LATENT HARDENING IN CRYSTAL PLASTICITY

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ABSTRACT: A new phenomenological latent hardening model for rate-dependent single crystal plasticity is proposed. The new law captures the experimentally observed path dependence and provides for better control of the orientation and material dependence associated with latent hardening in single crystal plasticity.

INTRODUCTION: Latent hardening refers to the influence of the active slip system on the hardening of the inactive slip systems and is an important concept in crystal plasticity as it controls the shape of the single crystal yield surface. In crystal plasticity, the critical shear strength of slip system \( \alpha \), \( \tau_{\alpha}^{(\alpha)} \), evolves according to \( \tau_{\alpha}^{(\alpha)} = h^{(\alpha)}|\dot{\gamma}^{(\alpha)}| \) where \( \dot{\gamma}^{(\alpha)} \) is the shearing rate and \( h^{(\alpha)} \) is the hardening modulus with diagonal terms related to self-hardening and non-diagonal terms related to latent hardening. Expressing the non-diagonal moduli as a latent hardening coefficient, \( q_{\alpha \kappa}^{(\alpha \kappa)} \), times a self-hardening modulus, \( h_{\alpha \alpha} \), (no summation over \( \kappa \) ) allows for a general framework from which to model the influence of latent hardening on single crystal plasticity.

Latent hardening is most commonly probed by measuring a latent hardening ratio (LHR). The LHR, \( q^{(\alpha \kappa)} \), is obtained by dividing the initial critical shear strength after a strain path change by the maximum critical shear strength prior to the strain path change. Only one slip system is to be activated during the secondary uniaxial test. Careful experimental measurements indicate that the LHRs evolve with deformation (Jackson and Basinski [1967], Franciosi et al. [1980]). In a well-annealed single crystal all systems initially have the same strength and therefore \( q^{(\alpha \kappa)} = 1 \). For single glide orientations, the LHRs quickly reach a maximum and then decay to a saturation value as other slip systems become activated. Fig. 1 shows this transition for the copper glissile system. The rate of hardening is also plotted on the secondary y-axis to confirm the general trend. Basinski and Basinski's [1979] comprehensive review provides further experimental evidence related to latent hardening.

Fig. 1 Evolution of the LHR for the copper glissile system as compared to the rate of hardening (Jackson and Basinski [1967] and Franciosi et al. [1980]).
PROCEDURES, RESULTS AND DISCUSSION: To capture the observed experimental latent hardening behaviour, a model is developed which considers latent hardening coefficients that evolve with deformation. Internal variables appropriate for the evolution equation are the critical primary shear strength and the maximum and saturation LHRs, $q_m$ and $q_s$. The latent hardening coefficient is separated into a non-constant, $\tilde{q}_m^{(\alpha \kappa)}$, and a constant, $\tilde{q}_s^{(\alpha \kappa)}$, component. In single glide, the latent critical shear stress rate can be expressed in terms of the primary critical shear stress rate as,

$$\tau_c^{(\alpha \kappa)} = \tilde{q}_m^{(\alpha \kappa)} \tau_c^{(\kappa)} = (\tilde{q}_m^{(\alpha \kappa)} + \tilde{q}_s^{(\alpha \kappa)}) \tau_c^{(\kappa)}, \quad \alpha = 1, n$$

where $(\bullet)^{-1}$ denotes single glide, $n$ is the total number of slip systems, and $\alpha \neq \kappa$ applies to Eqns. (1) through (6). A suitable decay function for $\tilde{q}_m^{(\alpha \kappa)}$ is,

$$\tilde{q}_m^{(\alpha \kappa)} = Z \sech^2 [c'(\tau_c^{(\kappa)} - \tau_o)], \quad \alpha, \kappa = 1, n$$

where $c' = c / (\tau_i - \tau_o)$, $c = 2$, and $\tau_o$ and $\tau_i$ are the initial and back-extrapolated critical yield stress in shear. The latent hardening coefficients are assumed to saturate to $q_s^{(\alpha \kappa)} = q_s$ at $\tau_i$. In single glide Eqn. (1) can be integrated, via substitution of Eqn. (2), to obtain the critical shear stress on the latent systems as,

$$\tau_c^{(\kappa)} = \tau_o + c'^{-1} Z \tanh [c'(\tau_c^{(\kappa)} - \tau_o)] + q_s (\tau_c^{(\kappa)} - \tau_o), \quad \alpha = 1, n$$

Furthermore, $q_m = \tau_c^{(\alpha \kappa)} / \tau_i$ is assumed and used to obtain $Z$ from Eqn. (3). This leads to a path-dependent formulation for the latent hardening coefficients, viz. Eqn. (4).

$$\tilde{q}_m^{(\alpha \kappa)} = [c' \tau_c^{(\kappa)} (q_m - q_s) + c' \tau_o (q_s - 1)] \sech^2 [c'(\tau_c^{(\kappa)} - \tau_o)] + q_s, \quad \alpha, \kappa = 1, n$$

An explicit statement for the LHRs, in single glide, follows from Eqn. (3),

$$q^{(\alpha \kappa)}(\tau) = (1 - q_s) \tau_o / \tau_c^{(\kappa)} + [\tau_i / \tau_c^{(\kappa)} (q_m - q_s) + \tau_o / \tau_c^{(\kappa)} (q_s - 1)] \tanh [c'(\tau_c^{(\kappa)} - \tau_o)] + q_s$$

where $\alpha = 1, n$. If $\tilde{q}_m^{(\alpha \kappa)} = 0$, Eqn. (5a) reduces to,

$$q^{(\alpha \kappa)}(\tau) = (1 - q_s) \tau_o / \tau_c^{(\kappa)} + q_s, \quad \alpha = 1, n$$

Most existing latent hardening laws used in rate-dependent crystal plasticity employ constant latent hardening coefficients, i.e. $\tilde{q}_m^{(\alpha \kappa)} = 0$. In the current notation, diagonal latent hardening is defined by $\tilde{q}_i^{(\alpha \kappa)} = \delta^{(\alpha \kappa)}$ where $\delta^{(\alpha \kappa)}$ is the Kronecker delta; isotropic latent hardening is defined by $\tilde{q}_i^{(\alpha \kappa)} = 1$; and multi-parameter latent hardening (Peirce et al. [1983]) is defined by $\tilde{q}_i^{(\alpha \kappa)} = q_i^{(\alpha \kappa)}$ where $q_i^{(\alpha \kappa)}$ is typically chosen to be $1 < q_i^{(\alpha \kappa)} < 1.4$. The difference between constant and variable latent hardening coefficients, as formulated in the new law, is shown by plotting the latent shear stress as a function of the primary shear stress (Fig. 2). A comparison of the resulting LHRs (Eqns. (5a) and (5b)) shows that the new law leads to LHRs that reach a maximum at $\tau_i$ and then decay to a saturation value, while the choice of a constant latent hardening coefficient leads to LHRs that evolve from one to the saturation value (Fig. 3). The increased control over the difficulty in activating secondary systems, as afforded by new law, allows for a latent hardening description that is in better agreement with the experimental observations.
Activation of multiple glide has two primary influences on the evolution of the LHRs. Firstly, regardless of the form of the self-hardening moduli, the effective latent hardening decreases with an increase in the slip system activity, defined by the number and intensity of active secondary systems. Secondly, multiple glide can lead to a rapid increase in the rate of hardening as compared to single (or coplanar) glide. The associated increase in the critical shear strengths propels the latent hardening coefficients to saturation and therefore the model is consistent with observations in single and multiple glide. In this regards, latent hardening coefficients are partly associated with the self-hardening modulus and therefore the new law is suited for models that capture the phenomenology of hardening in single and multiple glide. Nevertheless, the new law allows for an effective means to better control the orientation and material dependence of latent hardening in rate-dependent crystal plasticity.

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REFERENCES: