Eleventh Workshop on Non-Perturbative Quantum Chromodynamics

l'Institut d'Astrophysique de Paris
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Hadron interactions from lattice QCD

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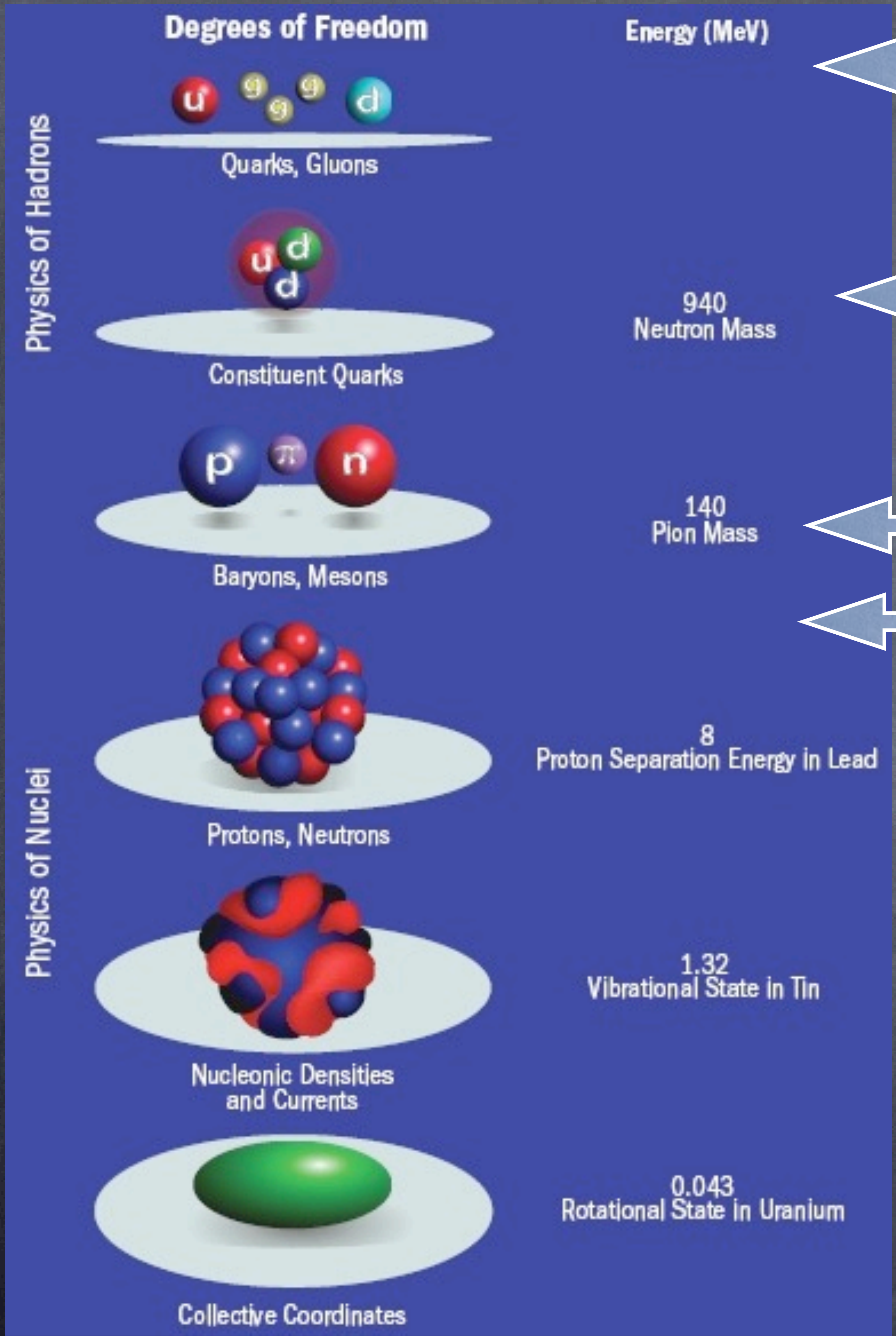

QCD

Hadron structure
and spectrum

Hadronic Interactions

Nuclear physics

Summary

- The problem
  - Hadron-Hadron scattering phase shifts
  - Binding energies
  - Study systems with more than 2 hadrons
- The calculation
  - Evaluation of Euclidean correlators
  - Extract the finite volume energy levels
    - They are related to phase shifts and binding energies in infinite volume
- Recent results

NPLQCD COLLABORATION

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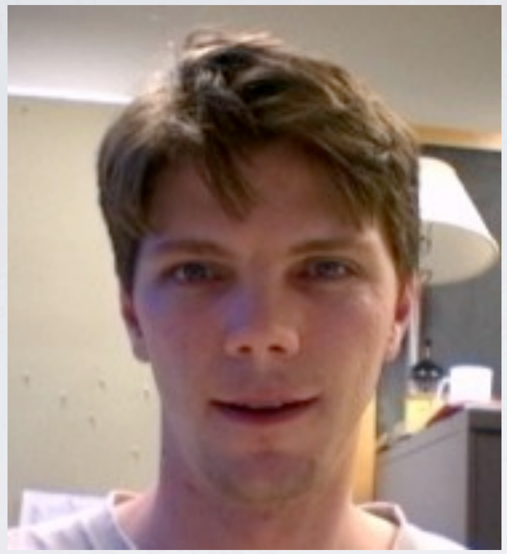

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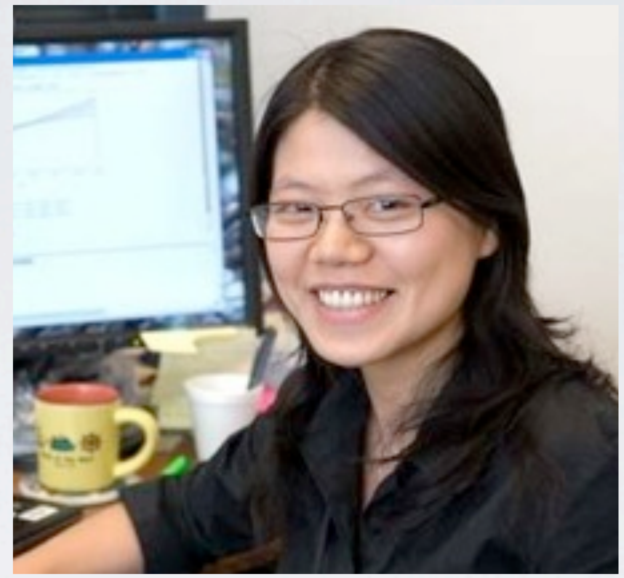

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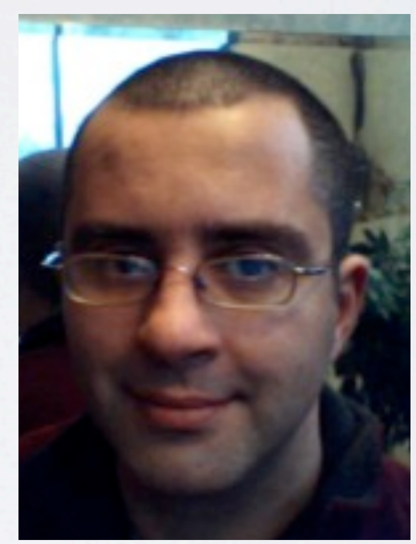

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Elastic Scattering Phases shifts

- Maiani-Testa no-go theorem
- Luscher: Finite volume two particle spectrum is related to elastic scattering phase shifts

Scattering in One dimension

In center of mass coordinates

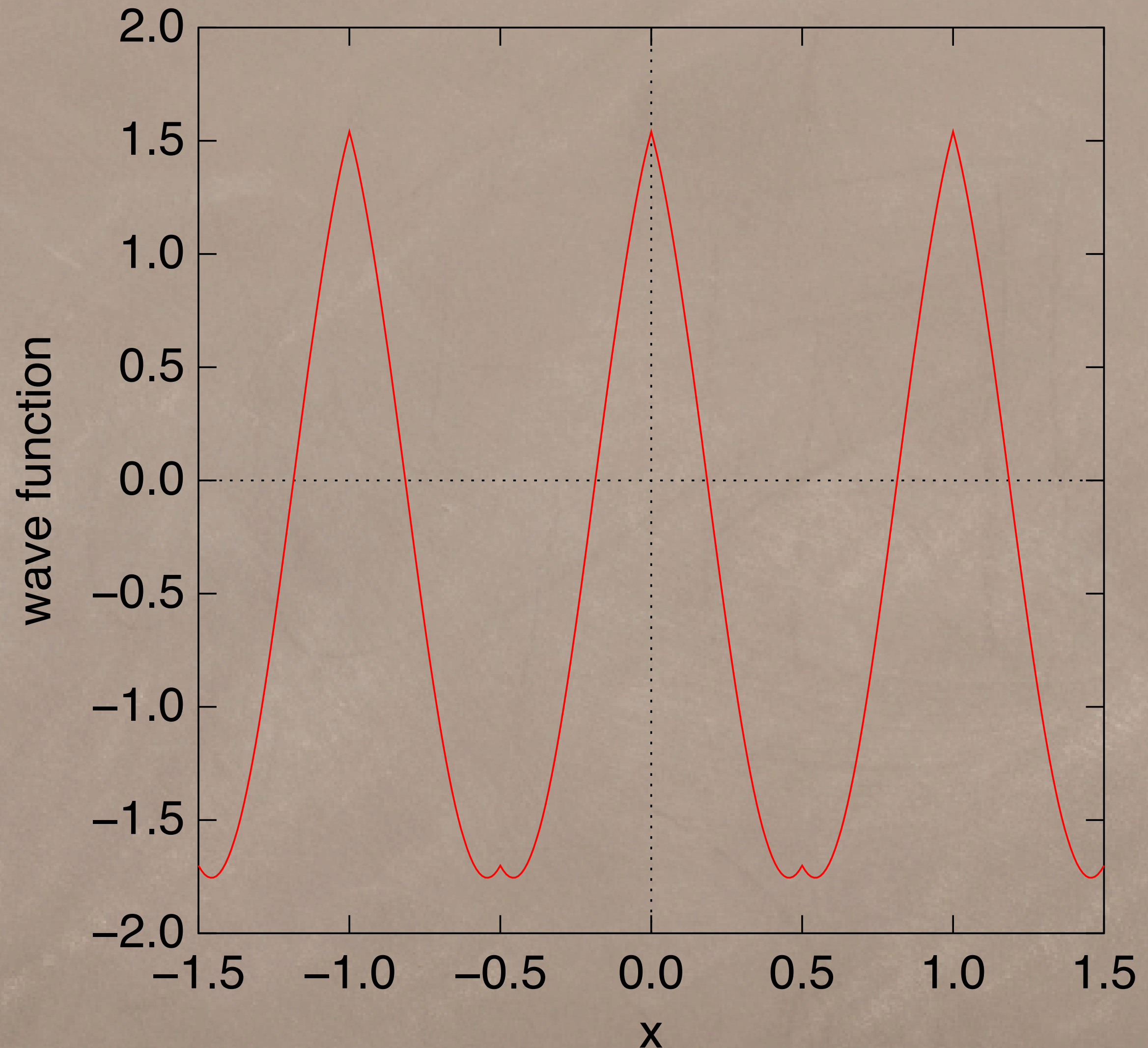
$$-\frac{1}{m}\frac{\partial^2\Psi}{\partial x^2} + c(k)\delta(x)\Psi = E\Psi$$

$$\Psi = A\left(e^{-ik|x|} + e^{ik|x|+2i\delta(k)}\right)$$

$$E = \frac{k^2}{m}$$

- Wave functions are almost plane waves
- Finite length with periodic boundary conditions
- Wave function needs to be periodic and even under x ---> -x (symmetric under particle exchange)

Scattering in One dimension



$$c(k) = -\frac{1}{mk}\tan\delta(k)$$

Scattering in One dimension

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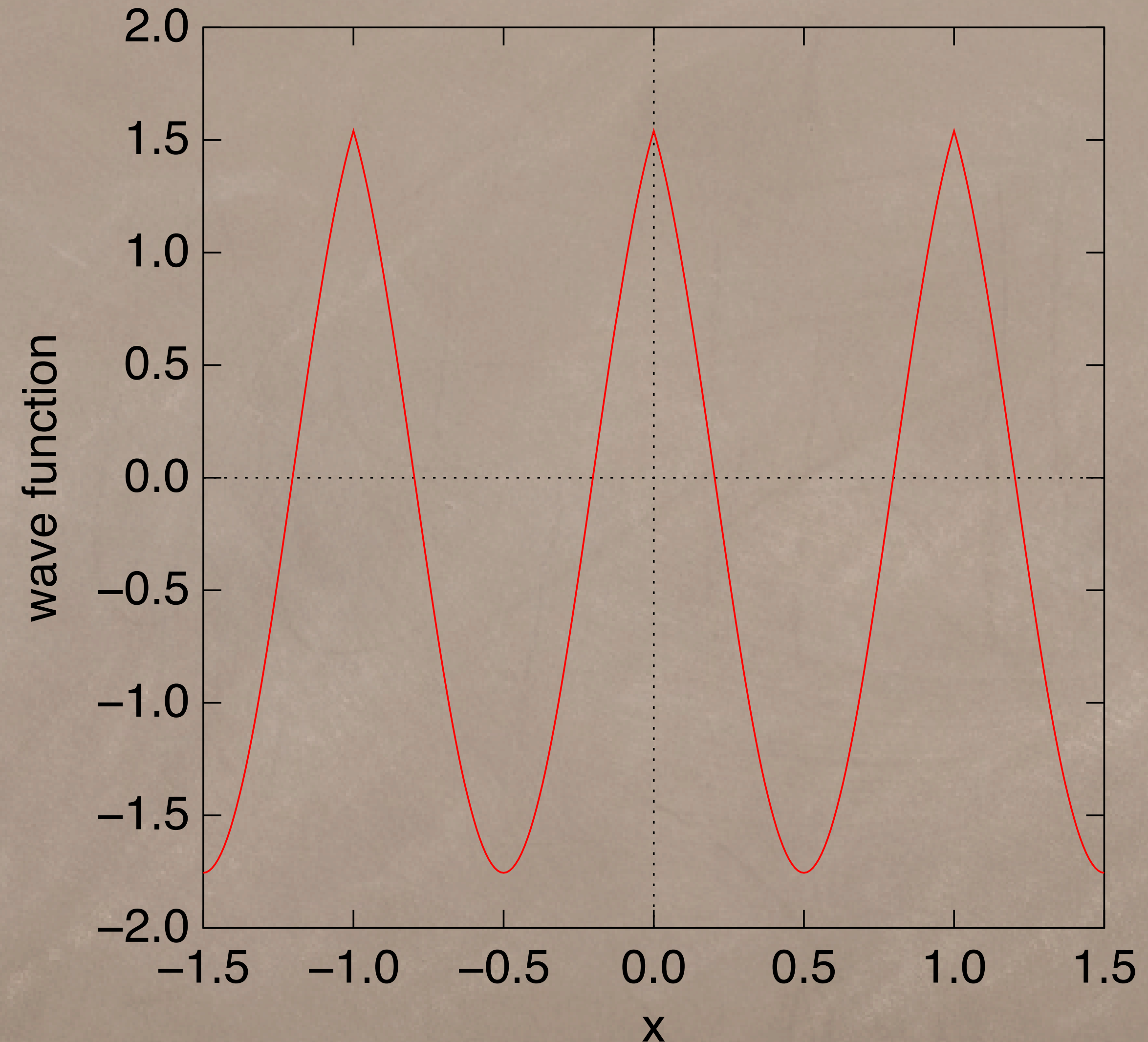
Scattering in One dimension



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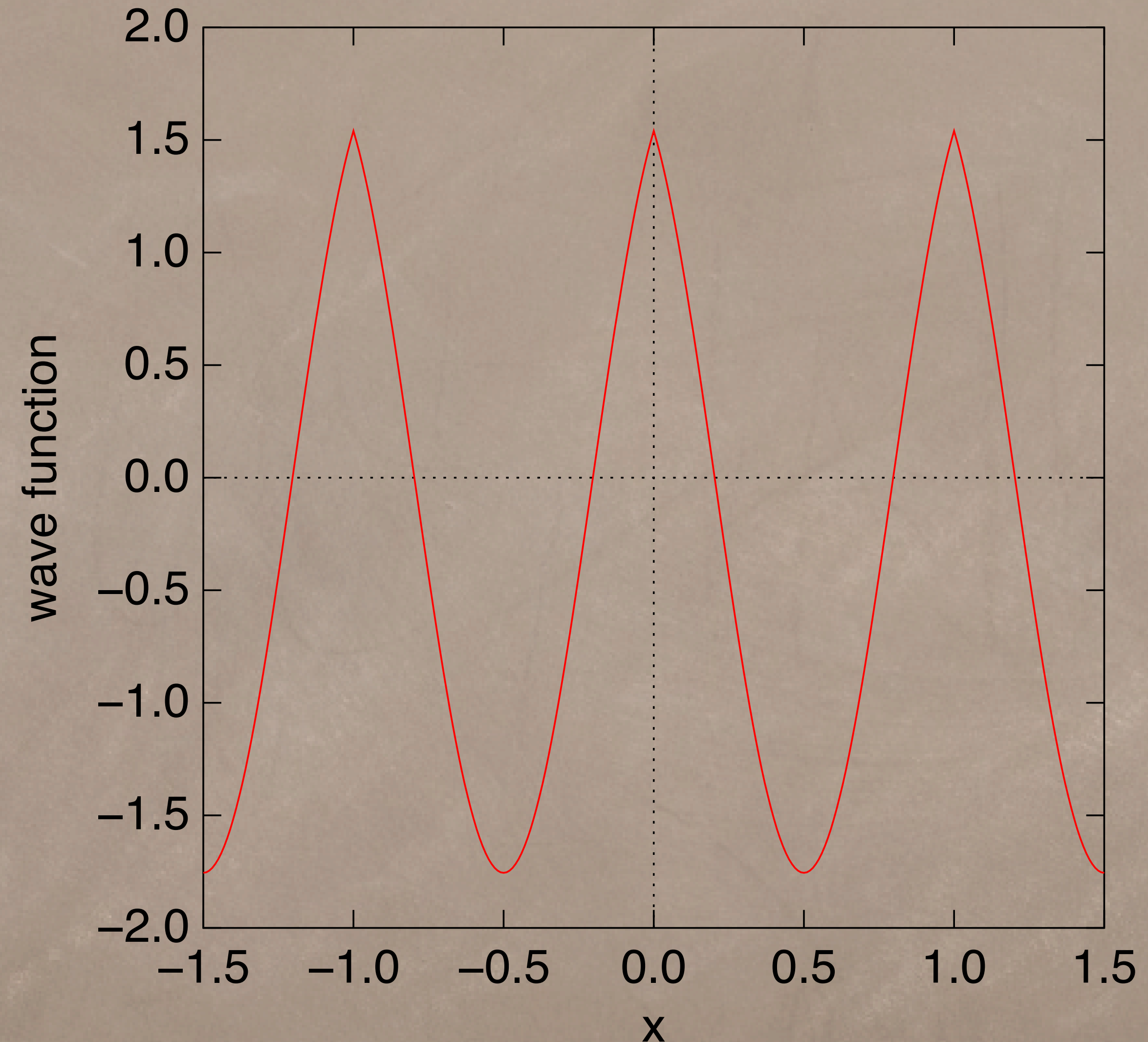
Scattering in One dimension



$$c(k) = -\frac{1}{mk}\tan\delta(k)$$

$$kL + 2\delta = 2n\pi$$

Luscher Formula

Energy level shift in finite volume:

$$\Delta E_n \equiv E_n - 2m = 2\sqrt{p_n^2 + m^2} - 2m$$

p_n solutions of:

$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S}\left(\frac{p^2 L^2}{4\pi^2}\right) \qquad \mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} - 4\pi\Lambda$$

$$p_n \cot \delta(p_n) = \frac{1}{a} + \cdots \qquad \frac{1}{a} = \frac{1}{\pi L} S\left(\frac{p_0^2 L^2}{4\pi^2}\right) + \cdots$$

Expansion at p->0 :

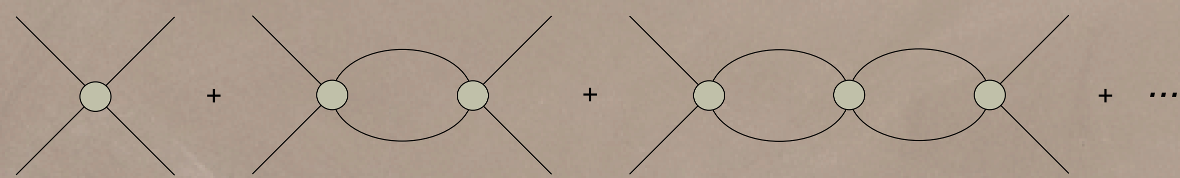
$$\Delta E_0 = -\frac{4\pi a}{m L^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2\right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

a is the scattering length

c_1 and c_2 are universal constants

Bound states

$A(p) =$



$$A(p) = \frac{4\pi}{m} \frac{1}{p\,cot\delta \; - \; i\,p}$$

$$E_{-1} = \sqrt{p^2 + m^2} - 2m \qquad\qquad p^2 < 0$$

$$E_{-1} = -\frac{\gamma^2}{m}\left[1 \; + \; \frac{12}{\gamma L} \; \frac{1}{1 - 2\gamma (p \cot \delta)'} \; e^{-\gamma L} \; + \; \ldots \right]$$

γ is the infinite volume binding momentum

Beane et.al. hep-lat/0312004v1

Scattering Phases shifts, Bound States on the Lattice

- Maiani-Testa no-go theorem
- Luscher: Finite volume two particle spectrum is related to elastic scattering phase shifts
- Computational problem: Calculate in Euclidean space and finite volume the two particle spectrum
- Extract energy levels from exponentially decaying correlation functions in Euclidean time
- Baryons: Signal to noise ratio grows exponentially with Euclidean time

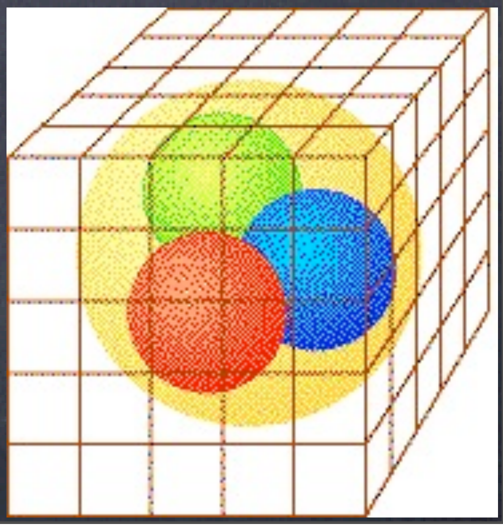
The Computation

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_{\mu,x} dU_\mu(x) \;\; \mathcal{O}[U, D(U)^{-1}] \;\; \det\left(D(U)^\dagger D(U)\right)^{n_f/2} \;\; e^{-S_g(U)}$$

Monte Carlo Evaluation

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U_i)$$

Statistical error $\frac{1}{\sqrt{N}}$



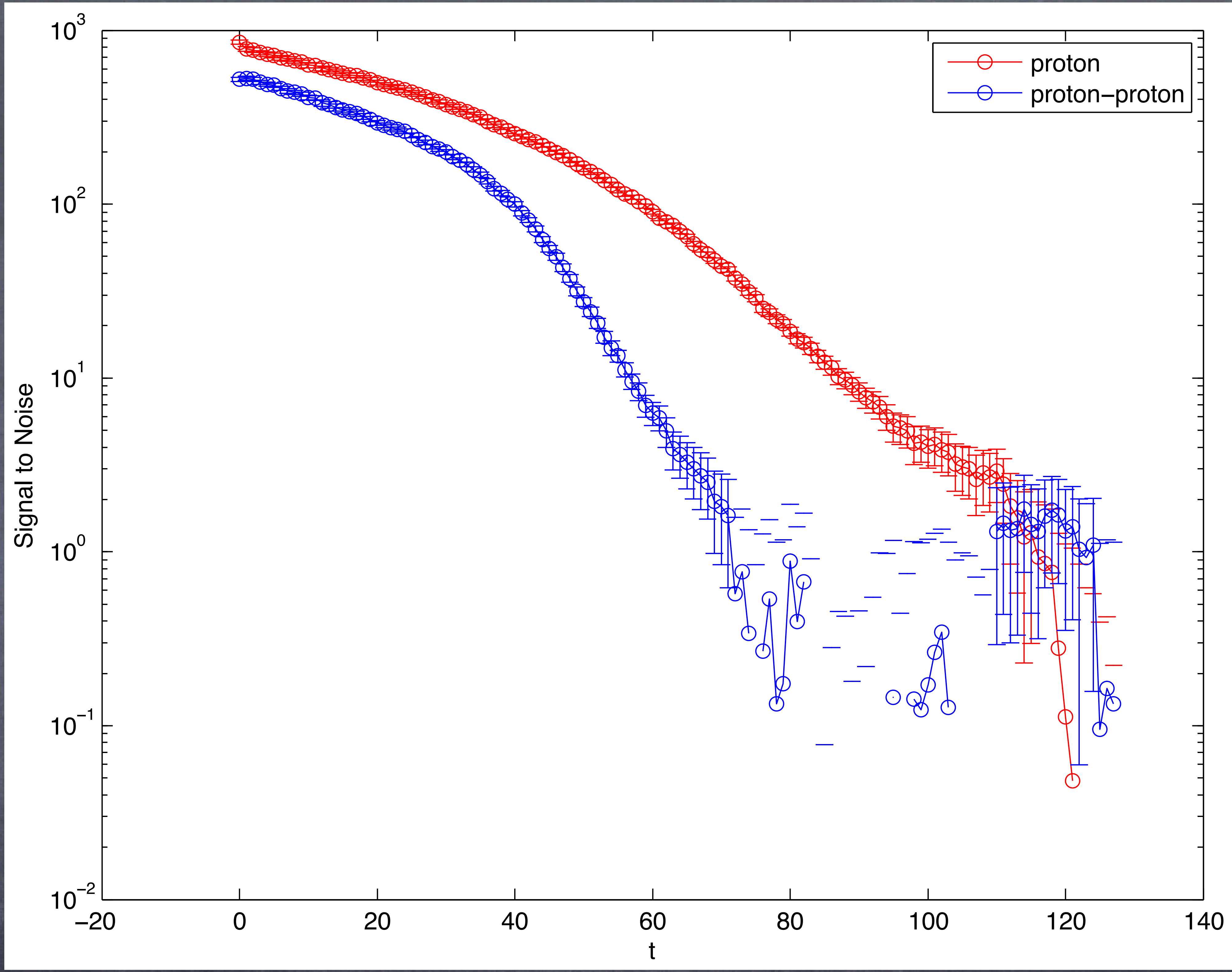
Signal to Noise ratio for correlation functions

$$C(t) = \langle N(t)\bar{N}(0)\rangle \sim Ee^{-M_N t}$$

$$var(C(t)) = \langle N\bar{N}(t)N\bar{N}(0)\rangle \sim Ae^{-2M_N t} + Be^{-3m_\pi t}$$

$$StoN = \frac{C(t)}{\sqrt{var(C(t))}} =\sim Ae^{-(M_N - 3/2 m_\pi)t}$$

- The signal to noise ratio drops exponentially with time
- The signal to noise ratio drops exponentially with decreasing pion mass
- For two nucleons: StoN(2N) = StoN(1N)^2

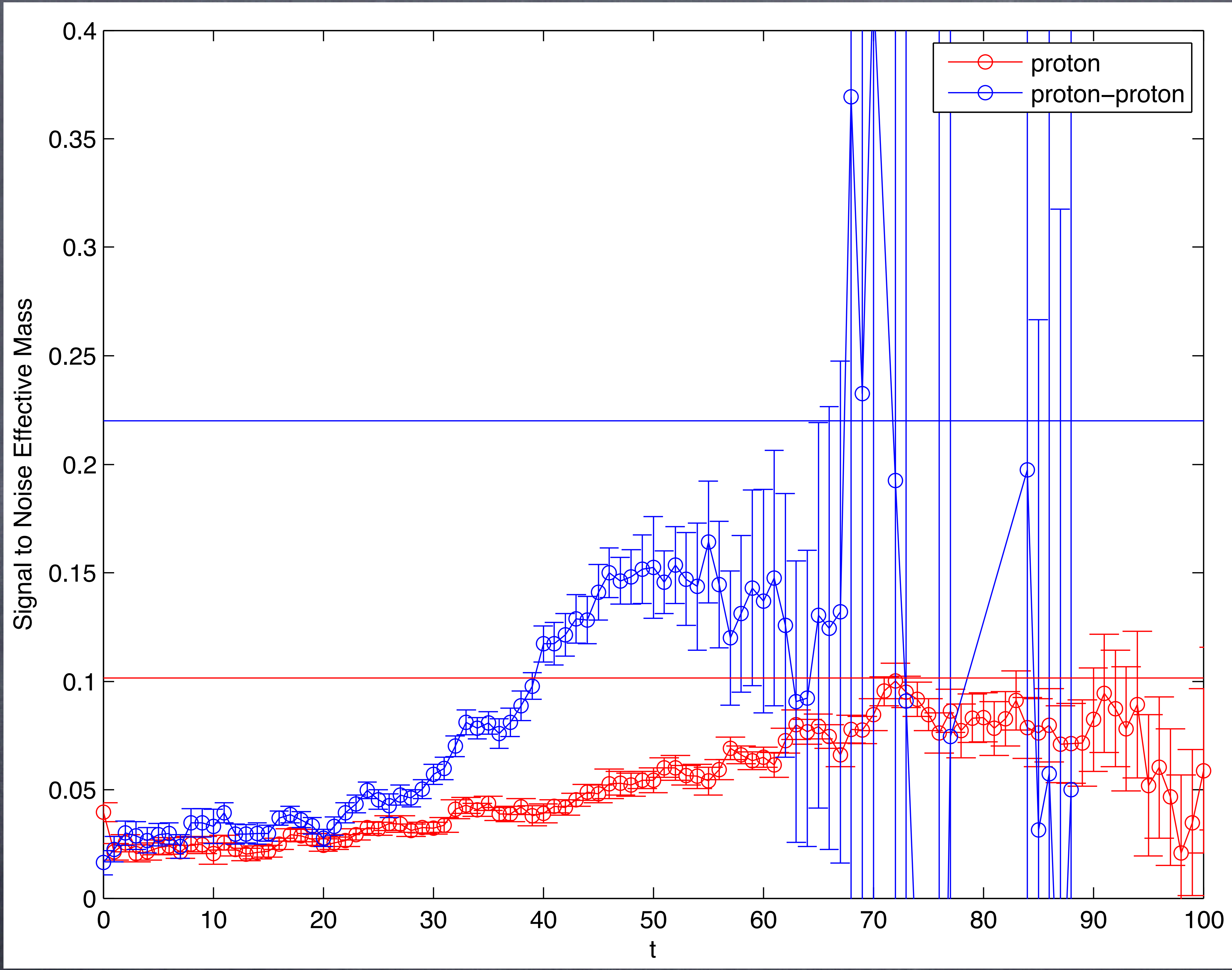


Signal to Noise

$32^3 \times 256$

M_π=390MeV

NPLQCD data



Signal to Noise Effective Mass

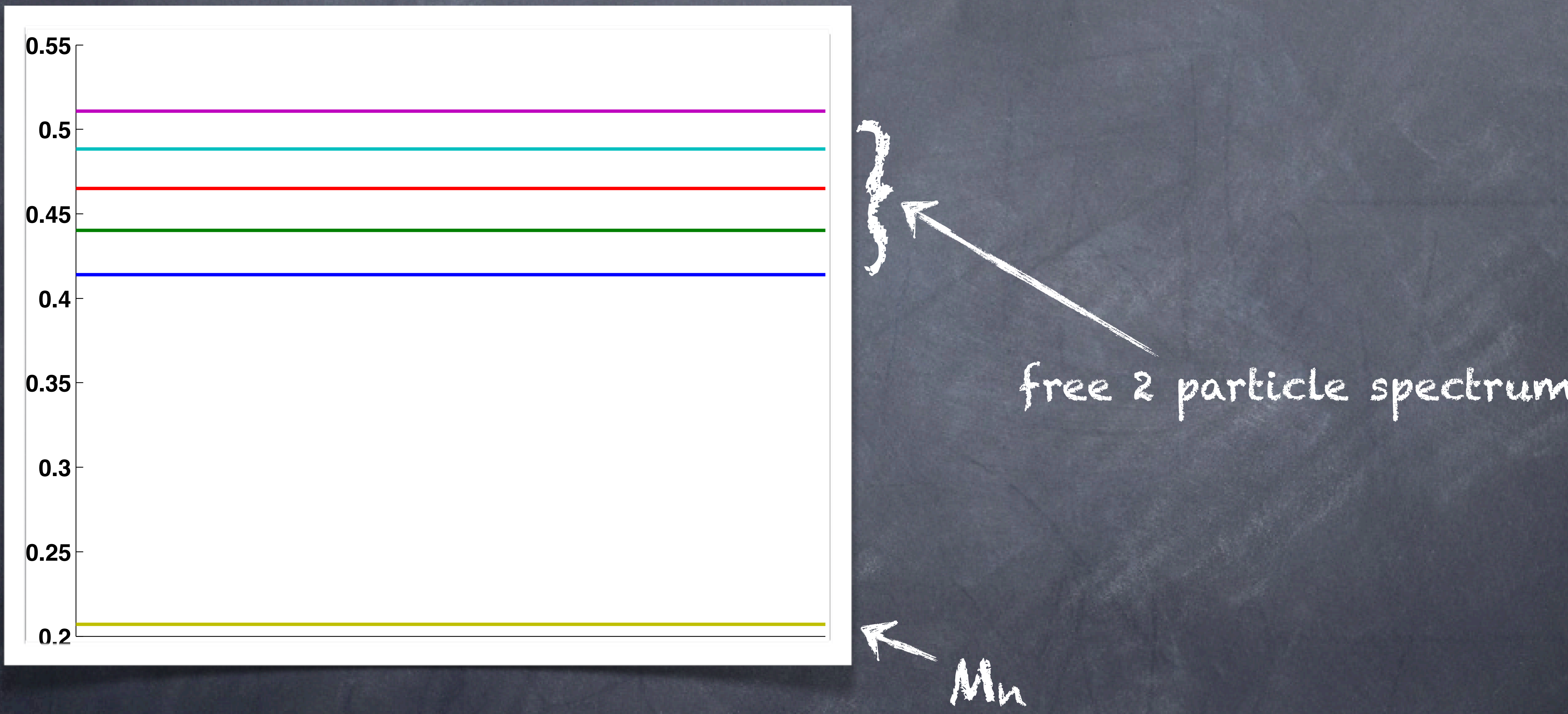
$32^3 \times 256$

M_π=390MeV

anisotropy factor 3.5

NPLQCD data

Expected Two Nucleon spectrum



24^3 box

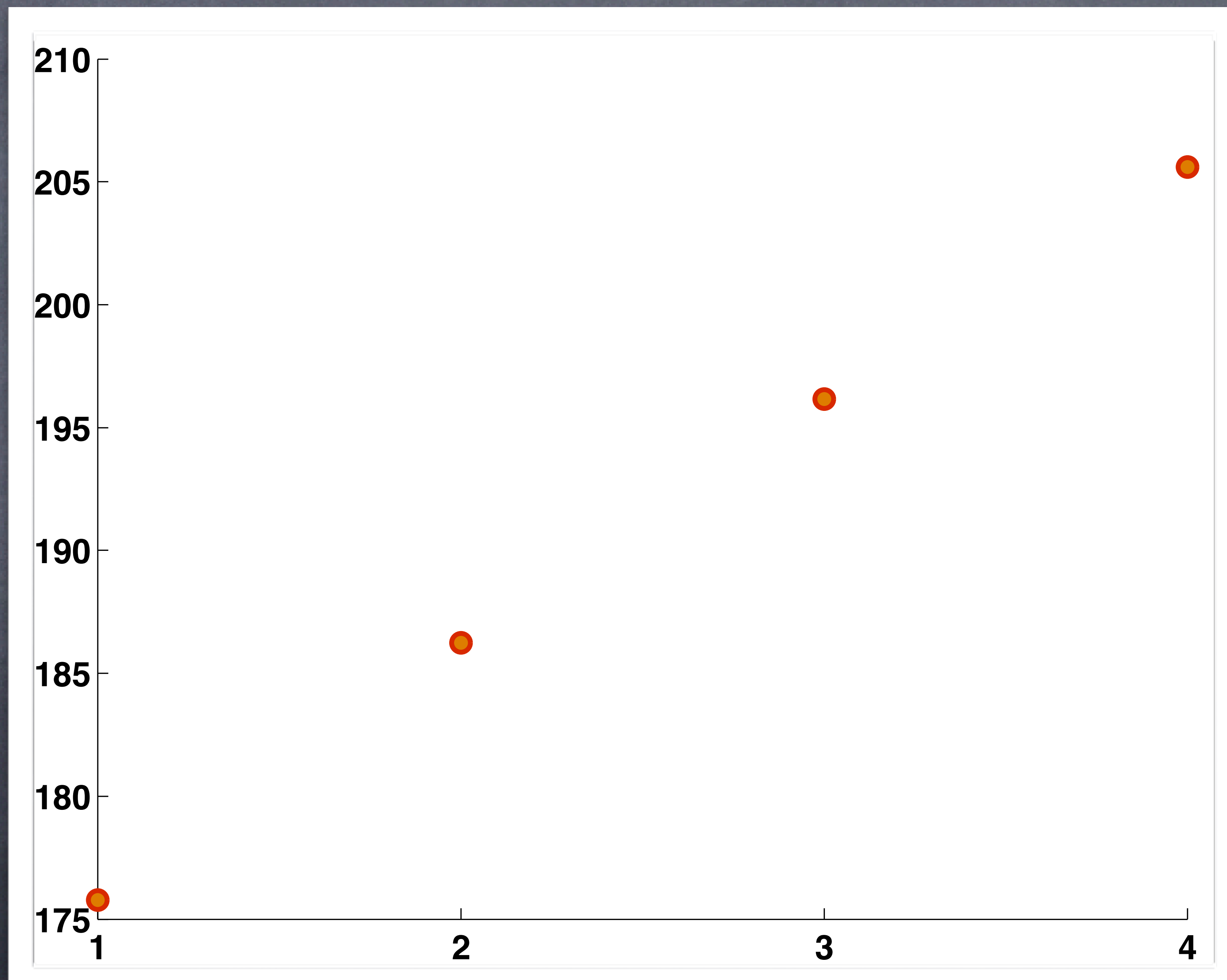
anisotropy factor 3.5

M_π=390 MeV

Needed Time Separation

$$e^{-\Delta E \delta t} \approx 10^{-2}$$

24^3 box



δt

Two particle state

anisotropy factor 3.5

Conclusion

We need to fit for several low lying states for reliable estimation of the ground state of the two particle system in a finite box

We need very high statistics to be able to resolve excited state contamination

Spectroscopy Methods

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Use multiple correlators and construct linear combinations that couple predominately to one state

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Use multiple correlators and construct linear combinations that couple predominately to one state

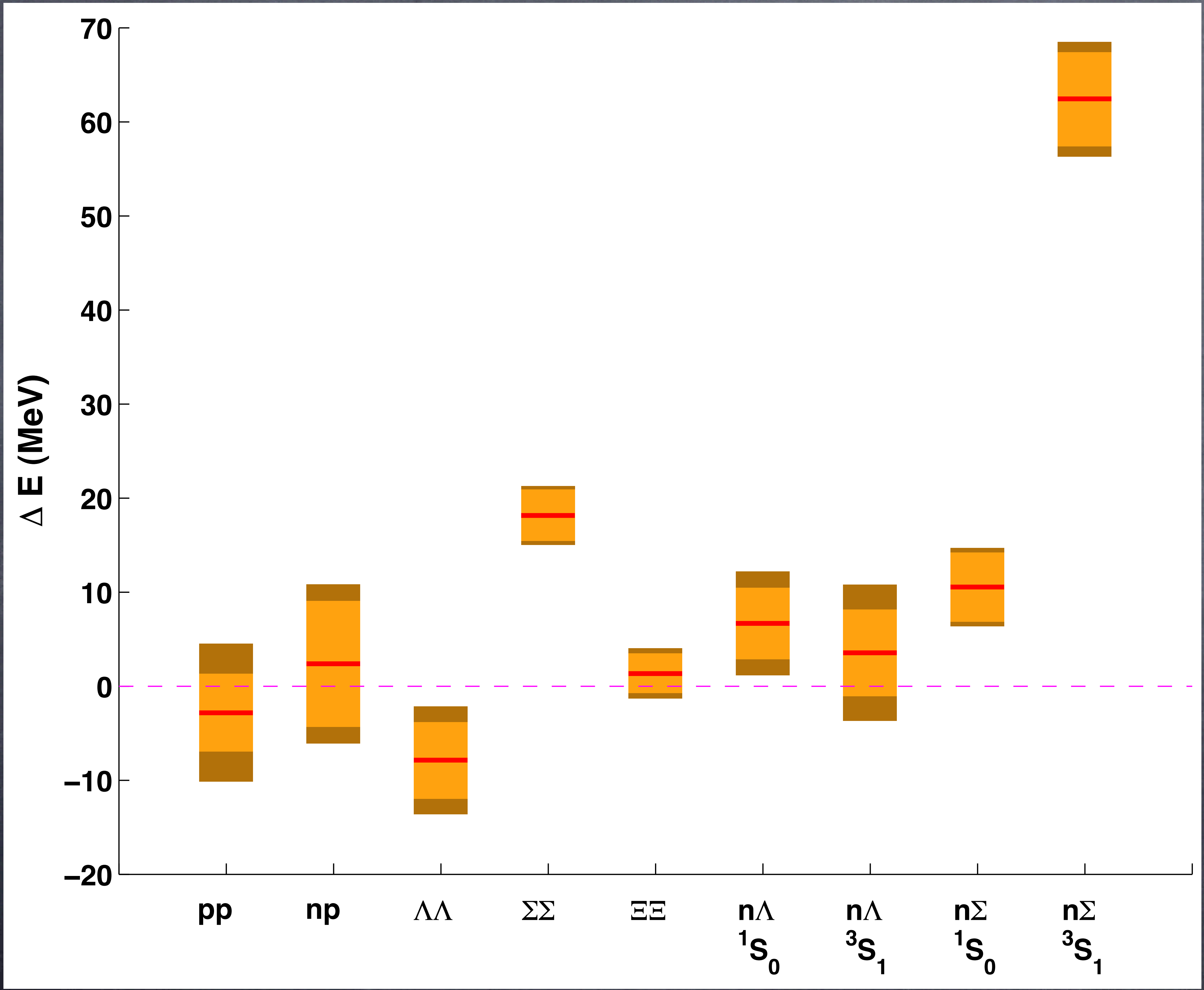
- "Variational": Symmetric positive definite matrix of correlators [C. Michael, '85; Luscher&Wolf '90; ...]
- Prony methods: [Fleming '04; NPLQCD '08; Fleming et.al. '09]
- Matrix Prony [NPLQCD '08]
- Generalized pencil of matrix [Aubin, KO'10]
  - "variational" for non symmetric matrices

Two Baryon Correlation functions

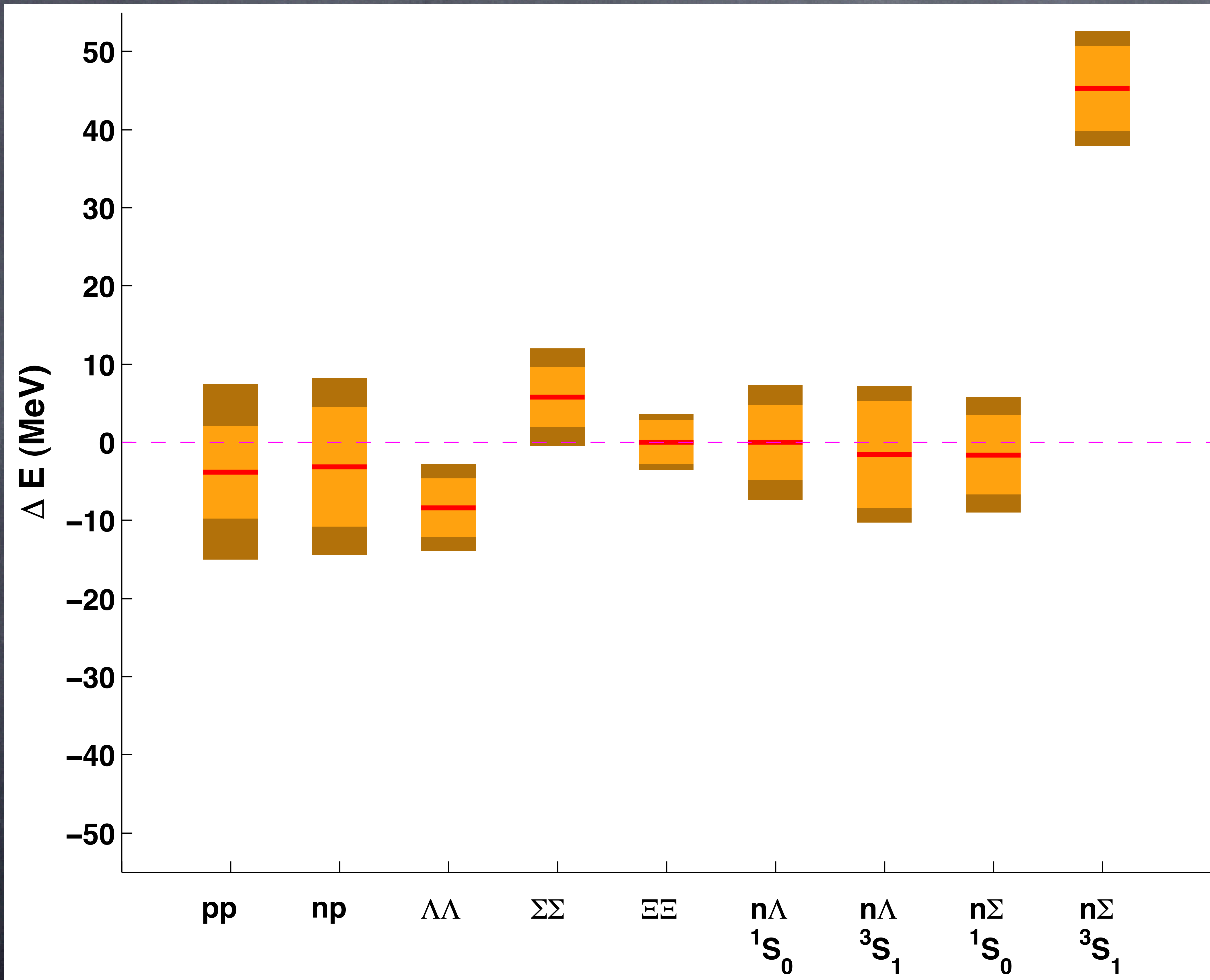
- Single smeared quark source
- Multiple sink interpolating fields
    - Smeared, Point and Smeared-Point
- Resulting a 3x1 matrix
- No-need for all-to-all propagators
- Very high statistics (300K correlation functions on 2K lattices)

NPLQCD data

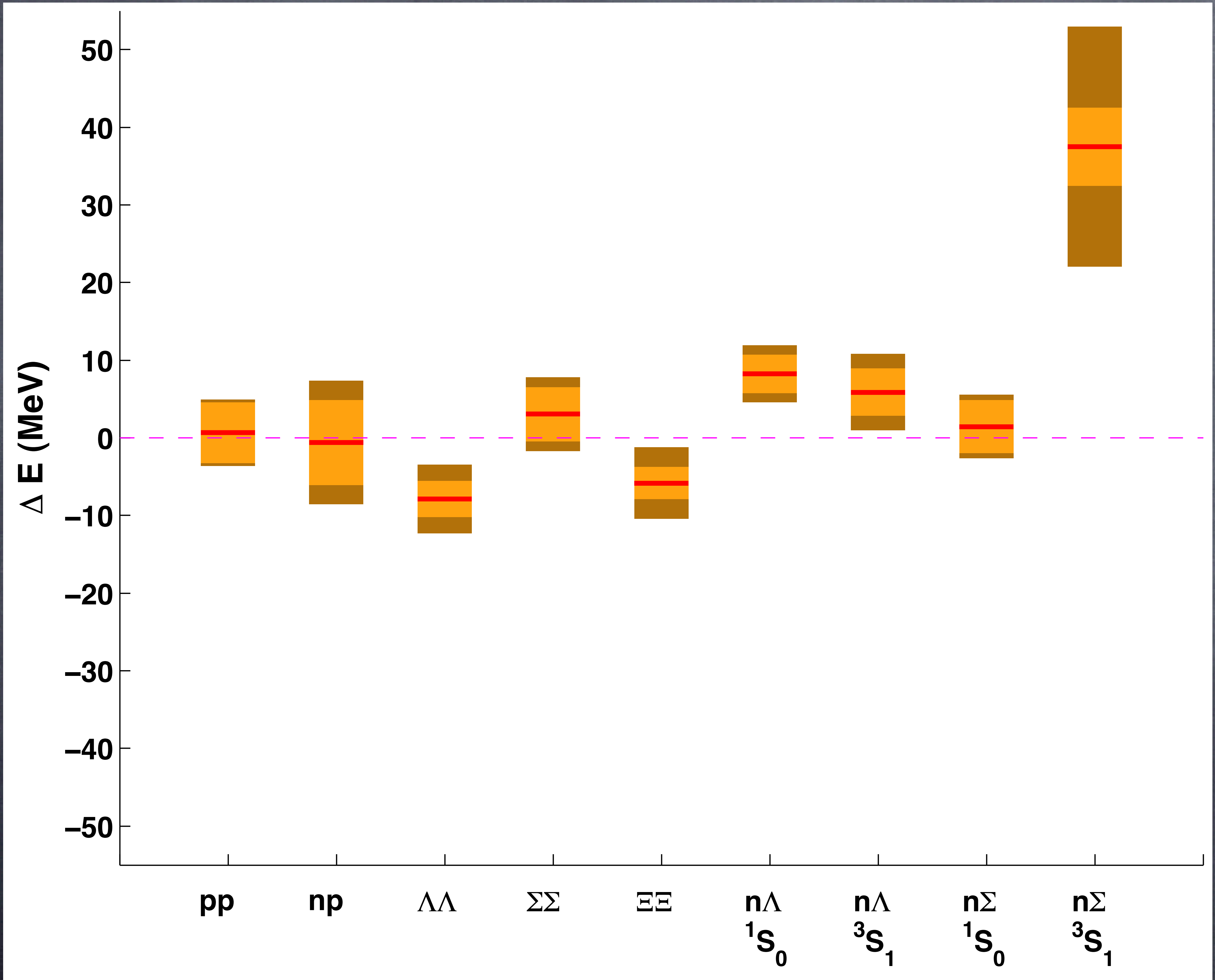
$20^3 \times 128$



$24^3 \times 128$

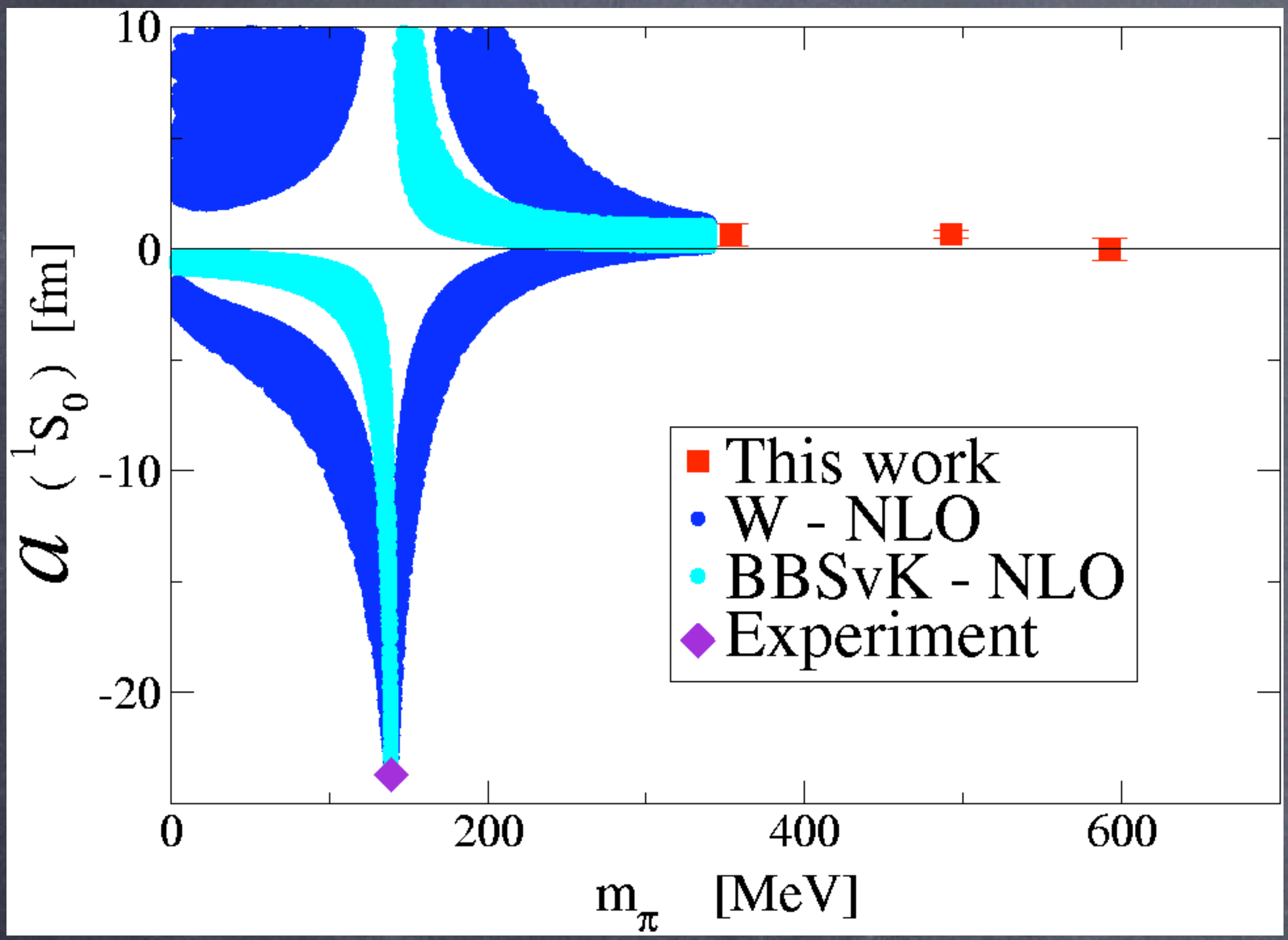


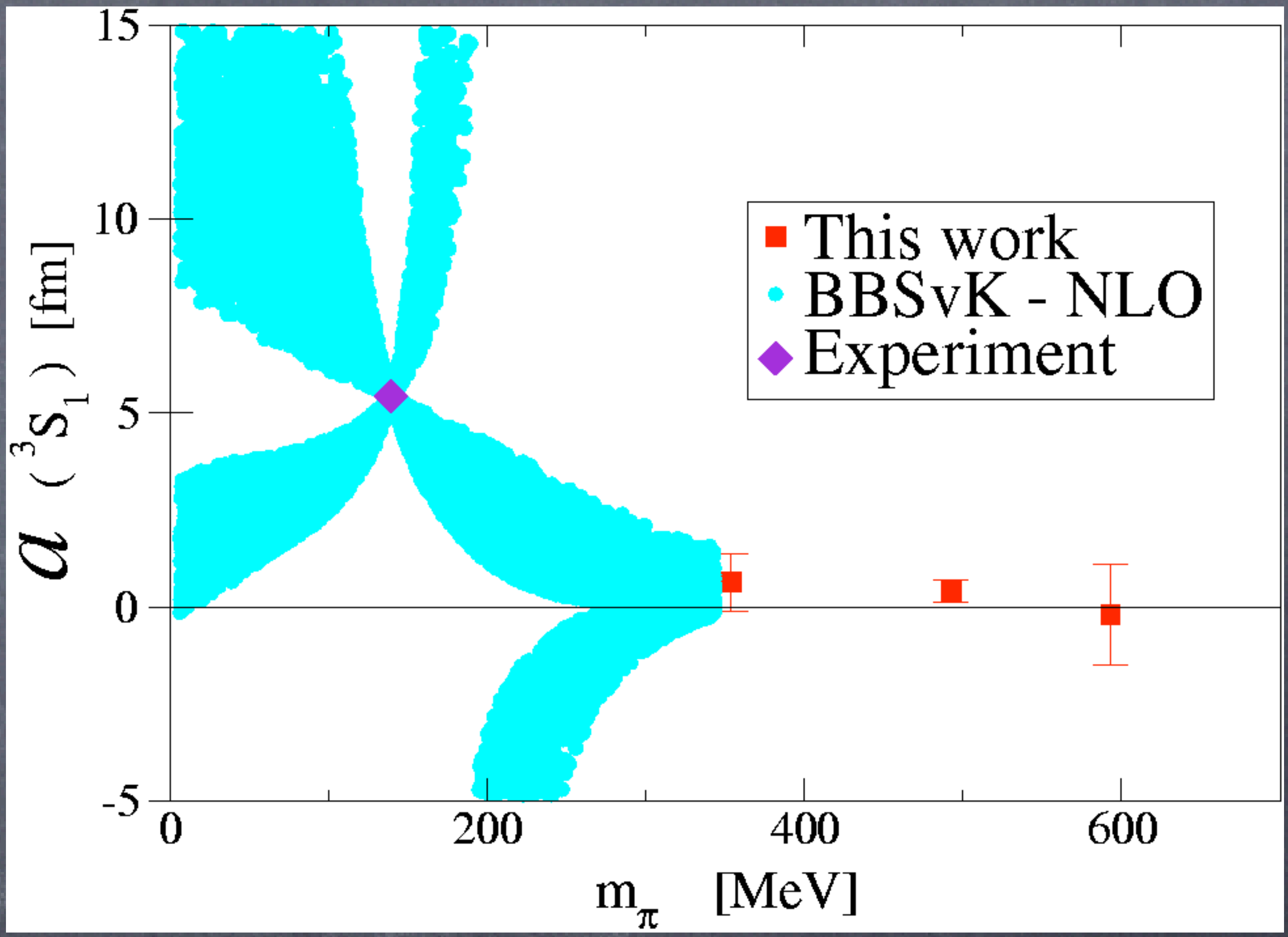
$32^3 \times 256$



Nucleon-Nucleon

NPLQCD: Phys.Rev.Lett.97 2006

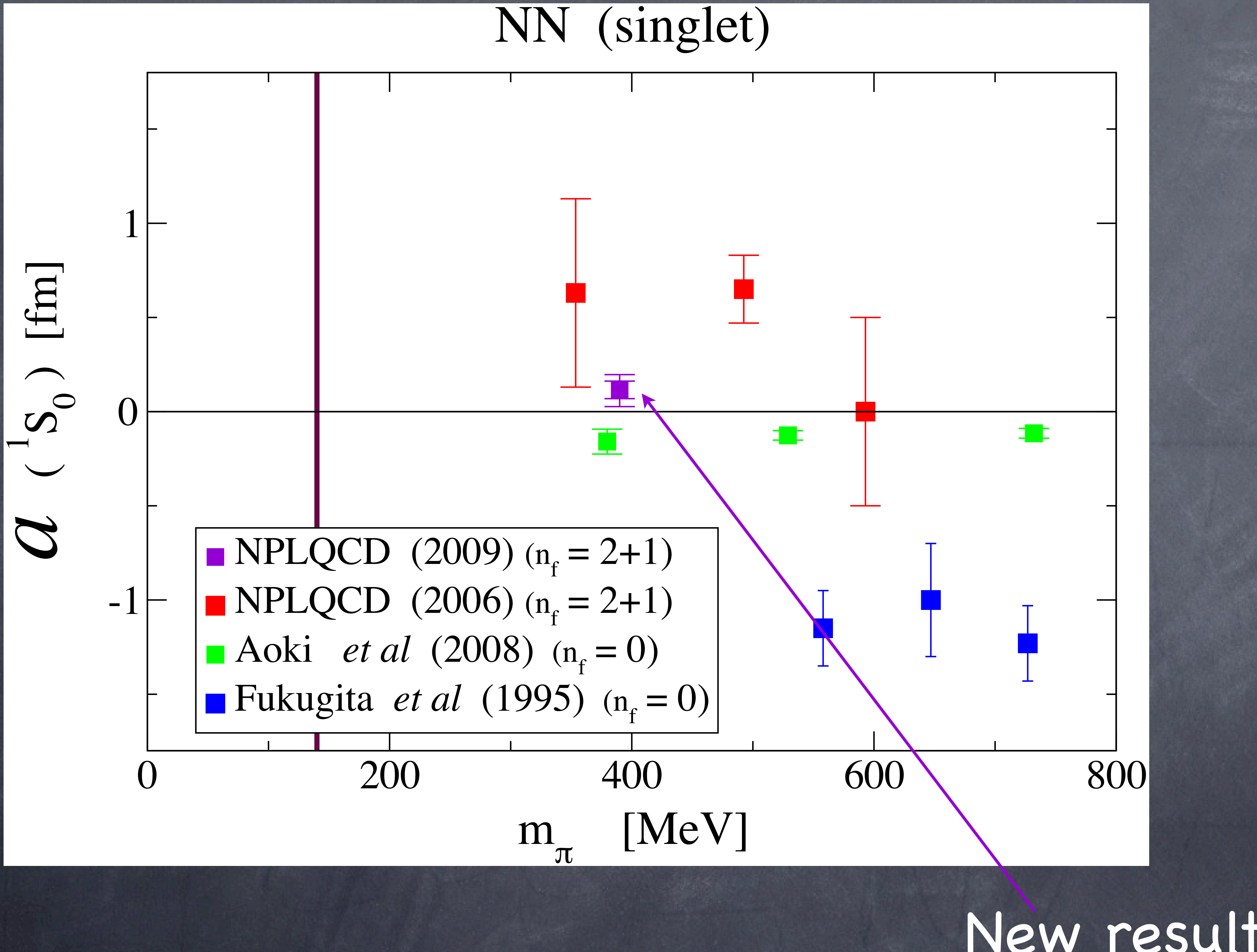




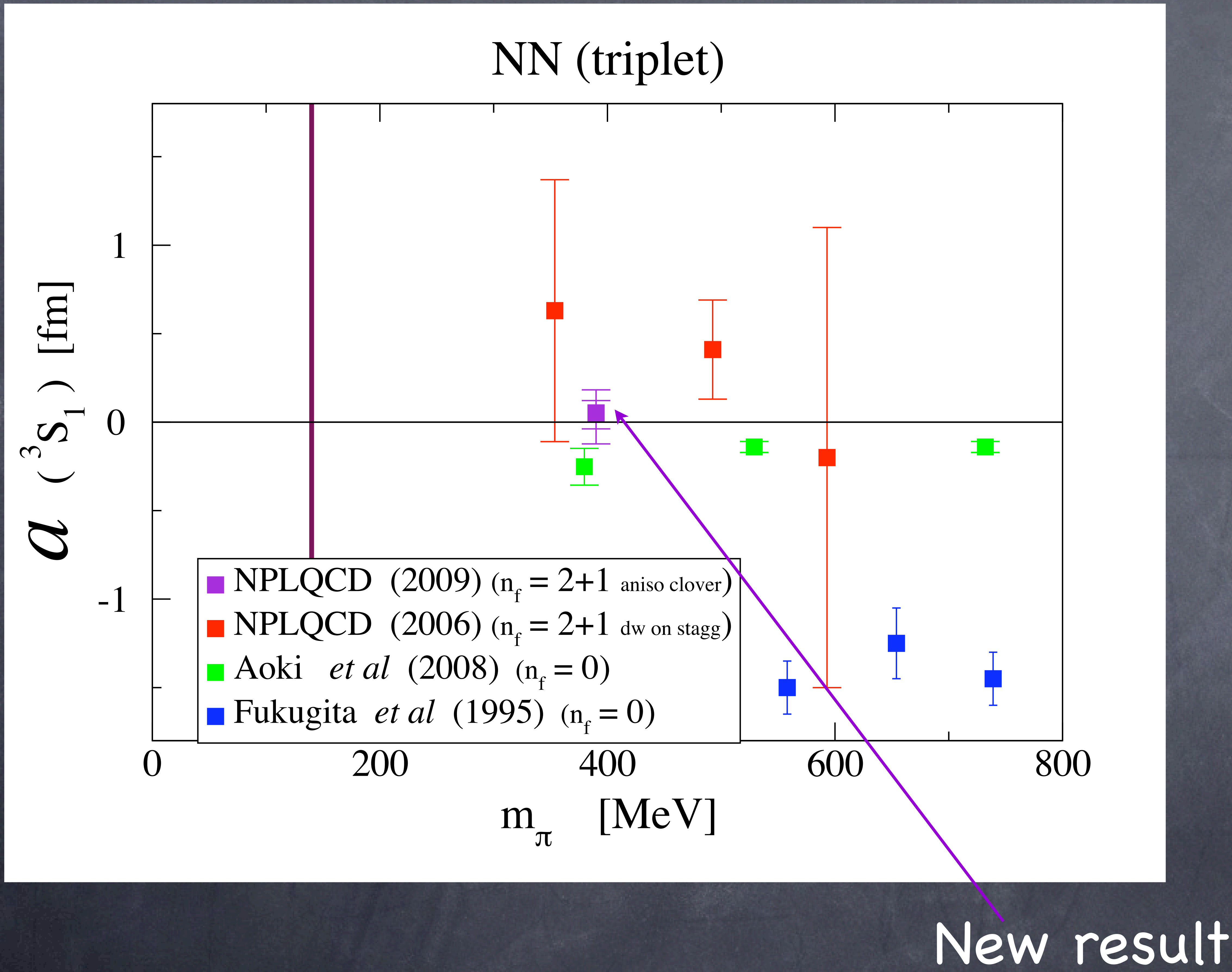
BBSvK: Beane Bedaque Savage van Kolck '02

W: Weinberg '90;Weingberg '91; Ordonez et.al '95

Fukugita et al. '95: Quenched heavy pions



NN (singlet)
a ($^1S_0$) [fm]
1
0
-1
0
200
400
600
800
m_π [MeV]
NPLQCD (2009) (n_f = 2+1)
NPLQCD (2006) (n_f = 2+1)
Aoki *et al* (2008) (n_f = 0)
Fukugita *et al* (1995) (n_f = 0)
New result


NN (triplet)
1
0
-1
a ($^3S_1$) [fm]
0
200
400
600
800
m_π [MeV]
NPLQCD (2009) (n_f = 2+1 aniso clover)
NPLQCD (2006) (n_f = 2+1 dw on stagg)
Aoki et al (2008) (n_f = 0)
Fukugita et al (1995) (n_f = 0)
New result

H-Dibaryon

proposed 1977 by R. Jaffe

Λ-Λ bound state (uuddss)

H-Dibaryon

- Negative energy shift is observed in finite volume
- Use multiple (large) volumes to extract infinite volume energy γ
- Finite volume corrections are big if binding energy is small

$$E_{-1} = -\frac{\gamma^2}{m}\left[1 + \frac{12}{\gamma L}\,\frac{1}{1-2\gamma(p\cot\delta)'}\,e^{-\gamma L} + \ldots\right]$$

H-Dibaryon

16^3x128	2.0fm	useless
20^3x128	2.5fm	marginal
24^3x128	3.0fm	good
32^3x256	4.0fm	excellent

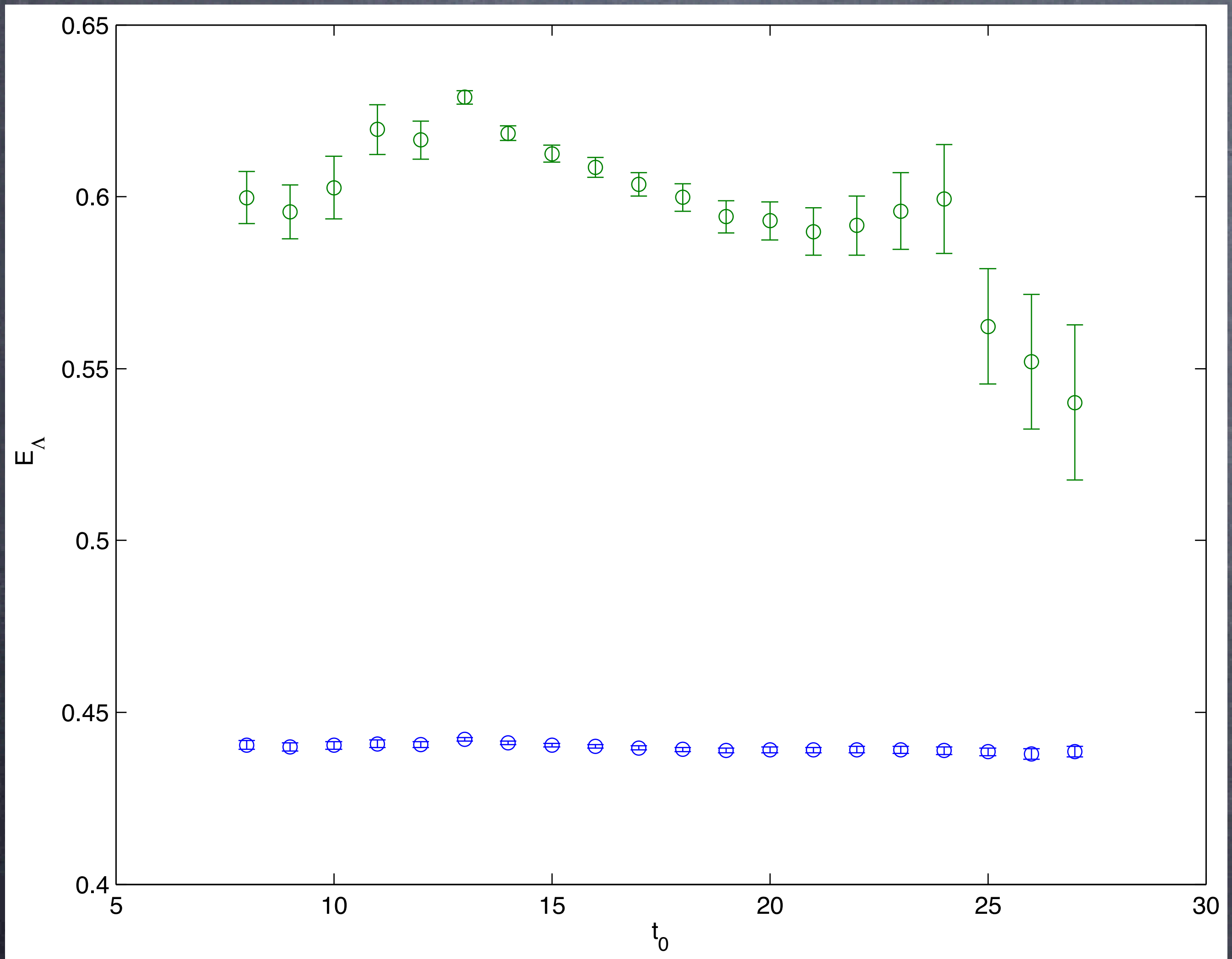
M_π=390MeV
2+1 Clover anisotropic fermions

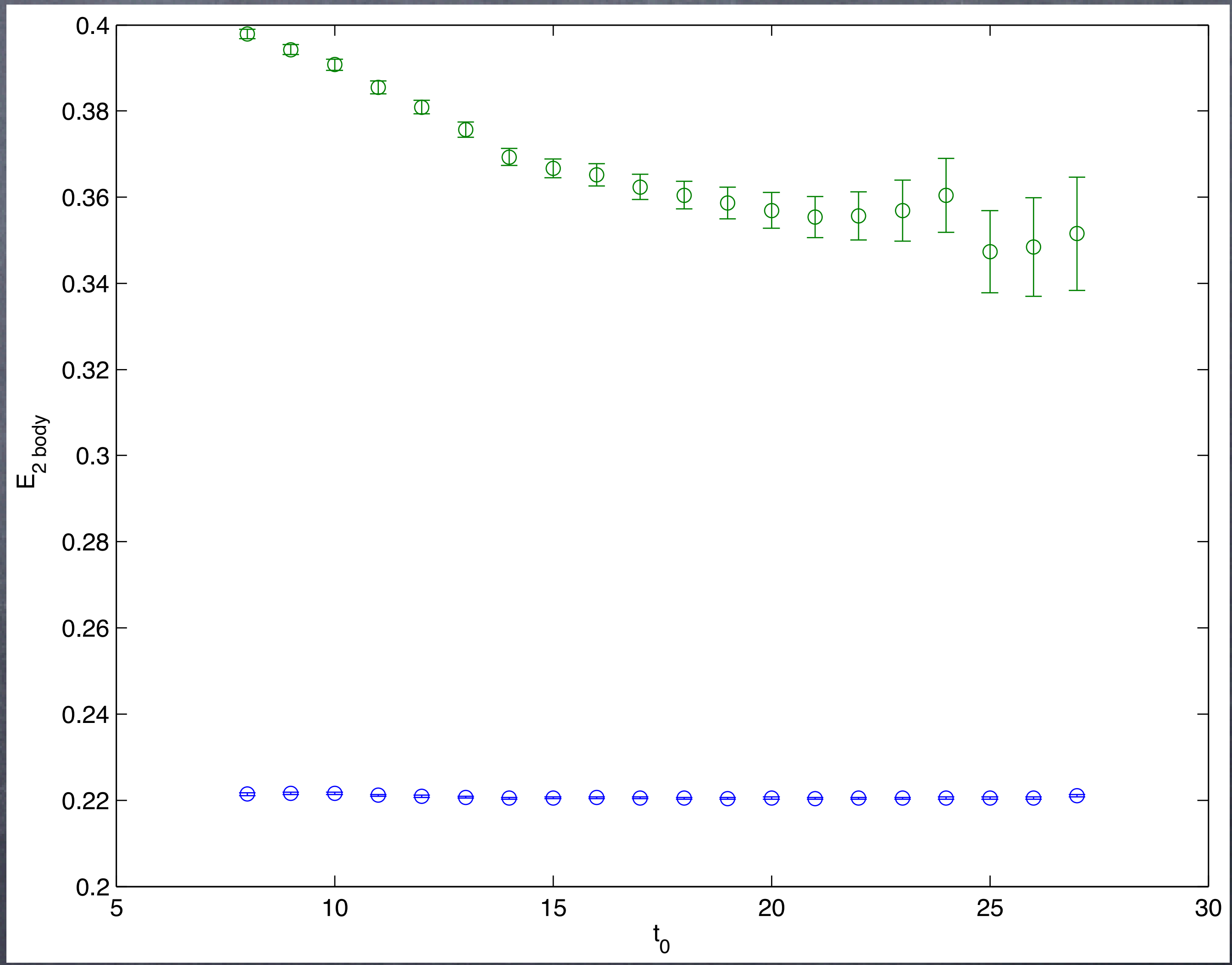
$$E_{-1} = -\frac{\gamma^2}{m}\left[1 + \frac{12}{\gamma L}\frac{1}{1-2\gamma(p\cot\delta)'}e^{-\gamma L} + \ldots\right]$$

H-Dibaryon

I=0 S=-2 two baryons

Lambda





H-Dibaryon

$$E_{-1} = -\frac{\gamma^2}{m}\left[1 + \frac{12}{\gamma L}\frac{1}{1-2\gamma(p\cot\delta)'}e^{-\gamma L} + \ldots\right]$$

B_H= 16.6 ± 2.1 ± 4.6 MeV

statistical systematic

NPLQCD: arXiv:1012.3812

Phys. Rev. Lett. **106**, 162001
(Published April 20, 2011)

M_π=390MeV
2+1 Clover anisotropic fermions

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statistical systematic

Continuum limit?

Physical pion mass?

Isospin breaking?

Electromagnetism?

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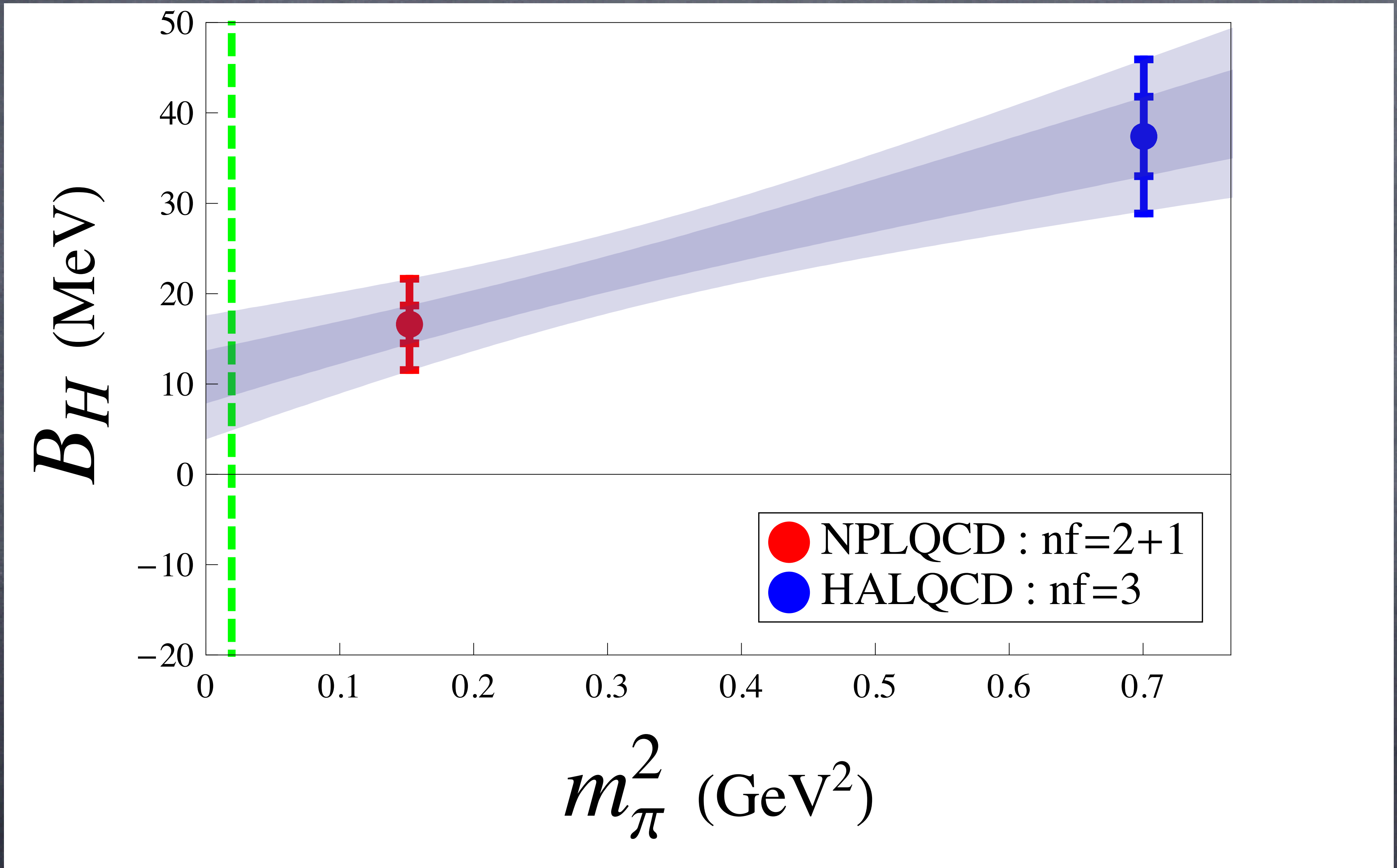


arXiv:1103.2821

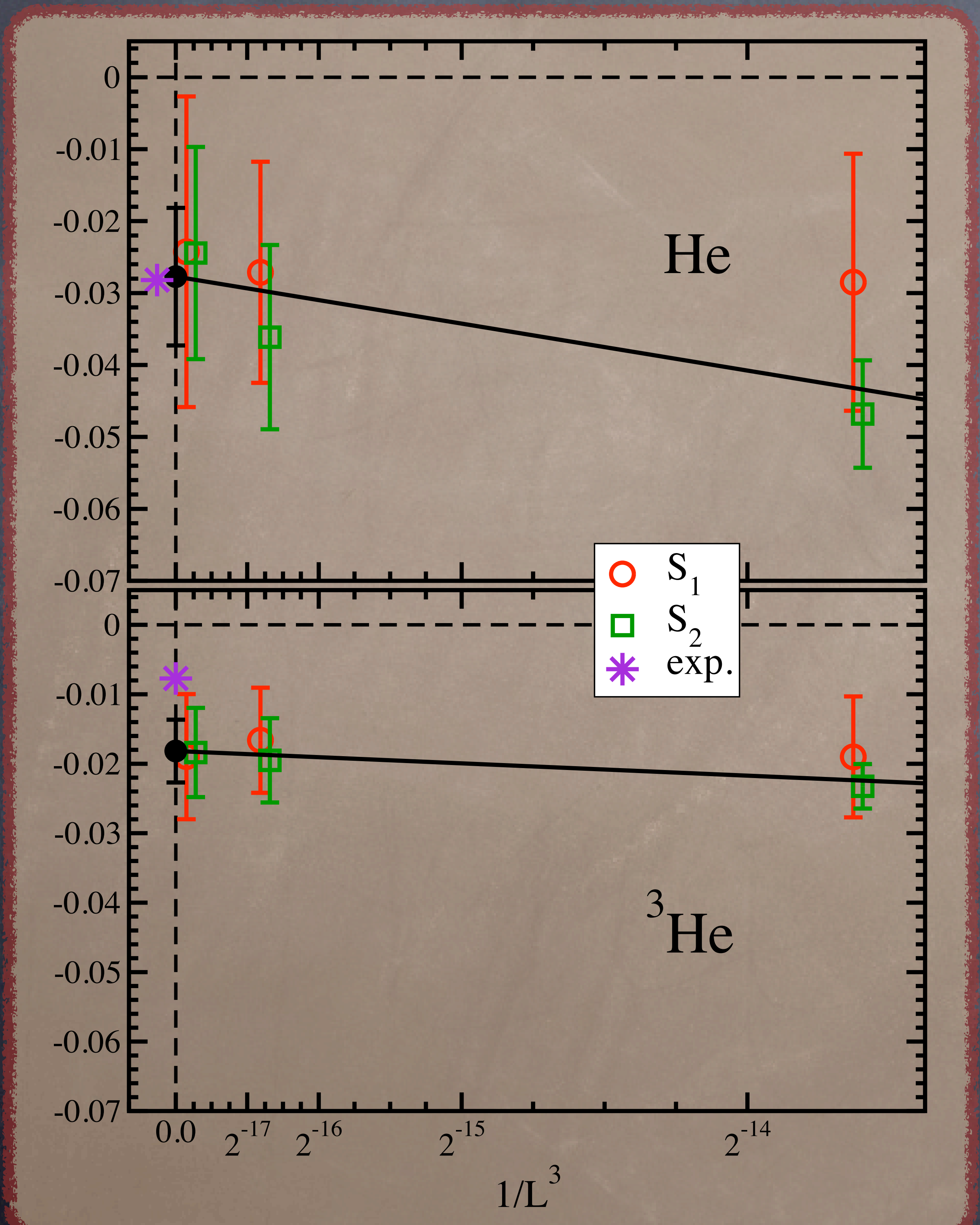
Conclusions

- Two nucleon systems are quite challenging
  - Deuteron has not been observed
  - Progress has been made in quenched QCD and very heavy pion masses (CP-PACS: arXiv:1105.1418, Phys.Rev.D81:111504,2010)
- Some evidence of bound h-dibaryon at heavy pion masses
  - What happens at the physical pion mass?
- Energy estimation methodology needs further development
  - Better interpolating fields
  - Cost of correlation function construction
- More than 2 baryon systems
- Realistic computations are still very expensive and it is difficult to make progress

Helium

CPPACS: Quenched heavy pion

Phys.Rev.D81:111504,2010



3He
NPLQCD: 2+1 dynamical
390MeV pion