The two-body and the many-body problem in QCD from the lattice

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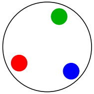
Scope of lattice QCD simulations

Physics of **color singlets**

- "One-body" physics: **confinement**
  hadron masses
  form factors, etc..



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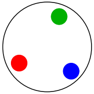
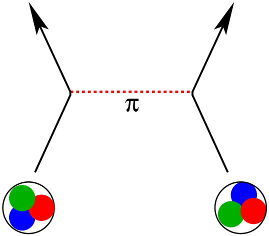
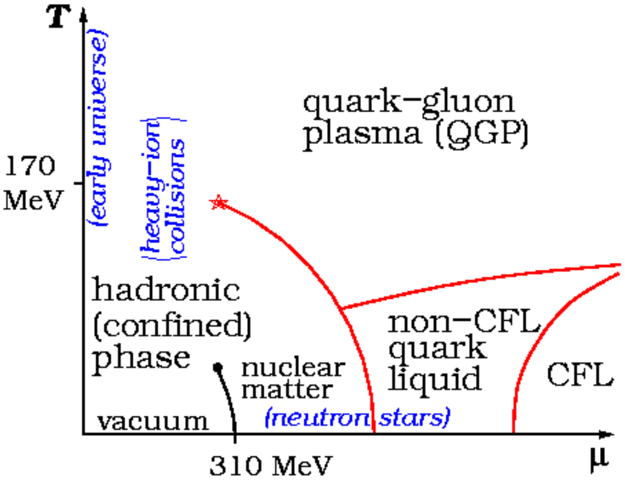


- "Two-body" physics: **nuclear interactions**
  pioneers Hatsuda et al, Savage et al



hard-core
+
pion exchange?

QCD phase diagram according to Wikipedia



Here: • “many-body” physics: **hadron $\leftrightarrow$ nuclear matter** transition
• “two-body”: $T = 0$ **nuclear interactions**

A different approach to the sign problem

$$Z = \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{N_f}\bar{\psi}_i(\not{D} + m_i + \mu_i\gamma_0)\psi_i\right)$$

$\det(\not{D} + m + \mu\gamma_0)$ complex $\rightarrow$ try integrating over the gauge field first!

- Problem: $-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \rightarrow \beta_{\text{gauge}}\mathrm{Tr}U_{\text{Plaquette}}$, ie. 4-link interaction 
- Solution: set $\beta_{\text{gauge}} = \frac{2N_c}{g^2}$ to **zero**, ie. $g = \infty$, strong coupling limit
- Then integral over gauge links *factorizes*: $\sim \int \prod dU \exp(\bar{\psi}_x U_{x,\hat{\mu}} \psi_{x+\hat{\mu}})$

- analytic 1-link integral $\rightarrow$ only color singlets survive

- perform Grassmann integration last $\rightarrow$ hopping of color singlets

$\rightarrow$ **hadron worldlines**

- sample gas of worldlines by Monte Carlo

Note: when $\beta_{\text{gauge}} = 0$, quarks are *always* confined $\forall(\mu, T)$, ie. **nuclear matter**

The price to pay: not continuum QCD

Strong coupling LQCD: why bother ?

Asymptotic freedom: $a(\beta_{\rm gauge}) \propto \exp(-\frac{\beta_{\rm gauge}}{4N_c b_0})$

ie. $a \to 0$ when $\beta_{\rm gauge} \equiv \frac{2N_c}{g^2} \to +\infty$. Here $\boxed{\beta_{\rm gauge} = 0}$!!

- Lattice “infinitely coarse”
- Physics not universal

Nevertheless:

- Properties similar to QCD: confinement and χSB
- Include (perhaps) next term in strong coupling expansion, ie. $\beta_{\rm gauge} > 0$

When $\beta_{\rm gauge} = 0$, sign problem is **manageable** $\to$ **complete solution**

Valuable insight? ***understand*** nuclear interactions

Further motivation

- 25^+ years of analytic predictions:

  80's: Kluberg-Stern et al., Kawamoto-Smit, Damgaard-Kawamoto

  $$\mu_c(T=0) = 0.66, \quad T_c(\mu=0) = 5/3$$

  90's: Petersson et al., $1/g^2$ corrections

  00's: detailed (μ, T) phase diagram: Nishida, Kawamoto,...

  now: Ohnishi et al. $\mathcal{O}(\beta)$ & $\mathcal{O}(\beta^2)$, Münster & Philipsen,...

  How accurate is mean-field $(1/d)$ approximation?

- Almost no Monte Carlo crosschecks:

  89: Karsch-Mütter $\rightarrow$ MDP formalism $\rightarrow$ $\mu_c(T=0) \sim 0.63$

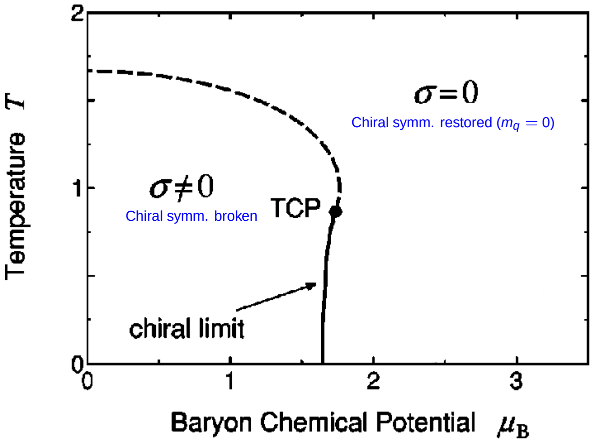
  92: Karsch et al. $T_c(\mu=0) \approx 1.40$

  99: Azcoiti et al., MDP ergodicity ??

  06: PdF-Kim, HMC $\rightarrow$ hadron spectrum $\sim 2\%$ of mean-field

Can one trust the details of analytic phase-diagram predictions?

Phase diagram from Nishida (2004, mean field, cf. Fukushima)



- Very similar to conjectured phase diagram of $N_f = 2$ **QCD**
- But no deconfinement here: high density phase is **nuclear matter**
- Baryon mass $= M_{\text{proton}}$ $\Rightarrow$ lattice spacing $a \sim 0.6$ fm not universal

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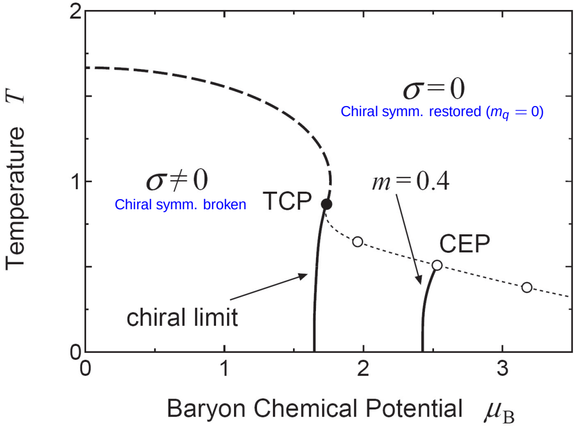

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Strong coupling $SU(3)$ with staggered quarks

$Z = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp(-\bar{\psi}(\not{D}(U) + m_q)\psi)$, no plaquette term ($\beta_{\text{gauge}} = 0$)

- **One** complex colored fermion field per site (no Dirac indices, spinless)
- $\not{D}(U) = \frac{1}{2}\sum_{x,\nu} \eta_\nu(x)(U_\nu(x) - U_\nu^\dagger(x - \hat{\nu}))$, $\eta_\nu(x) = (-)^{x_1 + .. + x_{\nu-1}}$
- Chemical potential $\mu \quad \to \exp(\pm a\mu) U_{\pm 4}$
- $\mathcal{D}U = \prod dU$ *factorizes* $\to$ integrate over links   Rossi & Wolff

$\to$ **Color singlet** degrees of freedom:

- **Meson** $\bar{\psi}\psi$: ***monomer***, $M(x) \in \{0,1,2,3\}$
- **Meson hopping**: ***dimer***, non-oriented $n_\nu(x) \in \{0,1,2,3\}$
- **Baryon hopping**: oriented $\bar{B}B_\nu(x) \in \{0,1\}$ $\to$ ***self-avoiding loops*** C

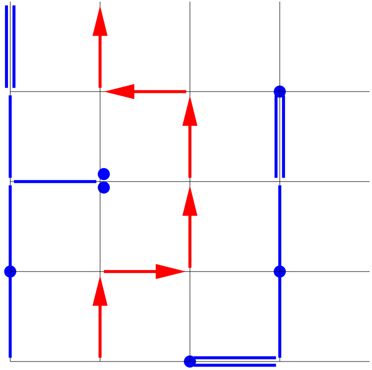
Point-like, hard-core baryons in pion bath

No πNN vertex

MDP Monte Carlo

$$Z(m_q,\mu) = \sum_{\{M,n_\nu,C\}} \prod_x \frac{m_q^{M(x)}}{M(x)!} \prod_{x,\nu} \frac{(3-n_\nu(x))!}{n_\nu(x)!} \prod_{\text{loops}\, C} \rho(C)$$

with **constraint** $(M + \sum_{\pm\nu} n_\nu)(x) = 3 \;\forall x \notin \{C\}$



Constraint: 3 blue symbols or a baryon loop at every site

MDP Monte Carlo

$$Z(m_q,\mu) = \sum_{\{M,n_\nu,C\}} \prod_x \frac{m_q^{M(x)}}{M(x)!} \prod_{x,\nu} \frac{(3-n_\nu(x))!}{n_\nu(x)!} \prod_{\text{loops}\, C} \rho(C)$$

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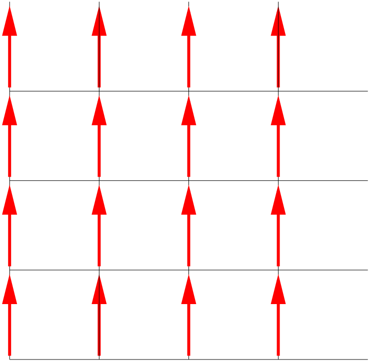

The dense (crystalline) phase: 1 baryon per site; no monomer $\rightarrow$ $\langle \bar{\psi}\psi \rangle = 0$

MDP Monte Carlo

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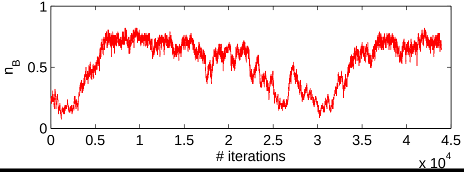
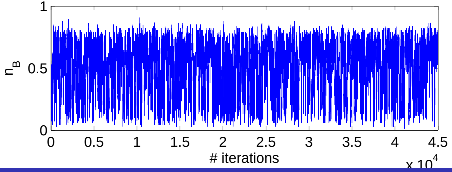
Remaining difficulties:

- Baryons are fermions: mild sign problem from $\rho(C)$ Karsch & Mütter
  $\rightarrow$ volumes up to $16^3 \times 4 \;\forall \mu$
- tight-packing constraint $\rightarrow$ local update inefficient, esp. as $m \rightarrow 0$
  Solved with **worm algorithm** (Prokof'ev & Svistunov 1998)
  Efficient even when $m_q = 0$

Local Metropolis, 4^3x2 at μ_c, m_q = 0.025



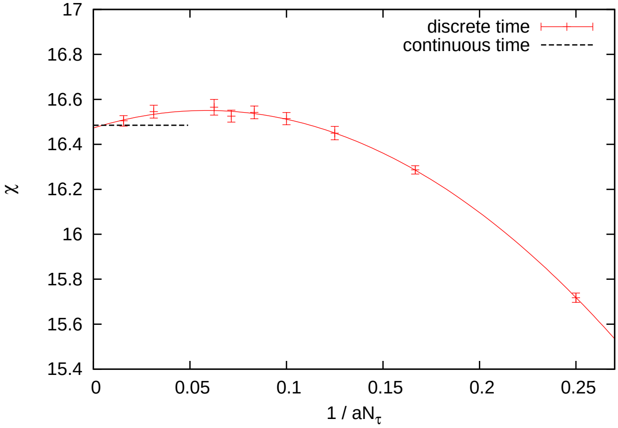
Worm, same parameter set



Continuous Euclidean time

Wolfgang Unger

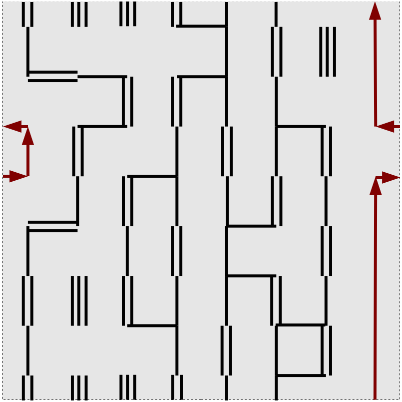
- Finite temperature: $T = \frac{1}{N_t a}$ can only vary in *discrete steps*
  $aT_c \sim 1.4 \quad \rightarrow$ need finer grid in Euclidean time
- Anisotropy γ in Dirac couplings: $\frac{a_s}{a_t}$ vs γ ? $\quad \frac{a_s}{a_t} = \gamma^2$ in mean-field
- Limit $\gamma \rightarrow \infty$, γ^2/N_t fixed is non-monotonic



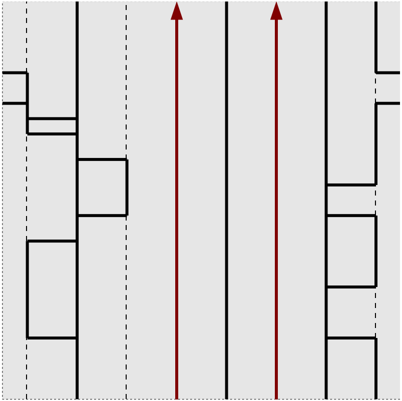
Continuous Euclidean time

Wolfgang Unger

Discrete time



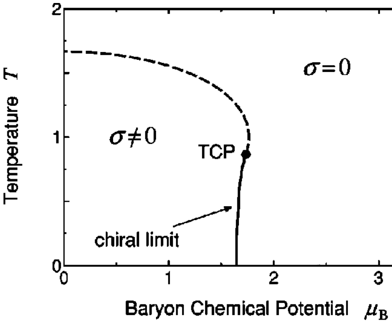
Continuous time

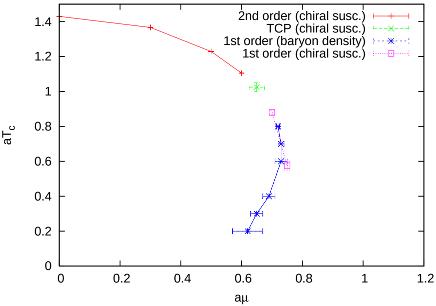


Bonus: - Simpler and faster: no multiple spatial dimers

- Sign problem *goes away* (baryons are static)
- Green's functions are *functions* of $\tau \rightarrow$ analytic continuation?

(μ, T) phase diagram in the chiral limit $m_q = 0$





- Phase boundary for breaking/restoration of $U(1)$ chiral symmetry
- 2nd order at $\mu = 0$: 3d O(2) universality class
- 1rst order at $T = 0$: ρ_B jumps from 0 to 1 baryon per site $\Longrightarrow$ **tricrit. pt. TCP**

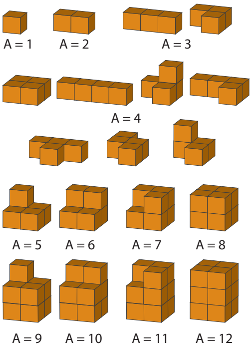
Finite-size scaling: $(\mu, T)_{TCP} = (0.65(3), 1.02(3))$ vs $(0.577, 0.866)$ (mean-field)

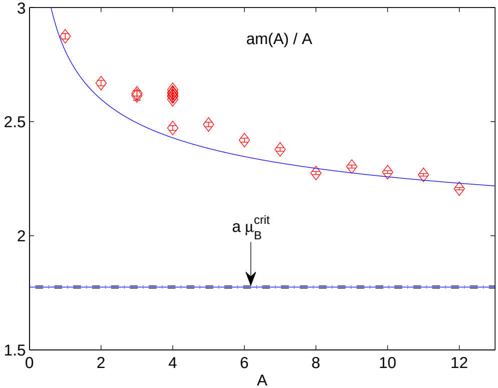
Accuracy of mean-field $\mathcal{O}(1/d)$ as expected

- Reentrance caused by *decreasing* entropy in dense phase (saturation)

  cf. Clausius-Clapeyron: $\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta S} \rightarrow$ vertical at $T = 0$

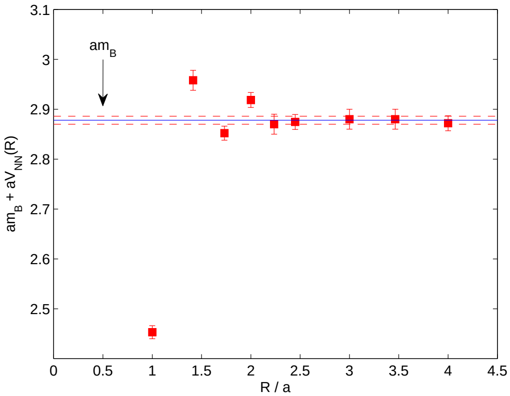
Nuclear matter: spectroscopy





- Can compare masses of differently shaped "isotopes"
- $E(B=2) - 2E(B=1) \sim -0.4$, ie. "deuteron" binding energy ca. 120 MeV
- $am(A) \sim a\mu_B^{\rm crit} A + (36\pi)^{1/3} \sigma a^2 A^{2/3}$, ie. (bulk + surface tension)
  Bethe-Weizsäcker parameter-free ($\mu_B^{\rm crit}$ and σ measured separately)
- "Magic numbers" with increased stability: $A = 4,\ 8,\ 12$ (reduced area)

Nuclear potential: more than hard core

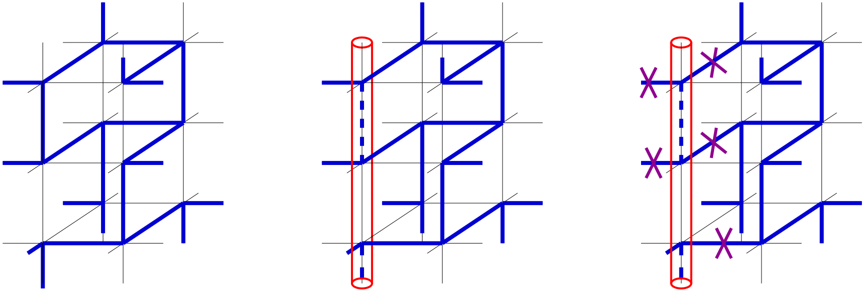


- Nucleons are point-like $\rightarrow$ no ambiguity with definition of static potential
- Nearest-neighbour attraction $\sim$ 120 MeV at distance $\sim$ 0.5 fm: cf. real world
- Baryon worldlines self-avoiding $\rightarrow$ no direct meson exchange (just hard core)

How do baryons interact beyond hard-core ?

How the nucleon got its mass

- Point-like nucleon **distorts pion bath** cf. Casimir



vacuum static baryon effect on pions

- Energy = nb. time-like pion lines

Constraint: 3 pion lines per site ($m_q = 0$) $\rightarrow$ energy density = 3/4 in vacuum

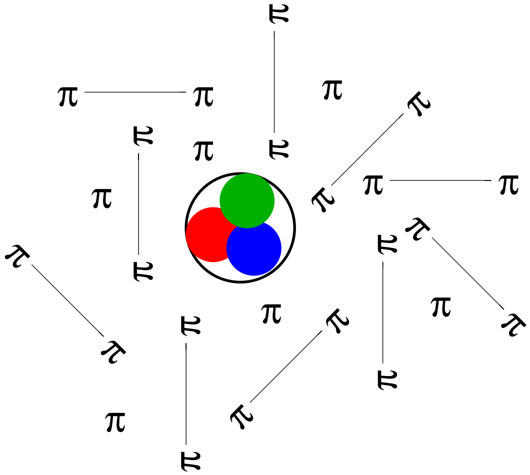
No spatial pion lines connecting to site occupied by nucleon $\rightarrow$ energy increase

Steric / Entropic effect

- $am_B \approx 2.88 = (3 - 0.75) + \Delta E_\pi$, ie. "valence"(78%) + "**pion cloud**"(22%)

So, in fact, nucleon is *not* point-like

Point-like “bag” of 3 valence quarks $\rightarrow$ macroscopic disturbance in pion vacuum



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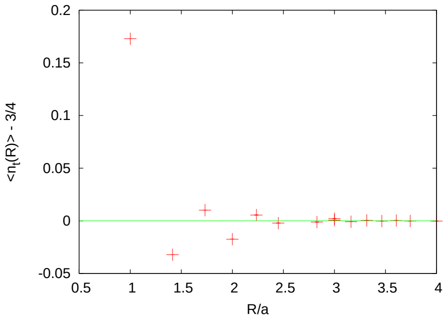
Static baryon prevents monomers = static (t-invariant) monomer “source”

Linear response $\propto$ Green’s fct. of lightest t-invariant meson, ie. rho/omega

(pion has factor $(-1)^t$)



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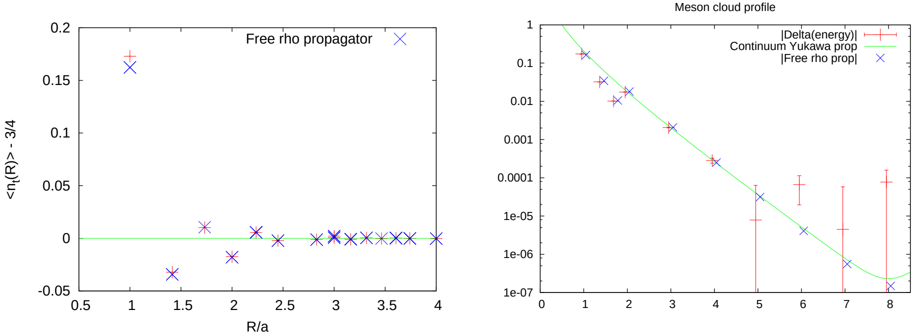


Pion cloud profile $\propto \frac{\exp(-\mathbf{m}_{\rho/\omega} r)}{r} \quad \times(-1)^{x+y+z}$

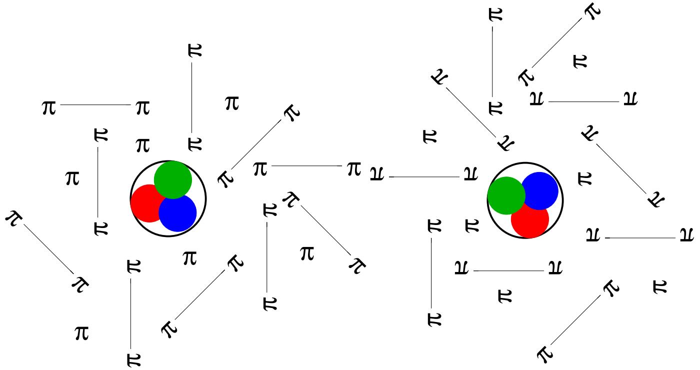
Nuclear interaction via pion clouds (thanks W. Weise)

- Here, baryons make self-avoiding loops $\rightarrow$ no direct meson exchange
- Interaction comes because of **pion clouds**

The two pion clouds can interpenetrate at $\approx$ constant energy (2nd order effect)

But each set of *valence* quarks disturbs pion cloud of other baryon

$$\Longrightarrow \quad V_{NN}(R) \approx -2 \times \Delta E_\pi(R) \; \propto \; \frac{\exp(-\mathbf{m}_{\rho/\omega} R)}{R} \times (-1)^{x+y+z}$$



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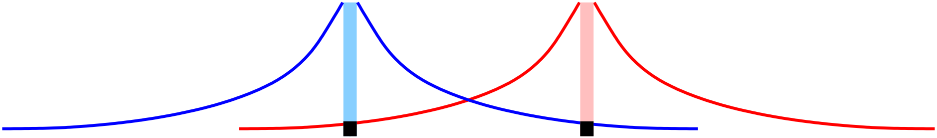

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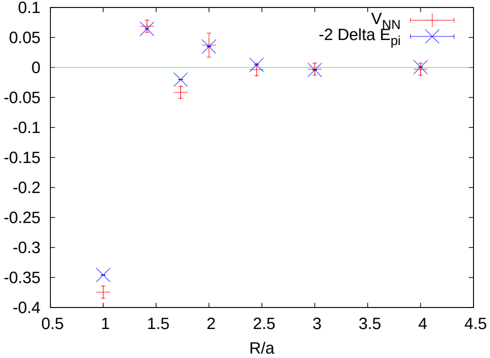
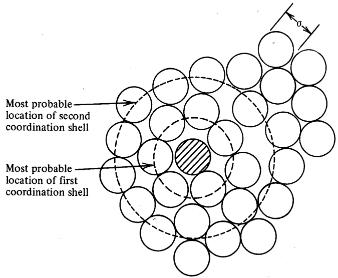
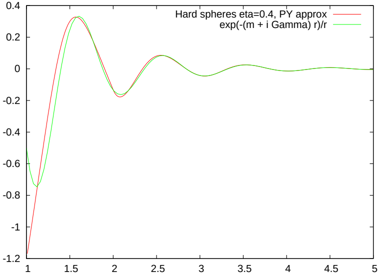


Meson exchange potential without meson exchange!

Recap: baryons and their interactions at $\beta = 0$

- Baryons are not point-like: pion cloud $\sim \exp(-m_{\rho/\omega} r)$
- Nuclear potential:
  - Hard-core from Pauli principle
  - Yukawa potential (times $(-1)^r$) from the two pion clouds
- Precisely like a **classical hard-sphere fluid**: entropic force
  - "Pion cloud" from ripples around tagged sphere
  - Density-density correlation: $V_{\rm eff}(r) \equiv -\log(g(r)) \sim \exp(-(m + i\Gamma)r)/r$





Conclusions

Summary: complete solution of strong coupling limit

- Phase diagram: take mean-field results with a grain of salt
- [Crude, crystalline] nuclear matter from QCD:

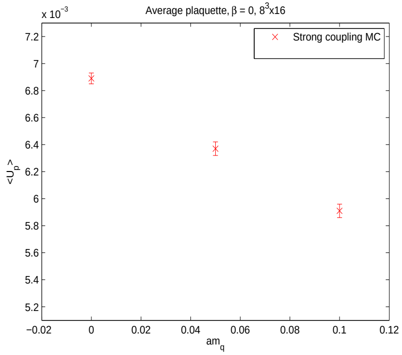
  tabletop simulations of first-principles nuclear physics
- Nucleon: point-like "bag" ($\rightarrow$ hard core) + large pion cloud

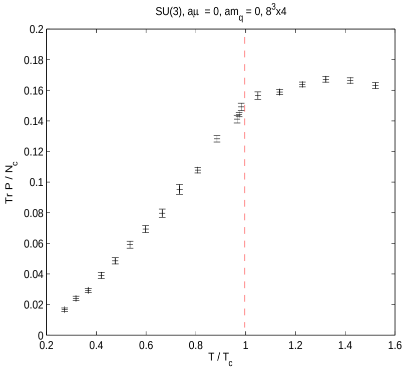
  cf. "little bag" model (Brown & Rho)
- Yukawa potential without meson exchange

Outlook

- Include $\mathcal{O}(\beta)$ effects
- Include second quark species $\rightarrow$ isospin ?

First step towards $O(\beta)$: gauge observables at $\beta = 0$





Plaquette vs m_q

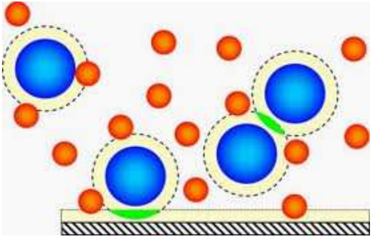
Polyakov loop vs T $(m_q = 0)$

Feedback of chiral transition $\rightarrow$ deconfinement

Entropic force

Soft condensed matter:
binary mixture of hard spheres

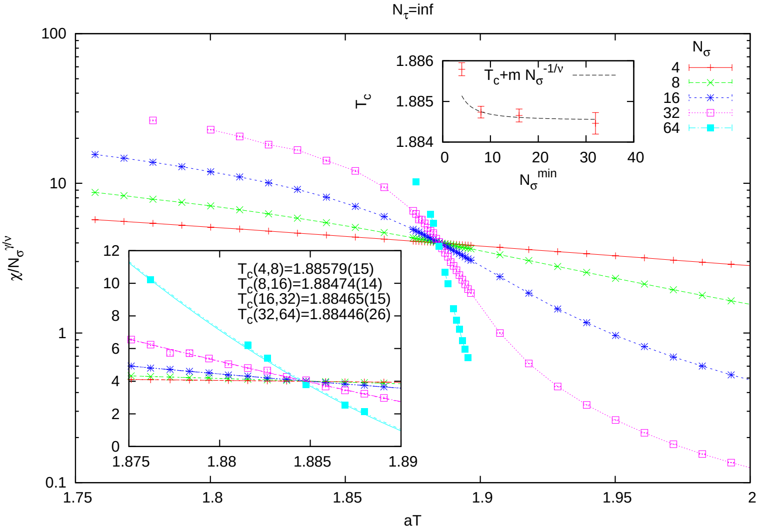
Excluded “halo” around each big sphere



Entropic force:

Total excluded volume decreases when halos overlap $\longrightarrow$ **attraction**

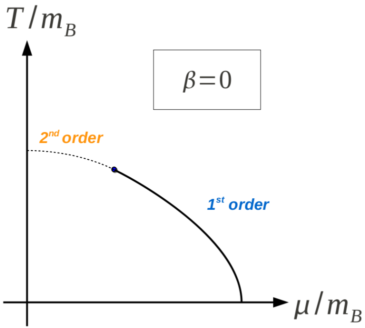
Continuous time: finite-T transition ($m_q = 0, \mu = 0$)



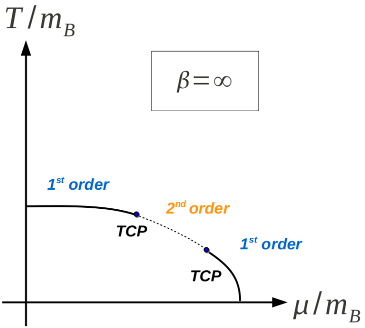
Avoid anisotropy γ $\rightarrow$ no error/ambiguity on T

Relating the strong-coupling and continuum phase diagrams

$N_f = 4$ continuum flavors, $m_q = 0$



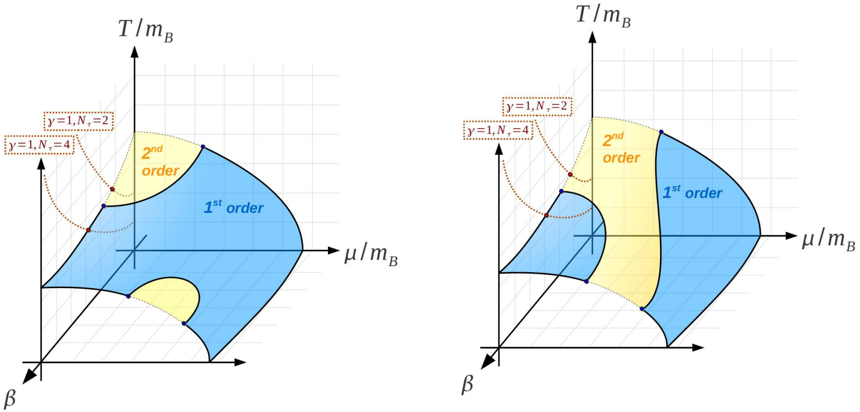
Strong coupling (known)



Continuum (minimal)

Relating the strong-coupling and continuum phase diagrams

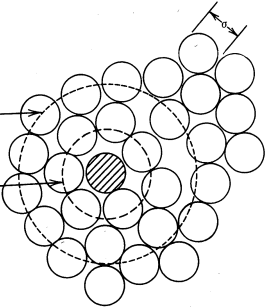
$N_f = 4$ **continuum flavors,** $m_q = 0$

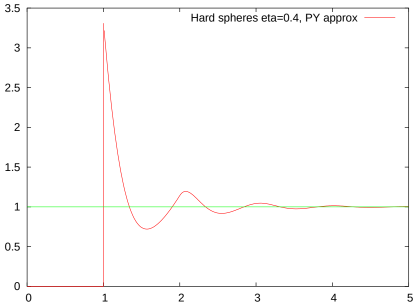


Can differentiate between the two cases

Case *right* favorable for quantitative continuum predictions

Backup: is pion bath essential? Classical hard spheres

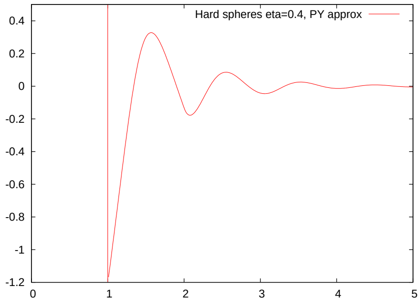




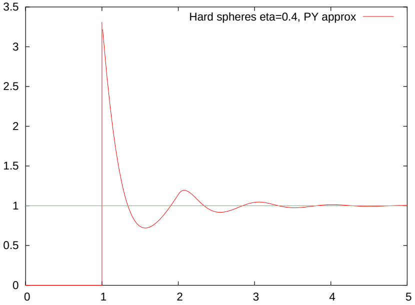
Density-density correlation
(Percus-Yevick approximation)

$g(r) \equiv \langle \rho(0)\rho(r) \rangle$ relaxes to $\langle \rho \rangle^2$ with damped oscillations $\rightarrow$ liquid

Backup: is pion bath essential? Classical hard spheres



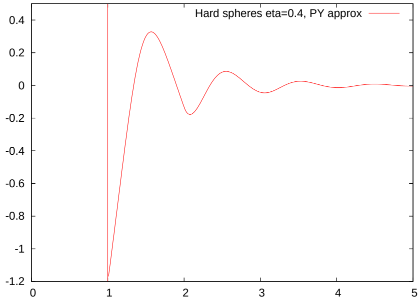
"Potential of mean force"



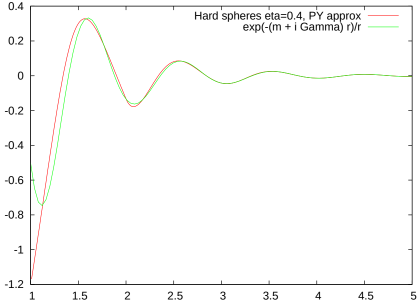
Density-density correlation

"Potential of mean force" $V_{\mathrm{eff}}(r) \equiv -\log(g(r))$ is hard-core + damped oscillatory

Backup: is pion bath essential? Classical hard spheres



"Potential of mean force"

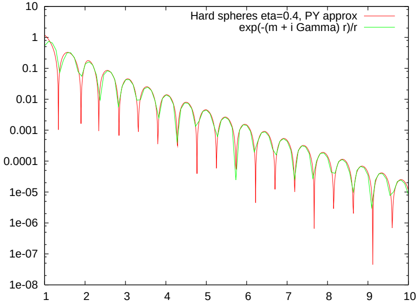


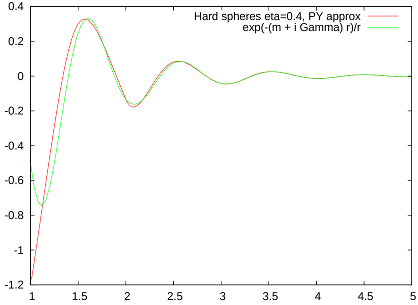
Compare with Yukawa potential

"Potential of mean force" $V_{\mathrm{eff}}(r) \equiv -\log(g(r))$ is hard-core + damped oscillatory

Consistent with Yukawa form $\frac{\exp(-mr)}{r} \times \cos(\Gamma r)$

Backup: is pion bath essential? Classical hard spheres





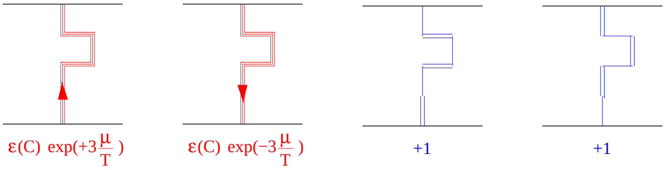
$\log(|V_{\text{eff}}(r)|)$ + Yukawa fit

Perfect fit at large distance

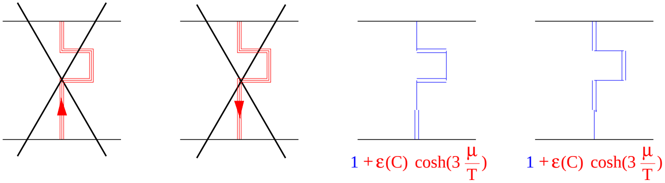
Hard-sphere "potential of mean force" is of Yukawa form

$$V_{\text{eff}}(r) = \text{Re}\left[\frac{e^{-(m+i\Gamma)r}}{r}\right]$$

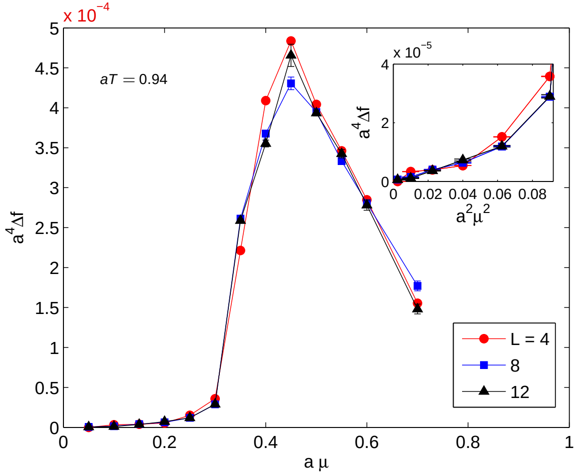
Backup: Karsch & Mütter's resummation



Karsch & Mütter: Resum into "MDP ensemble" $\rightarrow$ sign pb. eliminated at $\mu = 0$



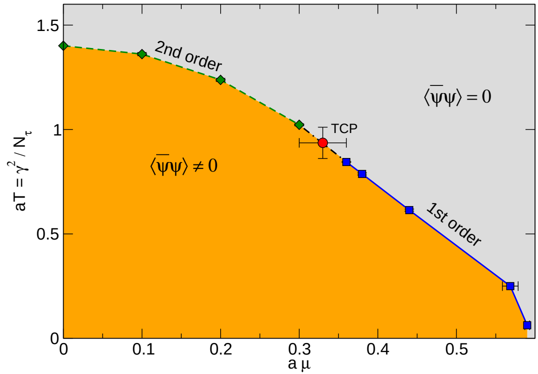
Backup: Sign problem? Monitor $-\frac{1}{V}\log\langle\text{sign}\rangle$



- $\langle\text{sign}\rangle = \frac{Z}{Z_{||}} \sim \exp(-\frac{V}{T}\Delta f(\mu^2))$ as expected; $\Delta f \sim \mu^2 + \mathcal{O}(\mu^4)$
- Can reach $\sim$ **$16^3 \times 4$** $\forall\mu$, ie. adequate

(μ, T) phase diagram in the chiral limit $m_q = 0$, and for $m_q \neq 0$

$m_q = 0$

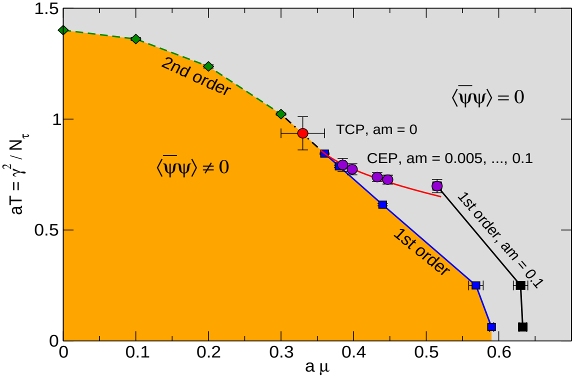


- Phase boundary for breaking/restoration of $U(1)$ chiral symmetry
- 2nd order at $\mu = 0$: 3d O(2) universality class
- 1rst order at $T = 0$: ρ_B jumps from 0 to 1 baryon per site $\Longrightarrow$ **tricrit. pt. TCP**

Finite-size scaling: $(\mu, T)_{TCP} = (0.33(3), 0.94(7))$ vs $(0.577, 0.866)$ (mean-field)

Beware of quantitative mean-field predictions for phase diagram

(μ, T) phase diagram in the chiral limit $m_q = 0$, and for $m_q \neq 0$



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Beware of quantitative mean-field predictions for phase diagram

- $m_q \neq 0$: liquid-gas transition $T_{CEP} \sim 200$MeV – traj. of CEP obeys tricrit. scaling