Special Edition

Design Project 3
Vibration Isolator

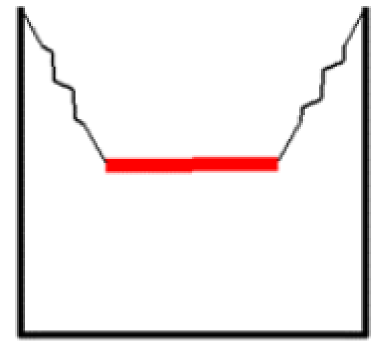
ENGN0040: Dynamics and Vibrations

Allan Bower, Yue Qi

School of Engineering
Brown University

Design Project 3: Vibration Isolator

Design a system to protect a sensitive object from ground borne vibrations



Constraints:

- Support mass between 100 and 400 grams
- Isolate horizontal vibrations in range $8<f<45$ Hz
- If possible, isolate vertical vibrations
- System must fit in an 8”x8”x8” cube, and its total mass must not exceed 300 grams
- System must attach to vibration test stand provided
- Must be made from materials provided

Design Project 3: Vibration Isolator

Materials and Supplies:

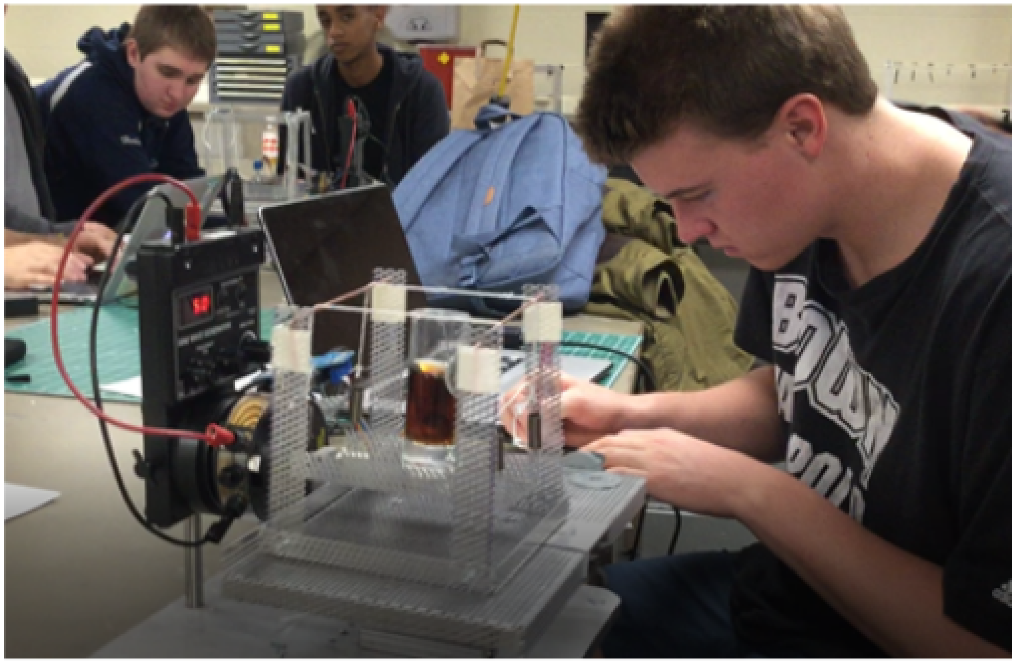
- Extension springs

Stiffness (lb/in)	Spring OD (in)	Unstretched lg (in)	Min load (lb)	Max load (lb)	Part Number
0.04	0.281	2.5	0.04	0.51	9654K49
0.06	0.25	1.875	0.04	0.51	9654K512
0.1	0.188	2.5	0.08	0.86	9654K816
0.5	0.188	1	0.11	0.75	9654K969
0.92	0.344	1.5	0.31	2.03	9654K912
1.59	0.125	1	0.26	1.1	9654K966

- Other:

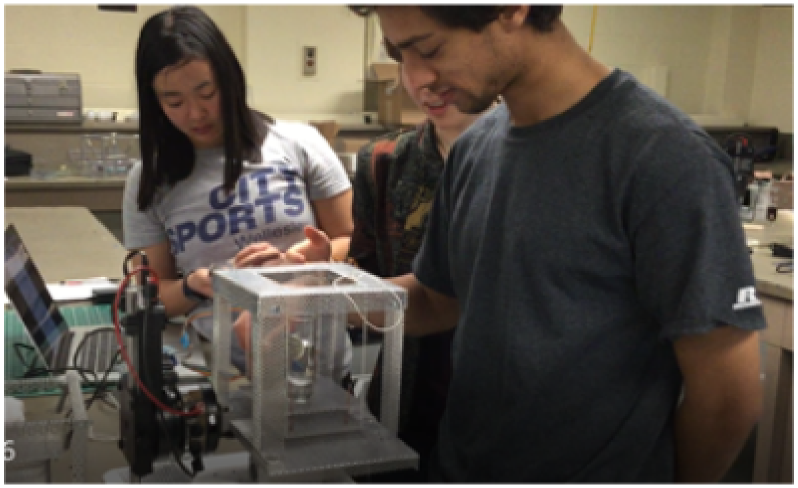
16 gage Perforated Al sheet (can be cut to size and bent into L or U sections)	McMaster 6232t181
Foam-core board	
Fishing wire	
3/16" Collars	McMaster 9414T5
4-40 Machine Screws; length between ¼" and 2"	McMaster 91792AXXX
4-40 Nuts	McMaster 90480A009
Star washers	McMaster 91114A009
Washers	McMaster 91114A005
Nylon Spacer, ¼"OD, ½" length	McMaster 94639A106
Nylon Spacer, ¼"OD, ¼ " length	McMaster 94639A103
Nylon Spacer, ¼"OD, ¾ " length	McMaster 94639A108
Nylon Spacer, ¼"OD, 1 " length	McMaster 94639A110

Design Project 3: Vibration Isolator



Deliverables:

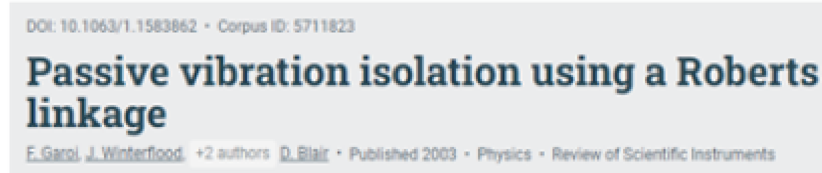
- Constructed isolation system
- Report describing design
  - Specifications
  - Predicted performance
  - Measured performance
- Demonstration of device in operation
  - Measure amplitude with test masses
  - Oral presentation



Organization:

- Groups of 4 or fewer
- One report per group

Design Project 3: Vibration Isolator


DOI: 10.1063/1.1583862 • Corpus ID: 5711823

Passive vibration isolation using a Roberts linkage

F. Garoi, J. Winterflood, +2 authors D. Blair • Published 2003 • Physics • Review of Scientific Instruments

How to do the design:

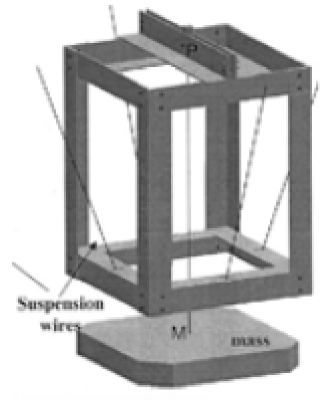
- Conceptual design
  - List as many ideas as possible (literature search, old HW problems, your own ideas)
  - Rank them, select one (or two) for final development

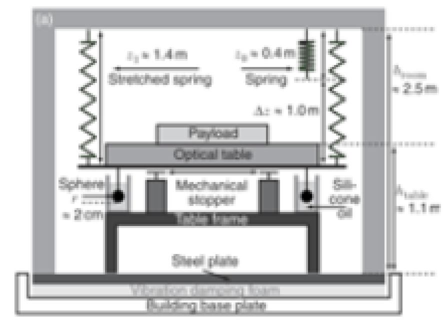

High performance passive vibration isolation system for optical tables using six-degree-of-freedom viscous damping combined with steel springs

Cite as: Rev. Sci. Instrum. 90, 015113 (2019); doi: 10.1063/1.5060707
Submitted: 21 September 2018 • Accepted: 27 December 2018 • Published Online: 24 January 2019 • Corrected: 25 October 2019

Gero L. Hermsdorf, Sven A. Szilagyi, Sebastian Rösch, and Erik Schäffer

- Detail design
  - Analyze idealized model to estimate performance and identify critical design variables
  - Select dimensions/specifications for components



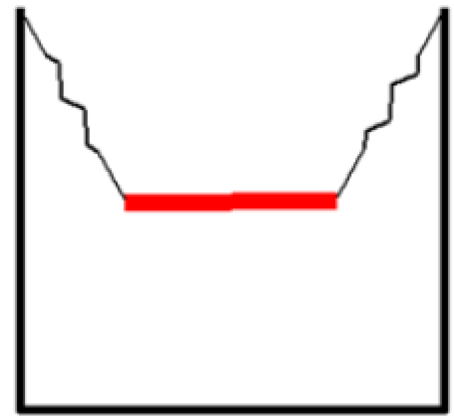


- Testing and iteration
  - Compare measured and predicted performance
  - Troubleshoot; modify design as needed

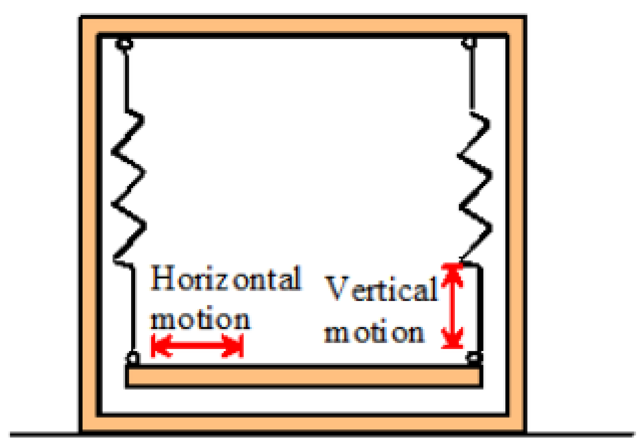
Design Project 3: Vibration Isolator

Design Calculations:

- Should include
  - Estimates of the natural frequency (horizontal, and if relevant, vertical) – may depend on mass
  - Estimates of the transmissibility – will depend on vibration frequency and may depend on mass



- Your system will have more than 1 DOF
  - If you assume a mode shape, assume the base is stationary, and neglect damping, you can use the energy method to estimate the natural frequency.



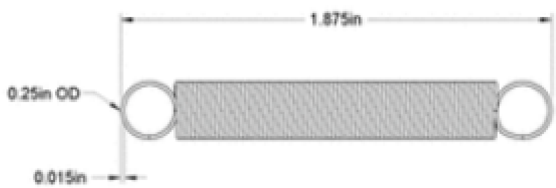
Design Project 3: Vibration Isolator

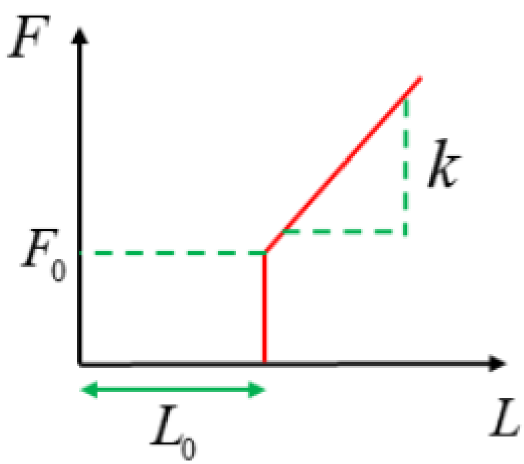
Design Calculations:

- The springs are non-standard

Stiffness (lb/in)	Spring OD (in)	Unstretched lg (in)	Min load (lb)	Max load (lb)	Part Number
0.04	0.281	2.5	0.04	0.51	9654K49
0.06	0.25	1.875	0.04	0.51	9654K512
0.1	0.188	2.5	0.08	0.86	9654K816
0.5	0.188	1	0.11	0.75	9654K969
0.92	0.344	1.5	0.31	2.03	9654K912
1.59	0.125	1	0.26	1.1	9654K966

k L_0 F_0



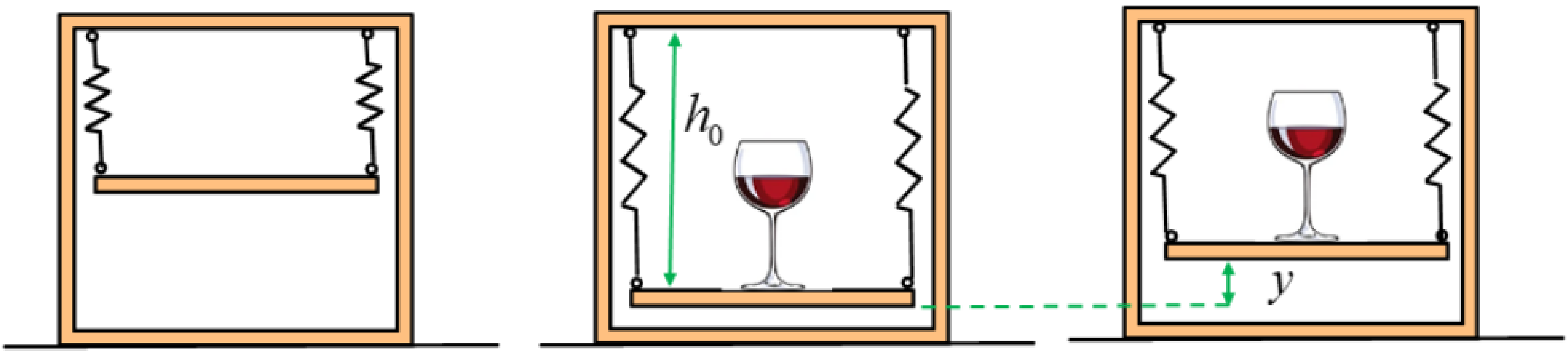


- Force $F = F_0 + k(L - L_0)$
- PE $U = F_0(L - L_0) + \frac{1}{2}k(L - L_0)^2$

Design Project 3: Vibration Isolator

Design Calculations:

- You may need to account for a static deflection



Static equilibrium

Small vibration

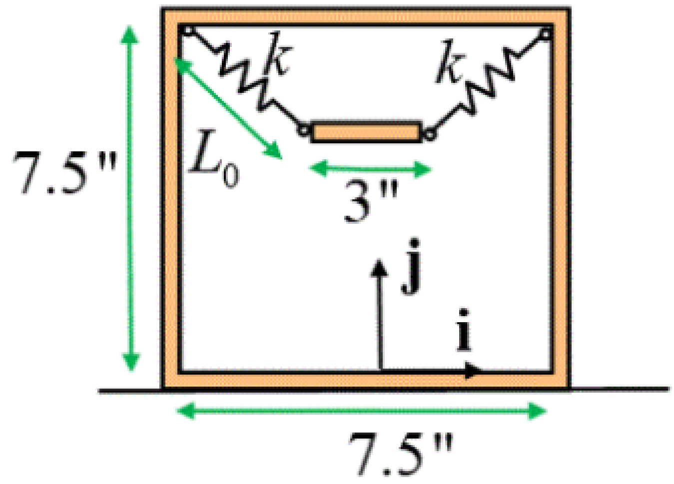
- Procedure to calculate natural frequency:
  - Find static equilibrium position (minimize potential energy)
  - Derive EOM for a small deflection from equilibrium (energy method or Newton's law or – for some designs – combine springs)
  - Linearize and arrange in standard form
  - Read off natural frequency

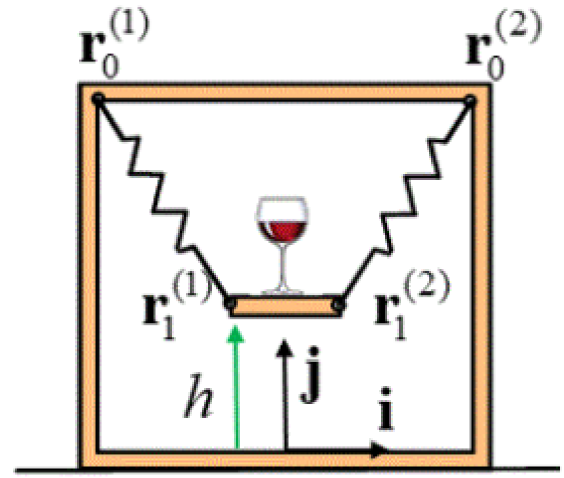
Example 1: Find the equilibrium height h of the platform

Spring properties:

- Stiffness $k = 0.1\ lb/in$
- Un-stretched Length $L_0 = 2.5"$
- Min Load $F_0 = 0.08\ lb$

Wine glass weight: $m = 0.882 lb$





Energy Method

Position Vectors:

$\underline{r}_0^{(1)} = -3.75\underline{i} + 7.5\underline{j}$ $\qquad$ $\underline{r}_0^{(2)} = 3.75\underline{i} + 7.5\underline{j}$

$\underline{r}_1^{(1)} = -1.5\underline{i} + h\underline{j}$ $\qquad$ $\underline{r}_1^{(2)} = 1.5\underline{i} + h\underline{j}$

Spring Lengths: $L_1 = |\underline{r}_1^{(1)} - \underline{r}_0^{(1)}|$ $\quad$ $L_2 = |\underline{r}_1^{(2)} - \underline{r}_0^{(2)}|$

Potential Energy
$$U = \tfrac{1}{2}k(L_1 - L_0)^2 + F_0(L_1 - L_0) + \tfrac{1}{2}k(L_2 - L_0)^2 + F_0(L_2 - L_0) + mgh$$

To find h: Solve $\frac{\partial U}{\partial h} = 0$

$\boxed{\Rightarrow h = 1.5" \text{ (MATLAB)}}$

```
syms h x real
weight = 0.882;
g = 32.2*12;
mass = weight/g;
k = 0.1; L0 = 2.5; F0 = 0.08;
r01 = [-3.75,7.5]; r02 = [3.75,7.5];
r11 = [-1.5,h];    r12 = [1.5,h];
L1 = sqrt(dot(r11-r01,r11-r01));
L2 = sqrt(dot(r12-r02,r12-r02));
U =  (k/2)*(L1-L0)^2 + F0*(L1-L0) ...
    +(k/2)*(L2-L0)^2 + F0*(L2-L0) ...
    + weight*h;
eq = diff(U,h)==0;
h = vpasolve(eq,h)
```

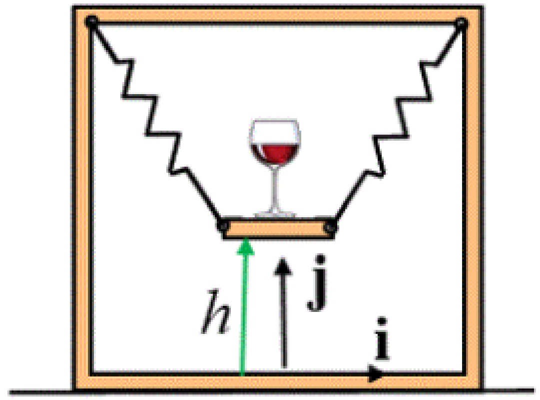
```
h = 1.49818092
```

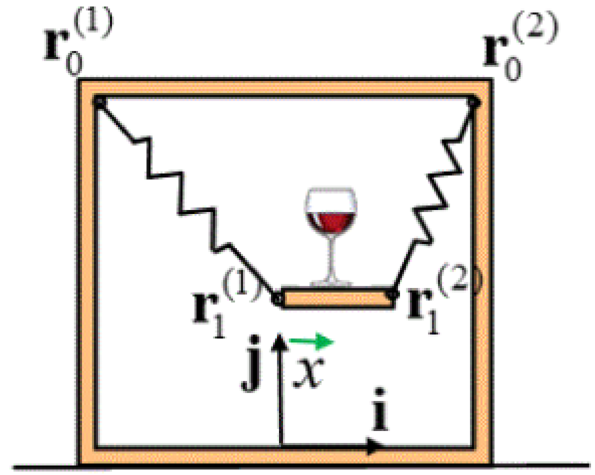
Example 2: Find the natural frequency of small amplitude horizontal vibrations

Spring properties:

- Stiffness $k = 0.1\ lb/in$
- Un-stretched Length $L_0 = 2.5"$
- Min Load $F_0 = 0.08\ lb$

Wine glass weight: $m = 0.882 lb$





Approach: (1) Find EOM

(2) Linearize for small x

Energy Method

$\underline{r}_1^{(1)} = (-1.5+x)\underline{i} + h\underline{j}$ $\quad \underline{r}_1^{(2)} = (1.5+x)\underline{i} + h\underline{j}$

Use formulas from Ex 1 for L_1, L_2, U

$T = \frac{1}{2} m (dx/dt)^2$

$$\frac{d}{dt}(T+U) \Rightarrow m\frac{d^2x}{dt^2} + \frac{\partial U}{\partial x} = 0$$

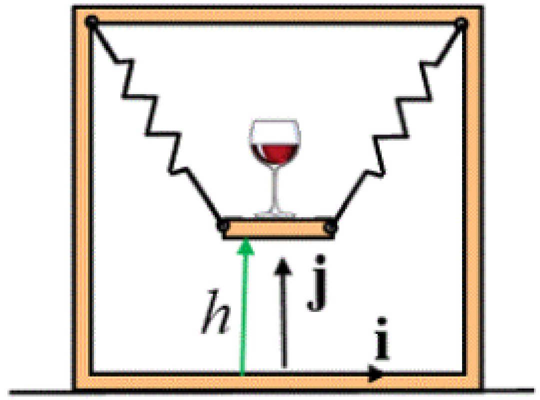
$-F_x$

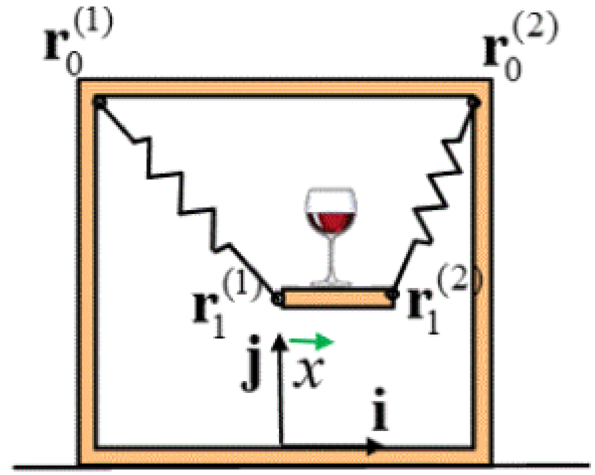
Example 2: Find the natural frequency of small amplitude horizontal vibrations

Spring properties:

- Stiffness $k = 0.1\ lb/in$
- Un-stretched Length $L_0 = 2.5"$
- Min Load $F_0 = 0.08\ lb$

Wine glass weight: $m = 0.882 lb$





$$m\frac{d^2x}{dt^2} - F_x = 0 \qquad F_x = -\frac{\partial U}{\partial x}$$

Linearize EOM:

$$F_x \approx F_x(0) + \left.\frac{\partial F_x}{\partial x}\right|_{x=0} x$$

$-k_x$

$$\Rightarrow \frac{m}{k_x}\frac{d^2x}{dt^2} + x = 0 \qquad k_x = -\frac{\partial F_x}{\partial x}$$

$$\Rightarrow \omega_n = \sqrt{\frac{k_x}{m}}$$

$$\boxed{\omega_n = 8.2\ rad/s \ \text{(MATLAB)}}$$

```
syms h x real
weight = 0.882;
g = 32.2*12;
mass = weight/g;
k = 0.1; L0 = 2.5; F0 = 0.08;
r01 = [-3.75,7.5]; r02 = [3.75,7.5];
r11 = [-1.5,h];    r12 = [1.5,h];
L1 = sqrt(dot(r11-r01,r11-r01));
L2 = sqrt(dot(r12-r02,r12-r02));
U =  (k/2)*(L1-L0)^2 + F0*(L1-L0) ...
    +(k/2)*(L2-L0)^2 + F0*(L2-L0) ...
    + weight*h;
eq = diff(U,h)==0;
h = vpasolve(eq,h)
```

h = 1.498180924640592439

```
r11 = [-1.5+x,h];    r12 = [1.5+x,h];
L1 = sqrt(dot(r11-r01,r11-r01));
L2 = sqrt(dot(r12-r02,r12-r02));
U =  (k/2)*(L1-L0)^2 + F0*(L1-L0) ...
    +(k/2)*(L2-L0)^2 + F0*(L2-L0) ...
    + weight*h;
Fx = simplify(diff(U,x));
kx = subs(diff(Fx,x),x,0)
wn = sqrt(kx/mass)
```

kx = 0.15349171045837716

wn = 8.20024313291429747

Design Project 3: Vibration Isolator

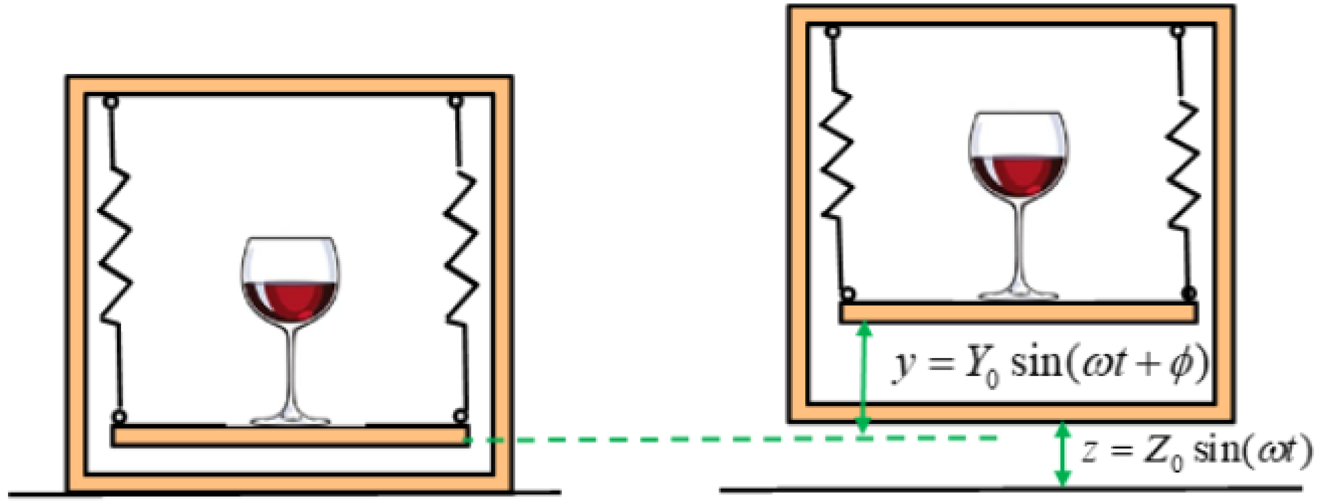
Design Calculations:

Transmissibility

$$\frac{Y_0}{Z_0} = KM(\omega/\omega_n, \zeta)$$

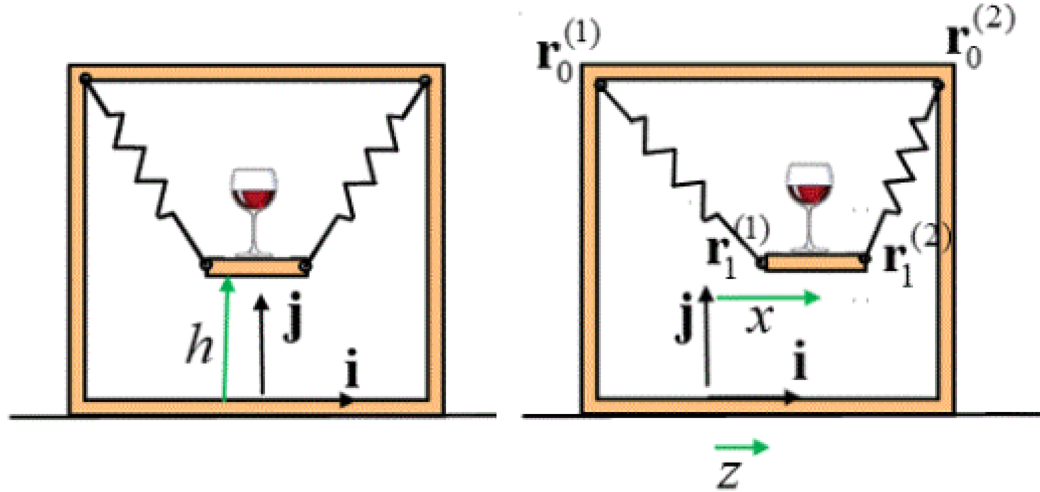
$$M = \frac{\sqrt{1+(2\zeta\omega/\omega_n)^2}}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2+(2\zeta\omega/\omega_n)^2}}$$

$$= \frac{1}{\left|\left(1-\omega^2/\omega_n^2\right)\right|} \quad \text{if} \;\; \zeta = 0$$



- Procedure to calculate transmissibility:
  - Find static equilibrium position (minimize potential energy)
  - Derive EOM for a small deflection from equilibrium, accounting for base motion (Newton's law, or energy method)
  - Linearize and arrange in standard form $\frac{1}{\omega_n^2}\frac{d^2y}{dt^2} + y = Kz$
  - Optional add 'modal damping' $\frac{1}{\omega_n^2}\frac{d^2y}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dy}{dt} + y = K(z + \frac{2\zeta}{\omega_n}\frac{dz}{dt})$ (will need to measure ζ)
  - Find magnification $M(\omega/\omega_n, \zeta)$

Example 3: Find the transmissibility of horizontal base-excited vibrations



Spring properties:

- Stiffness $k = 0.1\ lb/in$
- Un-stretched Length $L_0 = 2.5"$
- Min Load $F_0 = 0.08\ lb$

Wine glass weight: $m = 0.882 lb$

Approach: Modify EOM to account for base motion
Add "Modal damping" (optional)
Use formula for X_0 / Z_0

Find EOM

$$\underline{r}_0^{(1)} = (-3.75 + z)\underline{i} + 7.5\underline{j}$$
$$\underline{r}_0^{(2)} = (3.75 + z)\underline{i} + 7.5\underline{j}$$

Use formulas from Ex 1, Ex 2 for L_1, L_2 U, etc

EOM: $$m\frac{d^2x}{dt^2} - F_x = 0 \qquad F_x = -\frac{\partial U}{\partial x}$$

Linearize EOM for small x & z

$$F_x \approx \underbrace{\frac{\partial F_x}{\partial x}}_{-k_x} x + \underbrace{\frac{\partial F_x}{\partial z}}_{k_z} z$$

$$\Rightarrow \frac{m}{k_x}\frac{d^2x}{dt^2} + x = \frac{k_z}{k_x} z \qquad k_x = -\frac{\partial F_x}{\partial x} \qquad k_z = \frac{\partial F_x}{\partial z}$$

$$\Rightarrow \frac{1}{\omega_n^2}\frac{d^2x}{dt^2} + x = \bar{K} z \qquad \omega_n = \sqrt{\frac{k_x}{m}} \qquad \bar{K} = \frac{k_z}{k_x}$$

$$\boxed{\begin{array}{l} k_x = k_z = 0.153 \ \text{lb/in} \\ \omega_n \approx 8.2 \ \text{rad/s} \qquad \text{(MATLAB)} \end{array}}$$

```
r01 = [-3.75+z,7.5]; r02 = [3.75+z,7.5];
L1 = sqrt(dot(r11-r01,r11-r01));
L2 = sqrt(dot(r12-r02,r12-r02));
U =  (k/2)*(L1-L0)^2 + F0*(L1-L0) ...
    +(k/2)*(L2-L0)^2 + F0*(L2-L0) ...
    + weight*h;
Fx = -simplify(diff(U,x));
kx = -subs(diff(Fx,x),[x,z],[0,0])
kz = subs(diff(Fx,z),[x,z],[0,0])
wn = sqrt(kx/mass)
```

```
kx =
0.1534917104

kz =
0.1534917104

wn =
8.200243132
```

Add "Modal Damping" (optional)

$$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dx}{dt} + x = \bar{K}\left(z + \frac{2\zeta}{\omega_n}\frac{dz}{dt}\right)$$

Need to measure ζ (eg free vibration decay test)

Transmissibility

$$\frac{X_0}{Z_0} = \frac{\sqrt{1 + (2\zeta\omega/\omega_n)^2}}{\sqrt{(1-\omega^2/\omega_n^2)^2 + (2\zeta\omega/\omega_n)^2}} = \frac{1}{|1-\omega^2/\omega_n^2|} \text{ for } \zeta = 0$$

$$\omega_n = 8.2 \text{ rad/s}$$

At 8Hz $\quad \frac{X_0}{Z_0} = 0.027 \quad$ (lower for larger ω)