel diameters to give you the optimum speed for any ramp angle)
2. Configure your vehicle to climb the steepest possible ramp angle (which may exceed 15 degrees if you wish) and demonstrate that it is capable of climbing the ramp. **We recommend that you do not test your vehicle on steep ramps – this subjects the transmission to large loads, and can cause failures which will be difficult, or impossible, to fix (eg the gears separate from the shafts). Leave this test to the test day.**

**THE SOLAR PANELS MUST BE AT LEAST 4” FROM THE LIGHTS THROUGHOUT THE TEST!**

#### 5. Report Guidelines

Your report should describe your design, and outline the calculations and experiments that you used to select critical parameters in the design.

Here is a suggested outline for the report:

1. Executive Summary - a short description of the purpose of the report; an outline of its contents; and a short summary of the conclusions
2. Design Specifications – a description of your design, highlighting special features and giving details of important design parameters (e.g. the total weight of the vehicle, and its transmission ratio).
3. Design Calculations – describe the procedure you used to determine the transmission ratio and any other critical design parameters.

#### 6. Grading Rubric

Your project will be graded as follows:

Successfully built and tested vehicle: (20 points)  
Oral presentation: (10 points)  
Project Report (10 points)  
Peer Evaluation (10 points)

**(50 Points)**

## Appendix A: Theory

### A1 Characteristics of solar panels

You can read a short summary of the features and operating principles of solar cells at <http://zone.ni.com/devzone/cda/epd/p/id/5918>. Here, we summarize the information that you will need to design your vehicle.

A photovoltaic cell is a semiconductor device that converts incident radiation into electrical current. Cells are usually connected together into a *module* or *array* of cells, to increase their electrical power output. You will be provided with six such modules, connected together in parallel. Each module is advertised as generating a nominal current of 100 mA under '1 sun irradiation' (specified by standard as 1360W/m<sup>2</sup> of solar energy) so in theory your panels could generate as much as 0.6A of current.

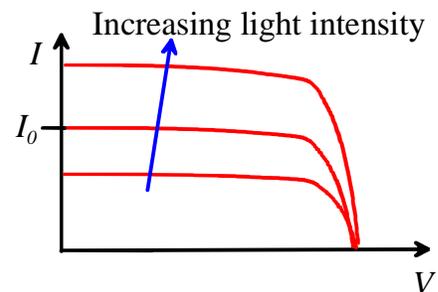
To design your vehicle, you will need to know the electrical current and voltage that your panel generates. The current generated by a solar module can be approximated by

$$I = I_0 - I_1(e^{qV/nkT} - 1) \quad (1)$$

Here:

1.  $I_0$  is the current generated by the photovoltaic effect. The value of  $I_0$  depends on how the panel is illuminated – for full sunlight, it should be 100mA for each module (600mA total). In your application, the value of  $I_0$  may be less than this, and will vary with the distance of the modules from the incandescent light bulbs that illuminate the cells.
2.  $I_1$  is the 'reverse saturation current' (the small electric current that flows through the cell when it is not illuminated, and an external power source such as a battery applies a negative voltage across its terminals). The value of  $I_1 \approx 30\mu A$
3.  $n$  is a dimensionless constant that depends on the number and characteristics of the photovoltaic cells in the module;
4.  $q$  is the magnitude of the charge of an electron  $1.609 \times 10^{-19} \text{ C}$
5.  $V$  is the voltage across the cell (this will depend on what is connected to the cell)
6.  $k$  is the Boltzmann constant  $1.380 \times 10^{-23} \text{ J/K}$
7.  $T$  is temperature, in Kelvin.

A typical series current-voltage curves for a module is sketched in the figure.



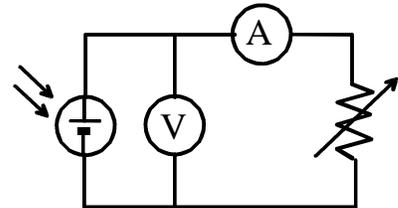
To use this formula, you will need to determine values for the constants  $I_0, n$  in the formula. You can do this using the data listed in the next section. The easiest way to do this is to plot the experimental data, and then find the values of  $I_0, n$  that make the theoretical curve fit (approximately) the data. You don't need to be too precise about this, because your design calculations are approximate and small errors in the values of  $I_0, n$  are negligible compared with the effects of other approximations.

### Measurement of solar panel characteristics

The TAs have measured the characteristics of the solar panels for you. If you don't trust them, you may repeat their measurements... If you choose to do so, the following equipment is provided for this purpose:

1. An ammeter (for measuring the current generated by the panels)
2. A voltmeter (to measure the voltage)
3. A variable resistor (to apply an electrical load to the panel)
4. An incandescent light bulb (identical to those that will illuminate your vehicle while it climbs the ramp).

You should connect the voltmeter, ammeter and resistor to the solar panel as illustrated in the circuit diagram. By varying the resistor, you can obtain a series of measurements of current and voltage generated by the panel. Repeat your measurement for several different distances of the panel from the light source.



Data obtained by the TAs are listed in the table below. These data are for *one* panel – if you connect six panels in parallel (by connecting the wires from each panel directly to the motor terminals) the current supplied to the motor will be equal to the sum of the current generated by each panel.

Panel 6" from light source	
Voltage (V)	Current (mA)
0.734	94.8
0.85	111
1.05	106.7
2.21	96.4
2.57	92
2.91	72
3	64.5
3.04	57.4
3.06	54.9
3.09	38.57
3.13	48
3.18	16.59
3.24	6.13
3.25	3.22
3.26	2.19
3.27	1.47
3.28	1.18
3.314	1.01
3.35	0.9

Panel 9" from light source	
Voltage (V)	Current (mA)
0.493	59
0.519	61.9
0.79	61.2
1.299	59.6
1.63	58.2
2.198	55
2.38	52
2.72	44.7
2.73	44
3.018	16.5
3.105	3.4
3.13	2.26
3.134	1.43
3.148	1.1
3.156	0.89
3.173	0.85

Panel 12" from light source	
Voltage (V)	Current (mA)
0.312	40.2
0.379	39.81
0.593	40
0.873	39.98
1.179	39.45
1.606	38.31
2.012	36.57
2.12	35.77
2.7	26.05
3	8.78
3.03	4.03
3.049	4.89
3.05	2.46
3.055	1.13
3.059	1.5
3.06	3.28
3.06	1.92
3.079	0.83

Note that these data are temperature sensitive – if the panels get too hot, they generate much less current than these measurements might suggest. So it's good to be conservative in your design calculations.

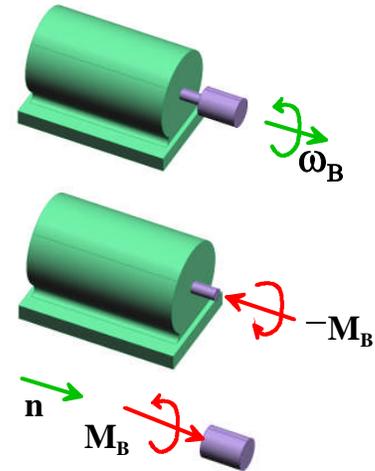
## A2 Characteristics of DC motors.

The behavior of a brushed DC permanent magnet motor is described in detail in Section 4.1.4 of the course notes. The most important characteristics of the motor are summarized briefly here, for convenience.

- An electric motor applies a *moment*, or *torque* to an object that is coupled to its output shaft. The moment is developed by electromagnetic forces acting between permanent magnets in the housing and the electric current flowing through the winding of the motor. In the following discussion, we shall assume that the body of the motor is stationary  $\omega_A = \mathbf{0}$ , and its output shaft rotates with angular velocity  $\omega_B = \omega \mathbf{n}$  where  $\mathbf{n}$  is a unit vector parallel to the shaft and  $\omega$  is its angular speed. In addition, we assume that the output shaft exerts a moment  $\mathbf{M}_B = T \mathbf{n}$ .

- Power to drive the motor will be supplied by connecting the solar panels directly to its terminals. The panel applies a voltage  $V$  and current  $I$  to the motor, which are related by the solar panel equations described in the preceding section.

- The moment applied by the output shaft is proportional to this electric current.



The electric current  $I$  flowing through the winding is related to the voltage  $V$  and angular speed of the motor  $\omega$  (in radians per second) by

$$I = (V - \beta\omega) / R \quad (2)$$

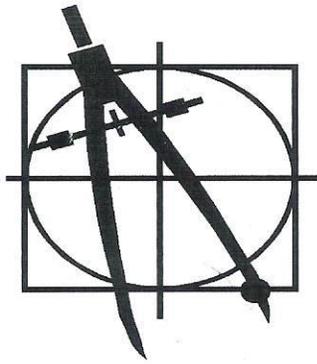
where  $R$  is the electrical resistance of the winding, and  $\beta$  is a constant that depends on the arrangement and type of magnets used in the motor, as well as the geometry of the wire coil.

The magnitude of moment  $T$  exerted by output shaft of the motor is related to the electric current and the speed of the motor by

$$T = \begin{cases} \beta I - T_0 - \tau_0 \omega & I > T_0 / \beta \\ 0 & I < T_0 / \beta \end{cases} \quad (3)$$

where  $T_0$  and  $\tau_0$  are constants that account for losses such as friction in the bearings, eddy currents, and air resistance.

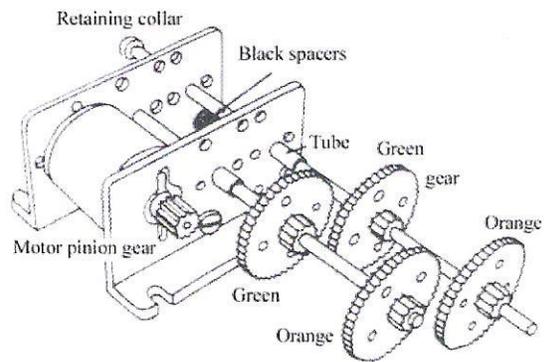
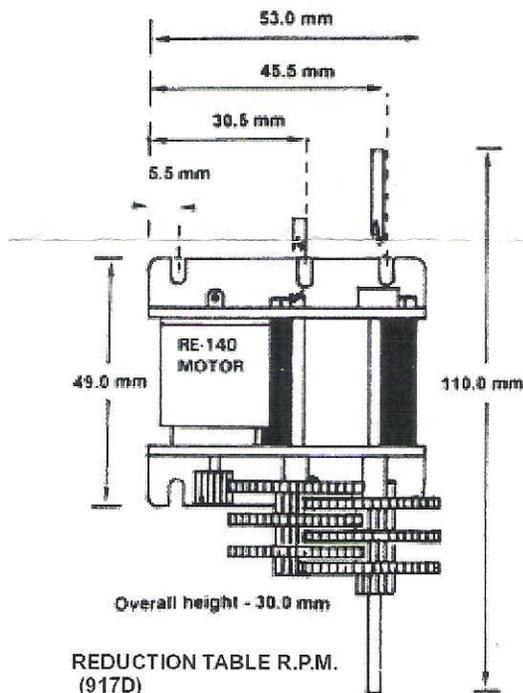
The manufacturers datasheet for the motor is shown overleaf. They do not provide a value for the stall current – we have measured this, and found 2.8A at 3V.



Images SI Inc.  
 109 Woods of Arden Road  
 Staten Island NY 10312  
 718.966.3694 Tel  
 718.966.3695 Fax  
 www.imagesco.com

# 917D Multi-Ratio Gearbox Datasheet

**GEARBOX DIMENSIONS.  
(917D)**



**REDUCTION TABLE R.P.M.  
(917D)**

	1.5V	3.0V
4:1	1850	3700
16:1	462	925
64:1	115	231
256:1	29	57
1024:1	7	14
4096:1	2	4

The unit operates on 1.5 & 3V DC power sources, either battery or suitable transformer. Its simple versatile design and sturdy construction make it suitable for a host of uses from powering models and robots to teaching the principles of mechanics. Current consumption depends on eventual load but is within the range of .2 to .8 amps. The output shaft is 3mm diameter.

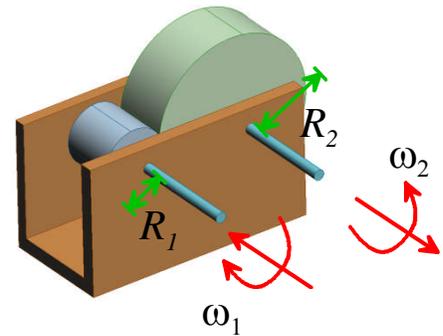
MODEL	VOLTAGE		NO LOAD		AT MAXIMUM EFFICIENCY						STALL TORQUE	
	OPERATING RANGE	NOMINAL	SPEED	CURRENT	SPEED	CURRENT	TORQUE		OUTPUT	EFF	STALL TORQUE	
			R.P.M.	A	R.P.M.	A	oz - in	g - cm	W	%	oz - in	g - cm
RE - 140	1.5 - 3.0	3.0v CONSTANT	14800	0.300	11500	1.05	0.152	10.92	1.29	41.03	0.68	49.0

### A3. Angular velocity relations for spur gears

The figure shows two gears, with radii  $R_1, R_2$ , which have  $N_1, N_2$  teeth and rotate with angular speed  $\omega_1, \omega_2$  (the gear teeth are not shown). The angular speeds are related by

$$\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{N_2}{N_1}$$

Note that if one gear rotates clockwise, the other must rotate counterclockwise.



To see this, note that

1. The contacting points on the two gears must have the same circumferential speed.
2. In one complete revolution, a point on the outside of the first gear travels around the full circumference of the gear – a total distance of  $2\pi R_1$ . The time taken for the gear to complete one revolution is  $2\pi / \omega_1$ . The speed of the outside of the gear is therefore  $2\pi R_1 / (2\pi / \omega_1) = R_1 \omega_1$
3. Similarly, a point on the outside of the second gear moves with speed  $R_2 \omega_2$ .
4. For (2) and (3) to be equal, it follows that  $R_1 \omega_1 = R_2 \omega_2$ .
5. Finally, note that for the gears to mesh, the distance between the teeth on the two gears must be equal. This means that  $2\pi R_1 / N_1 = 2\pi R_2 / N_2$ .

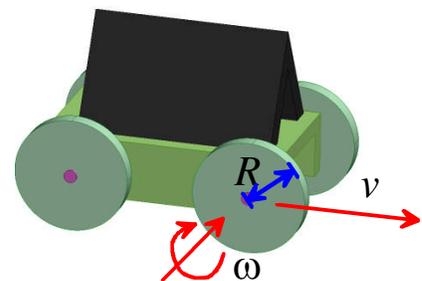
### A4. Relationship between the speed of a vehicle and the angular speed of its wheels

The figure shows a vehicle traveling at speed  $v$ . Its wheels have radius  $R$  and rotate with angular speed  $\omega$  (radians per second). The angular speed of the wheels is related to the vehicle's speed by

$$v = \omega R$$

To see this:

1. Visualize the wheel turning with the axle stationary. In one revolution, a point on the circumference of the wheel travels a total distance of  $2\pi R$ . The time taken for the wheel to complete one revolution is  $2\pi / \omega$ . The speed of the circumference relative to the axle is therefore  $2\pi R / (2\pi / \omega) = R\omega$
2. The point where the wheel touches the ground is stationary.
3. The axle must therefore move with speed  $v = \omega R$ .



**APPENDIX B: List of materials and supplies**

<b>MATERIALS AND SUPPLIES FOR SOLAR CAR PROJECT</b>		
Part	Source	Mass
6 Solar Panels	<a href="http://www.solarmade.com/SolarMini-Panels%26Motors.htm">http://www.solarmade.com/SolarMini-Panels%26Motors.htm</a> 4.-3.-100	27.4 grams (for one panel). The panel dimensions are 3¾" x 2½" x ¼"
Gearbox Motor	Edmunds Scientific <a href="http://scientificsonline.com/">http://scientificsonline.com/</a> Product # 30391-18	55.6 grams (with all 6 gears – each gear has mass 1.4grams)
Coupling	Custom-made by Brian Corkum	14.7 grams
Perforated Al sheet (can be cut and bent to make the chassis)	McMaster 9232T191	0.217 grams/cm <sup>2</sup>
8-32 Threaded Rod, ½" length	McMaster 91565A333	
8-32 Threaded Rod, 1" length	McMaster 91565A834	
8-32 Threaded Rod, 1½" length	McMaster 91565A835	
8-32 Threaded Rod, 2" length	McMaster 91565A836	
8-32 Threaded Rod, 2½" length	McMaster 91565A837	
8-32 Threaded Rod, 3" length	McMaster 91565A838	
8-32 Threaded Rod, 4" length	McMaster 91565A839	
Poster board – you can cut lightweight circular wheels of any diameter.		0.06255 grams/cm <sup>2</sup>
Rubber O rings 3/32" thick buna (for use as tires); ID 1", 1¼", 1½", 1¾", 2", 2¼", 2½", 2¾", 3", 3½"	See, e.g. McMaster 9452K85 (for 1" ring) and catalog page 34243	
Nylon Miter gear 24 teeth	McMaster 7297K15	
Nylon Miter gear 30 teeth	McMaster 7297K16	1.7 grams
Nylon Miter gear 36 teeth	McMaster 7297K17	7.5 grams
Rubber insulated rivet nut (thread onto 8-32 rod to mount miter gears onto axle)	McMaster 93495A130	
3/16" Collars	McMaster 9414T5	
8-32 Nuts	McMaster 90480A009	
Star washers	McMaster 91114A009	
Washers	McMaster 91114A005	
Nylon Spacer, ¼"OD, ½" length	McMaster 94639A106	
Nylon Spacer, ¼"OD, ¼" length	McMaster 94639A103	
Nylon Spacer, ¼"OD, ¾" length	McMaster 94639A108	
Nylon Spacer, ¼"OD, 1" length	McMaster 94639A110	