

## Homework \#2

- Available on course webpage
- After today's lecture you should be able to do all the problems.
- Prof. Bahar will be holding office hours Tuesday from 10noon to make up for missed office hours today

Note that HW\#2 is due this Friday by 5pm



- The delay of the $\mathrm{j}^{\text {th }}$ inverter stage is

$$
\begin{array}{ll} 
& t_{p, j}=t_{p 0}\left(1+C_{g, j+1} /\left(\gamma C_{g, j}\right)\right)=t_{p 0}\left(1+t_{j} / \gamma\right) \\
\text { and } & t_{p}=t_{p, 1}+t_{p, 2}+\ldots+t_{p, N} \\
\text { so } & t_{p}=\sum t_{p, j}=t_{p 0} \sum\left(1+C_{g, j+1} /\left(\gamma C_{g, j}\right)\right)
\end{array}
$$

- If $C_{L}$ and $C_{g, 1}$ are given, we have 2 different optimizations
- How should the inverters be sized to minimize delay?
- How many stages are needed to minimize the delay?


## Optimal number of inverters

- What is the optimal value for $N$ given $F$ ? (where $F=f^{N}$ )
- if the number of stages is too large, the intrinsic delay of the stages dominates
- if the number of stages is too small, the effective fan-out of each stage dominates

$$
\begin{aligned}
& t_{p}=N t_{p 0}(1+\sqrt[N]{F} / \gamma) \\
& \partial t_{p} / \partial N=\gamma+\sqrt[N]{F}-\frac{\sqrt[N]{F} \ln F}{N}=0 \\
& \rightarrow f=e^{(1+\gamma / f)}
\end{aligned}
$$

fan out (tapering factor) becomes $f=e=2.718$

- For $\gamma=1$ (the typical case) the optimum effective fan-out can be solved numerically and turns out to be close to 3.6

- Too many stages has a substantial negative impact on delay
- Choosing f slightly larger than optimum has little effect on delay and reduces the number of stages (and area).
- Common practice to use $\mathrm{f}=4$ (for $\gamma=1$ )


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Sizing the inverters in the chain

- After a bit of calculus, we find that for minimum delay:

$$
C_{g, j+1} / C_{g, j}=C_{g, j} / C_{g, j-1} \quad \text { for } i=2 \ldots N
$$

- What does this imply?
- All gates have the same effective fanout, $f$
- Each gate should be scaled up by the same factor w.r.t. its preceding gate
- What is the effective fanout for a gate given $C_{L}$ and $C_{g, 1}$ ?
- With a bit of algebra and inductive reasoning we find that:

$$
f=\sqrt[N]{C_{L} / C_{g, 1}}=\sqrt[N]{F}
$$

- $F=C_{L} / C_{g, 0}$ is the overall effective fanout
- What is the minimum delay through the chain?

$$
t_{p}=N t_{p 0}(1+\sqrt[N]{F} / \gamma)
$$

$\Longrightarrow$ Fanout of 4 (FO4) rule of thumb delay metric is based on this result


## The rest of today's lecture

- How do we develop design rules for sizing CMOS gates in general?
- The 2:1 ratio for an inverter doesn't necessary work best for other types of gates
- How should we go about planning the layout of these more complex CMOS gates?
- Gates with multiple inputs means more complex routing
- How to you order inputs and draw out the active area to minimize total area?




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## Layout planning for complex gates

- Want layout to be as dense (area efficient) as possible.
- Try to realize all NMOS and PMOS transistors in unbroken row of devices
- Requires only single strip of diffusion in both wells
- Careful ordering of inputs is important to help achieve this
- Use a systematic approach identifying Euler paths




## Homework \#2

- Should now have all the background to do all the problems for this homework
- Due Friday, by 5pm
- Hardcopy due to me or Marc
- /gpfs/data/engn 1600 directory needs to contain all relevant files
- Let me know if you have any issues meeting this deadline



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## Wire delay models

- Ideal wire
- same voltage is present at every segment of the wire at every point in time - at equi-potential
- only holds for very short wires, i.e., interconnects between very nearest neighbor gates
- Lumped C model
- when only a single parasitic component $(C, R$, or L$)$ is dominant the different fractions are lumped into a single circuit element
- When the resistive component is small and the switching frequency is low to medium, can consider only $C_{\text {; }}$ the wire itself does not introduce any delay; the only impact on performance comes from wire capacitance

- good for short wires; pessimistic and inaccurate for long wires


## Chain network Elmore delay

- A typical wire is a chain network with (simplified) Elmore delay of

$$
\tau_{\mathrm{DN}}=\sum \mathrm{c}_{\mathrm{i}} \mathrm{r}_{\mathrm{ij}}=\sum^{\mathrm{N}} \mathrm{c}_{\mathrm{i}} \sum^{\mathrm{i}} \mathrm{r}_{\mathrm{j}}
$$

- Where $\sum^{\mathrm{i}} r_{j}=r_{1}+r_{2}+\ldots+r_{i}$


