Quasi-particle degrees of freedom in finite temperature SU(N) gauge theories.

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11° workshop on Non-Perturbative Quantum Chromodynamics

Institut d'Astrophysique de Paris - France - 6 June 6 2011

INTRODUCTION

- **Analysis of SU(Nc) gauge theories at finite temperature for Nc = 3 , 4 , 6 above the critical temperature.**

- **Lattice data in the region Tc < T < 5 Tc show a rapid change in the quantity ε - 3 p and therefore a sudden rearrangement of the effective degrees of freedom.**

- **Phenomenological interpretation of these features in terms of: quasi-particle degrees of freedom + gluon condensate contribution**

and indications from the lattice data about some properties of the quasi-particle mass .

P.Castorina, D.Miller, H.Satz, ArXiv:1101.1255 ;

P.Castorina, V.Greco, D.Jaccarino, D.Z., ArXiv:1105.5902

SU(Nc) Gauge theories at finite T

Phase transition at T**c** (first order for Nc > 2)

At T**c** onset of deconfinement. Simplified: glueball gas → free gluon gas

BUT: Stefan - Boltzmann limit reached only for T / T**c** >> 1

Region around Tc highly non perturbative and Lattice computations are required.

G. Boyd et al., Nucl. Phys B 469 (1996) 419
M. Panero, Phys Rev. Lett. 103 (2009) 23001
S. Datta, S. Gupta, Phys Rev D82 (2010) 114505

Focus on the interaction measure $\Delta(T) = \Theta^{\mu}{}_{\mu}(T) / T^4$

with normalized e.-m. tensor trace: $T^{\mu}{}_{\mu}(T) = \Theta^{\mu}{}_{\mu}(T) + \Theta_o{}^{\mu}{}_{\mu}$

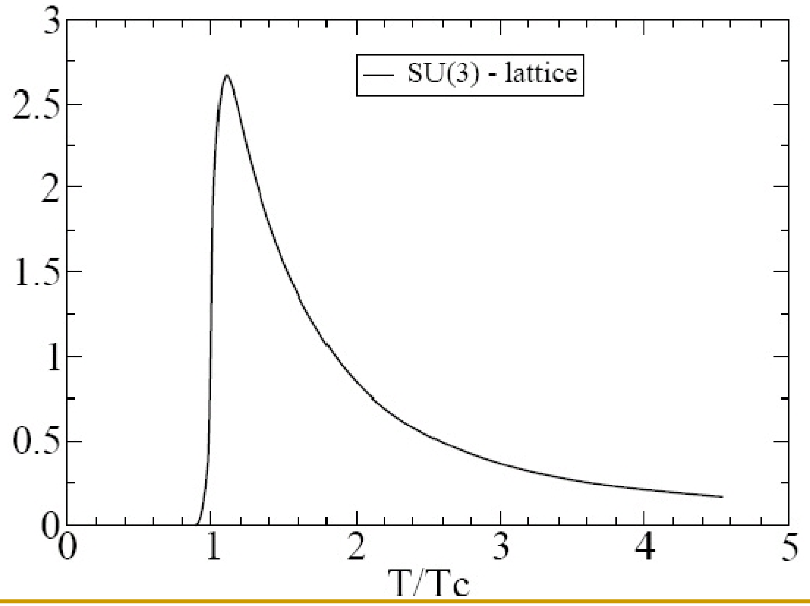
Δ is related to the energy density and pressure: $\Delta(T) = (\varepsilon - 3p) / T^4$

where ε and p vanish at T=0.

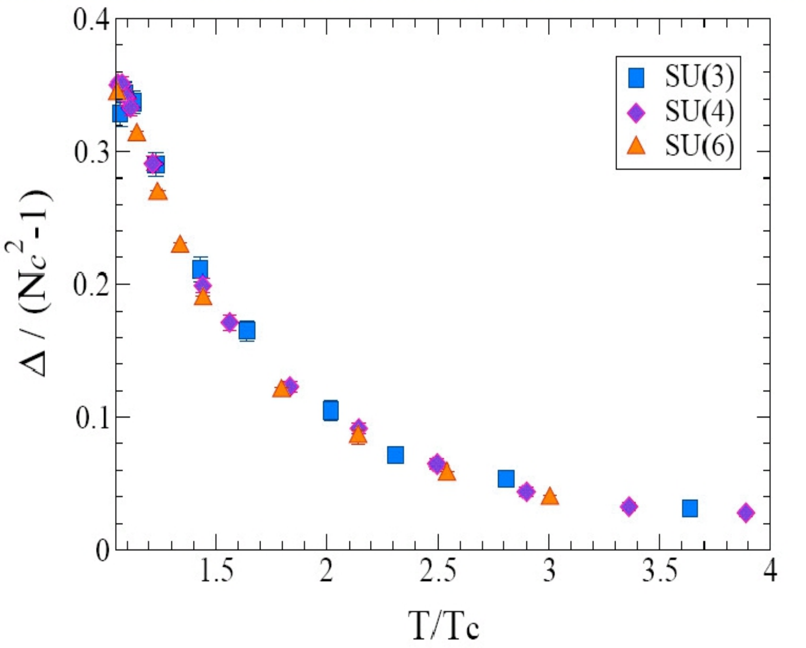
Δ (T) measures the evolution with T of the scale invariance breaking and is an indication of the residual interaction of the medium.

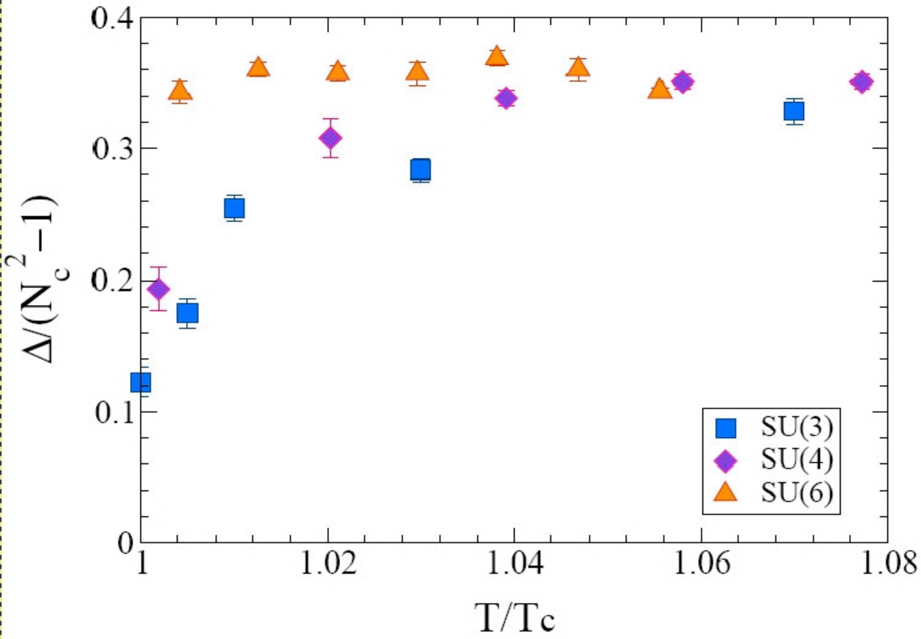
Δ = 0 corresponds to conformal invariance and is recovered at high T (S-B limit).

Just above the transition, the energy density grows much more rapidly than the pressure developing a peak at about **T= 1.05 Tc** and suggesting the appearance of new degrees of freedom.



Scaling with color number N_c





Above the peak clear scaling of Δ with the number of gluonic degrees of freedom

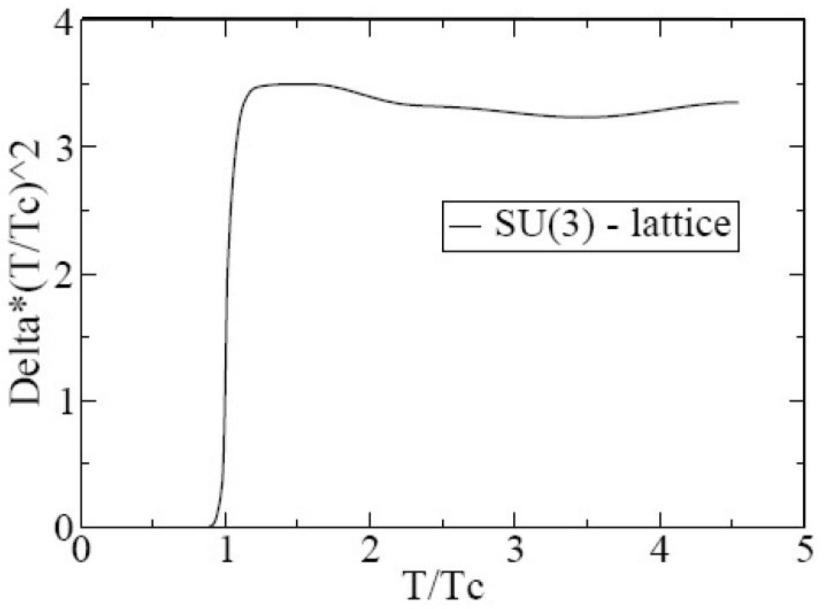
$$N_c^2 - 1$$

Below the peak deviation from scaling.

At T**c** approximate scaling of Δ with

$$(N_c^2 - 1)\ N_c$$

Scaling with the temperature T



Above the peak and up to 4 – 5 T**c**

$$\frac{\Delta \cdot T^2}{(N_c^2 - 1)}$$ **is almost T (and Nc) independent.**

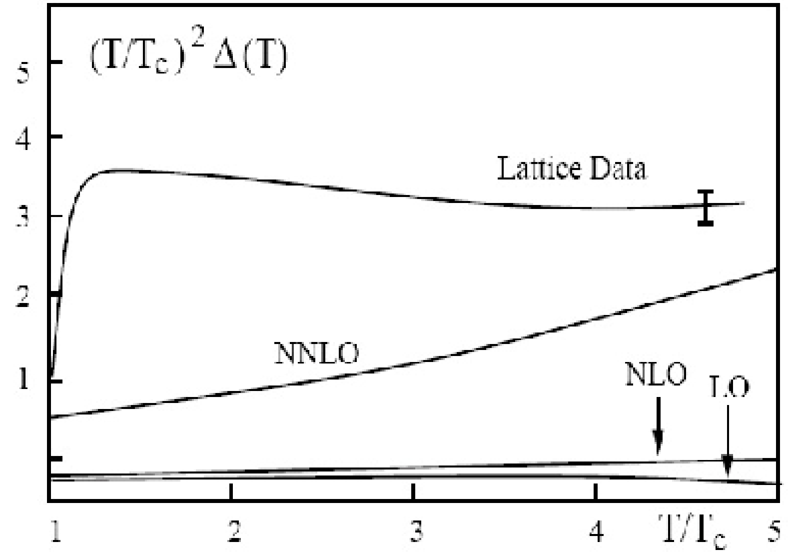
Substantial breakdown of perturbative calculations around the peak but also at **T = 4 - - 5 Tc**

Leading order $\Delta = a\,(N_c^2-1)N_c^2 g^4(T)$ with $N_c g^2(T) = \frac{b}{\ln(T/\Lambda)}$

which does not reproduce the **T** scaling.

Even the Hard Thermal Loop resummation fails in this temperature range.

J. Andersen, M.Strickland, N.Su, JHEP1008(2010) 113



Gluon condensate contribution to Δ

At finite temperature, with the adopted normalization:

H. Leutwyler, in "QCD 20 years later", World Scientific (1993) 693.

$$\Theta^{\mu}_{\mu}(T) = \epsilon(T) - 3p(T) = \langle G^2 \rangle_0 - \langle G^2 \rangle_T$$

with $G^2 = -\frac{\beta(g)}{2g} G^a_{\mu\nu} G^{\mu\nu}_a$ and $\beta(g) = -\frac{11 N_c}{48\pi^2} g^3 + O(g^5)$

Then the condensate has the desired Nc dependence (above the peak) :

$$G^2 = \frac{11 N_c g^2}{96\pi^2} G^a_{\mu\nu} G^{\mu\nu}_a = \frac{11 N_c g^2}{96\pi^2} (N_c^2 - 1) \overline{G}_{\mu\nu} \overline{G}^{\mu\nu}$$

as long as one is in the t' Hooft scaling regime : $N_c g^2$ independent of Nc.

→ Input $\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu}_a \rangle_0 \simeq 0.012 \pm 0.006 \ GeV^4$

The SU(3) gluon condensate at finite temperature has been computed on lattice:
It is almost constant below Tc.
Above Tc its c-electric component vanshes; its c-magnetic component stays const.

M.D'Elia, A.Di Giacomo,E.Meggiolaro, Phys.Rev. D 67 (2003) 114504.

$$G^2 = G_e^2 + G_m^2 \qquad \langle G_m^2\rangle_0 \simeq \langle G_e^2\rangle_0 = \frac{1}{2}\langle G^2\rangle_0$$

$$\frac{\langle G_m^2\rangle_T}{\langle G_m^2\rangle_0} \simeq 1 \qquad \frac{\langle G_e^2\rangle_T}{\langle G_e^2\rangle_0} = c(T) \simeq -\theta_H(T)$$

c(T) goes from 1 to 0 in the range (**0.95 -- 1.1) Tc**

Above 1.1Tc , the gluon condensate contributes to Δ with $\frac{\langle G^2\rangle_0}{2T^4}$

Below Tc and close to the transition the gluon condensate is essential to fit lattice data on Δ because the glueball gas contribution is too small.

F. Buisseret, Eur. Phys. J C68 (2010) 473.

Above Tc the gluon condensate **does not show the correct T behavior** but can still play a role very close to Tc.

Another contribution is necessary to explain the behavior of Δ in the range Tc < T < 4 Tc

Simplest picture: colored quasi-particle dof avaliable above the transition and the interaction effects can be described by a T-dependent mass.

Thermodynamics of a massive gas of particles:

$$\ln \mathcal{Z}(T,V) = 2V(N_c^2-1)\int \frac{d^3k}{(2\pi)^3}\,\ln\left[f_T(k)\,\exp\left(\sqrt{\vec{k}^2+m^2(T)}/T\right)\right]$$

$$f_T(k) = \left[\exp\left(\sqrt{\vec{k}^2+m^2(T)}/T\right)-1\right]^{-1}$$

$$p = T\,\frac{\partial \ln \mathcal{Z}}{\partial V} \qquad \epsilon = \frac{T^2}{V}\,\frac{\partial \ln \mathcal{Z}}{\partial T} \qquad \Delta = \frac{(\epsilon-3p)}{T^4} \qquad s = \frac{(\epsilon+p)}{T}$$

In the high temperature limit, lattice data approach the Stefan-Boltzmann limit of a weakly interacting gluon gas.

Accordingly, in that limit the number of color and polarization degrees of freedom of are those of gluons.

In particular 2 (rather than 3) polarization states for the quasi particle excitations are required.

Note that there are mechanisms of dynamical mass generation which do not introduce a third polarization state.

V. Gribov, Nucl. Phys. B 139(1978)1;
D. Zwanziger, Phys. Rev. Lett. 94 (2005) 182301.

J. M. Cornwall, Phys. Rev. D 26 (1982) 1453;
D. Binosi, J. Papavassiliou, Phys. Rep. 479 (2009) 1.

Mass Ansatze

At sufficiently large temperature, T only scale available → **m grows with T**

Previous attempts to extend the thermal mass form $m \sim N_c g^2(T)T$ down to Tc.

A. Peshier et. al. , Phys. Rev. D54 (1996) 2399.

But, around the Δ peak and close to Tc, due to strongly non-perturbative effects the perturbative anzsatze could be insufficient.

Suggestion: mass of the excitations associated to the energy contained in a volume whose size is determined by the correlation length ξ .

$$m(t) \simeq \epsilon(t) V_{cor} = \epsilon(t) \int dr\ r^2 \frac{\exp[-r/\xi(t)]}{r^{1-\eta}} \qquad t = T/T_c$$

In a critical regime (Second Order Ph. Tr.) **ε and ξ** show power-law scaling in the variable (t -1). Then, **m diverges as an inverse power of (t-1).**

P.Castorina, D.Miller, H.Satz, ArXiv:11011255

$$m(t) = \frac{a}{(t-\delta)^c} + bt$$

Critical behavior at Tc corresponds to **$\delta = 1$.**

For Nc = 3, 4, 6, we expect large finite m(t=1) i.e. **$\delta < 1$.**

When T approaches Tc from above , m grows and
the colored excitations tend to decouple from the spectrum.

m(t) grows linearly for t >> 1.

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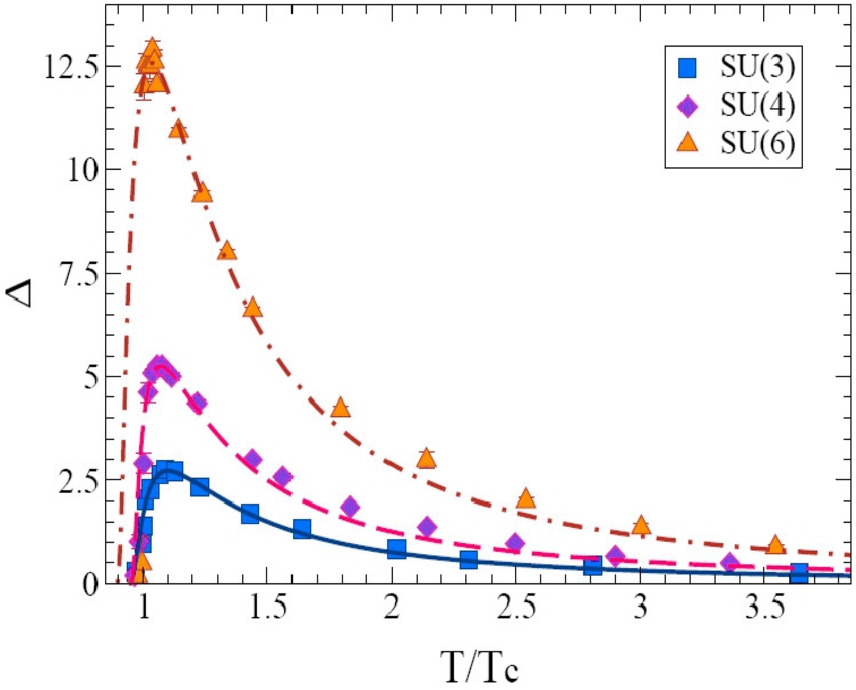
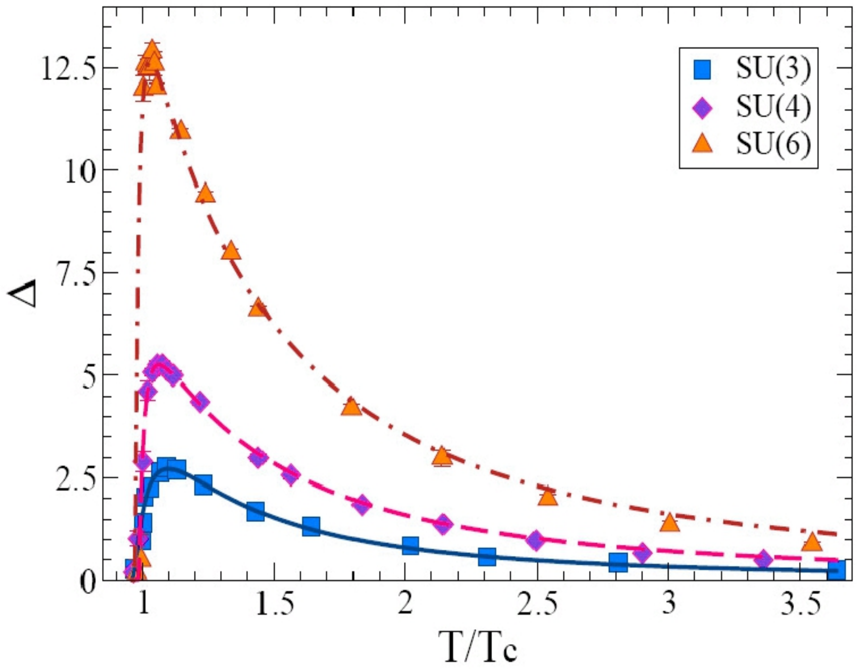
m(t) has a totally different behavior from the Debye screening mass m_D. Their relation is :

A. Peshier et. al. , Phys. Rev. D54 (1996) 2399.

$$m_D = \frac{g^2 N_c}{\pi^2 T} \int_0^\infty dk\ k^2 f_T^2(k) \exp \frac{\sqrt{\vec{k}^2 + m(T)^2}}{T}$$





fit to a, b, δ **c=0.5**

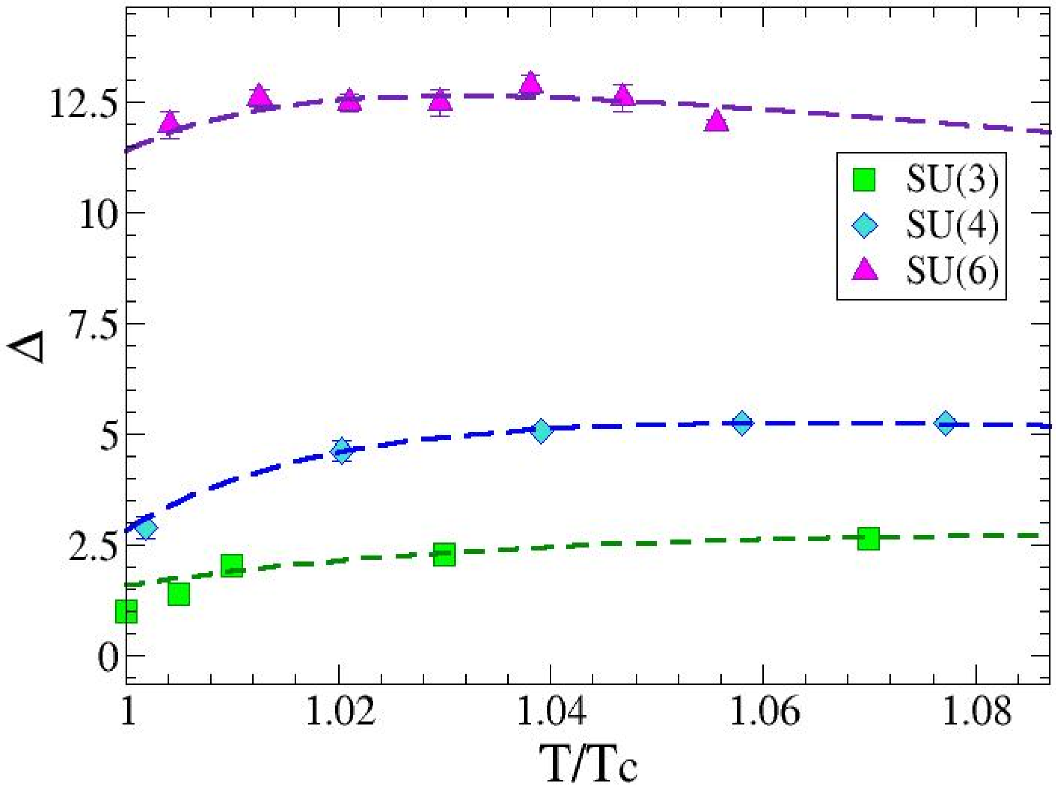
Nc	**3**	**4**	**6**
δ	0.94	0.94	0.89
c	-	-	-
χsq/dof	2.6	7.7	6.5

fit to a, b, c, δ

3	**4**	**6**
0.95	0.98	0.97
0.46	0.35	0.33
2.0	0.8	3.1

In all cases δ <1 and it shows small changes with Nc.

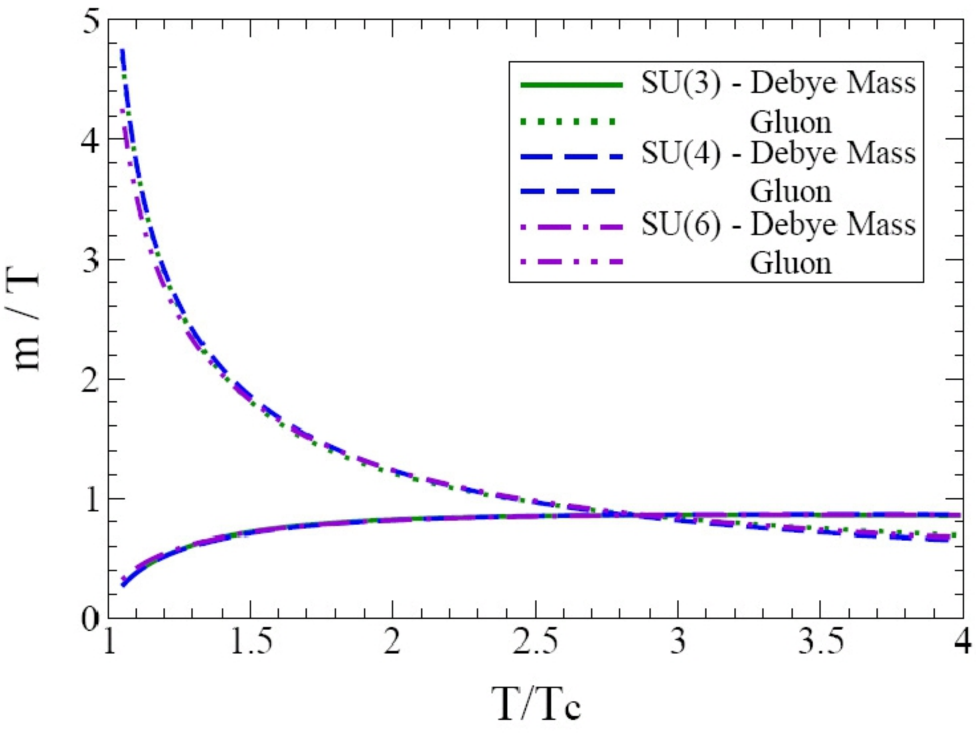
Close up around Tc



m(t) satisfactorily accounts for the correct T and Nc behavior

both above and below the peak.

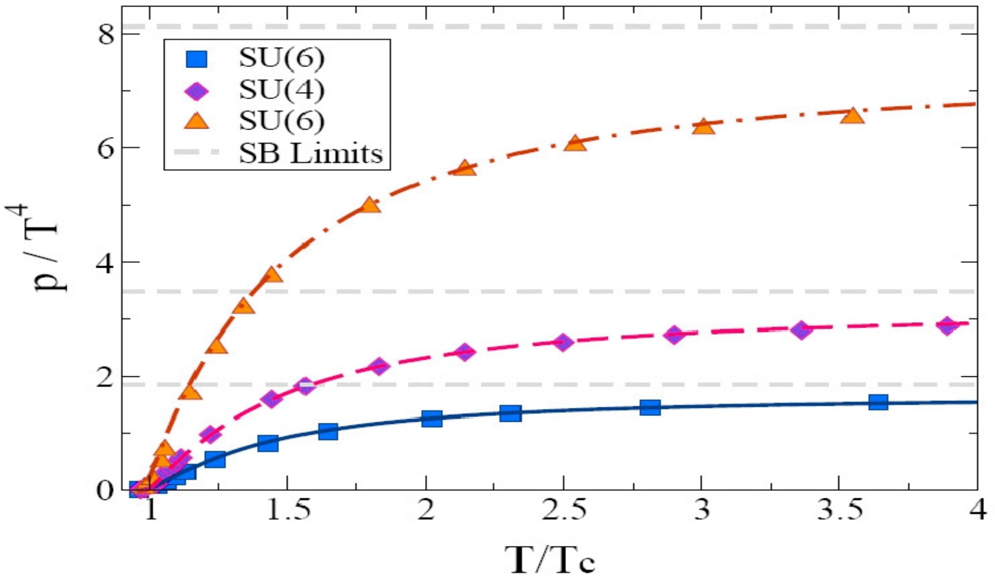
Quasiparticle mass m(t) and Debye mass m_D (t)



Due to **$\delta < 1$** , m(Tc) does not diverge and m_D (Tc) does not vanish.

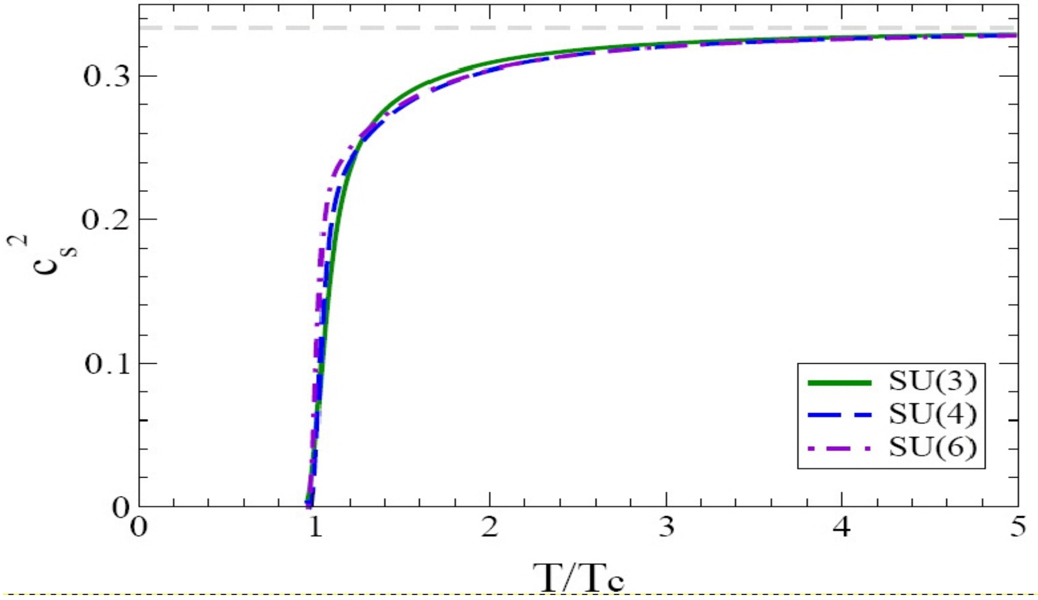
For Nc = 3, 4 ,6 → m(Tc) = 6.6 , 7.4 , 4.4

With Tc = 0.264 GeV the SU(3) mass is 1.7 GeV. Order of the glueball mass.



Check on other thermodynamical quantities.

← PRESSURE



← SPEED OF SOUND

$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_V$$

reaches 1/3 in the S-B limit.

Does the inclusion of the condensate modify this picture?

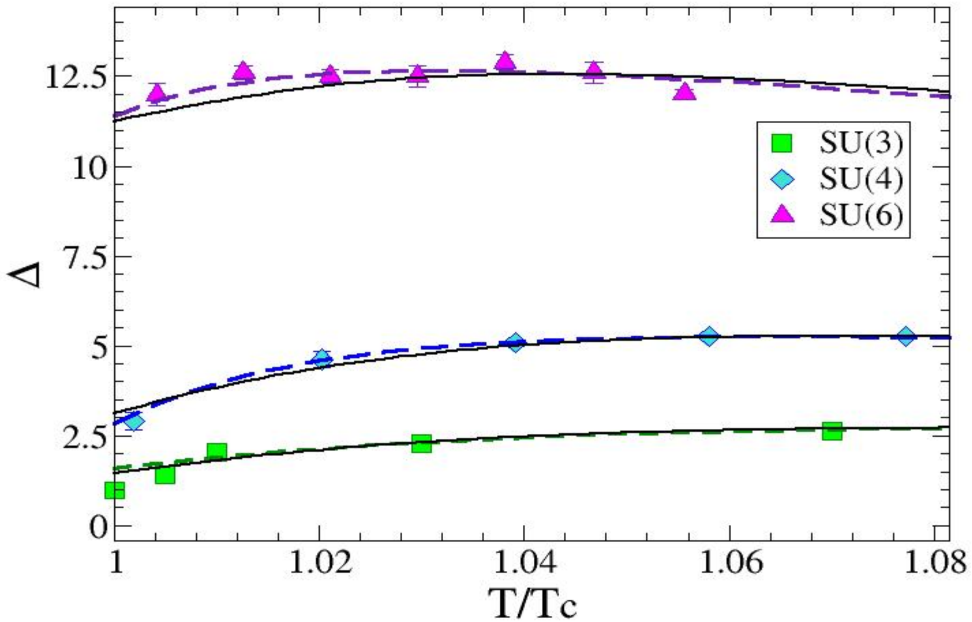
In the t' Hooft scaling regime, it scales as $N_c^2 - 1$

Its effect is suppressed at large T, but possibly relevant around Tc.

By including the gluon condensate, we observe only small additional effects.

fit to a, b, c, δ

Nc	**3**	**4**	**6**
δ	0.95	0.96	0.73
m (1)	7.7	7.5	2.6
χsq/dof	2.1	1.1	4.9



Conclusions

- Thermodynamics of the interaction measure Δ, above the transition temperature, shows a rapid appearance of new degrees of freedom in SU(3,4,6) gauge theories.

- They are very well described by quasi-particles with colors and polarizations of the gluons and with T dependent mass.

- Gluon condensate, numerically small, yields small additional changes below the peak of Δ.

- Mass rapidly increasing close to Tc, and of the order of the glueball mass at Tc.

- Decreasing trend of m(Tc) with Nc.

The suggestion of a critical behavior ($\delta = 1$) of m(Tc) needs to be confirmed by a direct analysis of the Nc=2 case.