Marco Ruggieri [魔留湖 (マルコ)　ルジエーーリ])
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QUARK MATTER WITH CHIRAL CHEMICAL POTENTIAL

Paris, *2011年 6月 6日*

Outline

- Chiral Chemical Potential: motivations
- The Model
- Phase Diagram with a Chiral Chemical Potential
  .) Chiral Symmetry Restoration
  .) Deconfinement
  .) Critical Endpoint
- Conclusions and Outlook

Motivations

Why am I interested to QCD with a chiral chemical potential?

Chirality in QGP

Chiral density (*chirality*):

$$N_5 = N_R - N_L$$

Imbalance between left- and right-handed quarks

Chirality in QGP

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Integrated Ward Identity: connecting chirality to winding number

$$\Delta N_5 = 2Q_W$$

$$Q_W = \frac{g^2}{32\pi^2} \int d^4x \; F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a = \Delta N_{CS}$$

Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number

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Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number

Moore and Tassler, **JHEP 1102 (2011) 105**

At high temperature, we expect copious production of *gluon* configurations with nonvanishing winding number (*strong –i.e. QCD- sphaleron*)

Chirality can be produced in the high temperature phase of QCD

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Chirality can be produced in the high temperature phase of QCD

Simplest way to treat quark matter with chirality in effective models:

$$\mu_5 \Leftrightarrow N_5$$

Chiral chemical potential, conjugated to chiral density

$$+\ \mu_5\bar{q}\gamma^0\gamma^5 q$$

Chiral density operator added to the Lagrangian density

Chirality in QGP

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Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number

Chirality can be produced in the high temperature phase of QCD

$$\mu_5 \Leftrightarrow N_5$$

$$+ \mu_5 \bar{q}\gamma^0\gamma^5 q$$

$$\mu \Leftrightarrow N$$

$$+ \mu \bar{q}\gamma^0 q$$

Chiral chemical potential ⟷ *Baryon chemical potential*

Chirality in QGP

Chiral density (*chirality*):

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Imbalance between left- and right-handed quarks

Integrated Ward Identity: connecting chirality to winding number

$$\Delta N_5 = 2Q_W$$

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Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number

Chirality can be produced in the high temperature phase of QCD

$$\mu_5 \Leftrightarrow N_5$$

$$+\, \mu_5 \bar{q}\gamma^0\gamma^5 q$$

$$n_5 = \frac{\mu_5^3}{3\pi^2} + \frac{\mu_5 T^2}{3}$$

Chiral chemical potential

K. Fukushima *et al*, **Phys.Rev. D78 (2008) 074033**

Remarks

Remark 1

We are aware that $\mu5$ is not a true chemical potential: chiral condensate mixes L and R components, thus making N5 a non-conserved quantity.

Treat $\mu5$ as a mere mathematical artifact

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Remark 2

Not affected by the sign problem, see for a nice explanation: K. Fukushima *et al*, **Phys.Rev. D78 (2008) 074033**

Grand Canonical Ensembles with a chiral chemical potential can be simulated on the Lattice with Nc=3, see for example: A. Yamamoto, **arXiv:1105.0385 [hep-lat]**

Another Motivation

Remarks

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Grand Canonical Ensembles with a chiral chemical potential can be simulated on the Lattice with Nc=3, see for example:
A. Yamamoto, **arXiv:1105.0385 [hep-lat]**

Remark 3

Relevant for the HICs phenomenology (Chiral Magnetic Effect):
D. Kharzeev et al., **Nucl.Phys. A803 (2008) 227**
K. Fukushima *et al*, **Phys.Rev. D78 (2008) 074033**
M. R. et al., **Phys.Rev. D81 (2010) 114031**

Some References

References

In **QGP context**, to mimic *chirality change* induced by *instantons* and *strong sphalerons* in Quark-Gluon-Plasma:

K. Fukushima *et al*, **Phys.Rev. D78 (2008) 074033**

K. Fukushima, R. Gatto and M. R., **Phys.Rev. D81 (2010) 114031**

M. Chernodub and A. Nedelin, **Phys.Rev. D83 (2011) 105008**

M. R., **arXiv:1103.6186 [hep-ph]**

A. Yamamoto, **arXiv:1105.0385 [hep-lat]**

C. A. Ballon Bayona et al., **arXiv:1104.2291[hep-th**

References

In **several non-QGP contexts**:

L. D. McLerran *et al*, **Phys.Rev. D43 (1991) 2027**

Nielsen and Ninomiya, **Phys.Lett. B130 (1983) 389**

A. N. Sisakian *et al*, **hep-th/9806047**

M. Joyce *et al.*, **Phys.Rev. D53 (1996) 2958**

The Microscopic Model

Description of the model I use in my concrete calculations.

M.R., **arXiv:1103.6186**

The Model

Nambu and Jona-Lasinio, **Phys. Rev. 122 (1961)**
M. Frasca, **arXiv:1105.5274 [hep-ph]**

NJL Model with the Polyakov Loop

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right) q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5 \boldsymbol{\tau} q\right)^2\right]$$

$$G = g\left[1 - \alpha_1 L L^\dagger - \alpha_2 (L^3 + (L^\dagger)^3)\right]$$

Polyakov Loop:
sensitive to confinement – deconfinement transition

In the model:
A_4 is background field

$$L = \frac{1}{3}\mathrm{Tr}_c \exp\left(i\beta\lambda_a A_4^a\right)$$

K. Fukushima, **Phys.Lett. B591 (2004) 277-284**
W. Weise *et al.*, **Phys.Rev. D73 (2006) 014019**
M. Yahiro *et al.*, **Phys.Rev. D82 (2010) 076003**

M.R., **arXiv:1103.6186**

The Model

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$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5\boldsymbol{\tau}q\right)^2\right]$$

$$G = g\left[1 - \alpha_1 L L^\dagger - \alpha_2(L^3 + (L^\dagger)^3)\right]$$

Coupling to quarks via:
.) Coupling constant
.) Covariant derivative

Polyakov Loop:
sensitive to confinement – deconfinement transition

$$L = \frac{1}{3}\mathrm{Tr}_c \exp\left(i\beta\lambda_a A_4^a\right)$$

K. Fukushima, **Phys.Lett. B591 (2004) 277-284**
W. Weise *et al.*, **Phys.Rev. D73 (2006) 014019**
M. Yahiro *et al.*, **Phys.Rev. D82 (2010) 076003**

Coupling dependent on L inspired by:
K. Kondo, **Phys.Rev. D82 (2010) 065024**

M.R., **arXiv:1103.6186**

The 1-loop TP

$$\mathcal{L} = \bar{q}\left(i\gamma^{\mu}D_{\mu} - m\right)q + G\left[\left(\bar{q}q\right)^{2} + \left(i\bar{q}\gamma_{5}\boldsymbol{\tau}q\right)^{2}\right]$$

M.R., **arXiv:1103.6186**

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Add a chiral chemical potential

M.R., **arXiv:1103.6186**

The 1-loop TP

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5\boldsymbol{\tau}q\right)^2\right] + \mu_5\bar{q}\gamma^0\gamma^5 q$$

One-loop Thermodynamic Potential

$$V = \mathcal{U}(L, L^\dagger, T) + \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm1}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\omega_s - \frac{N_c N_f}{\beta}\sum_{s=\pm1}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\log\left(F_+ F_-\right)$$

Minimization of V leads to physical values of
.) σ (chiral condensate)
.) L

$$F_- = 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$

$$F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s+\mu)} + 3Le^{-2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)}$$

1- and 2-quark states suppression in the confinement phase

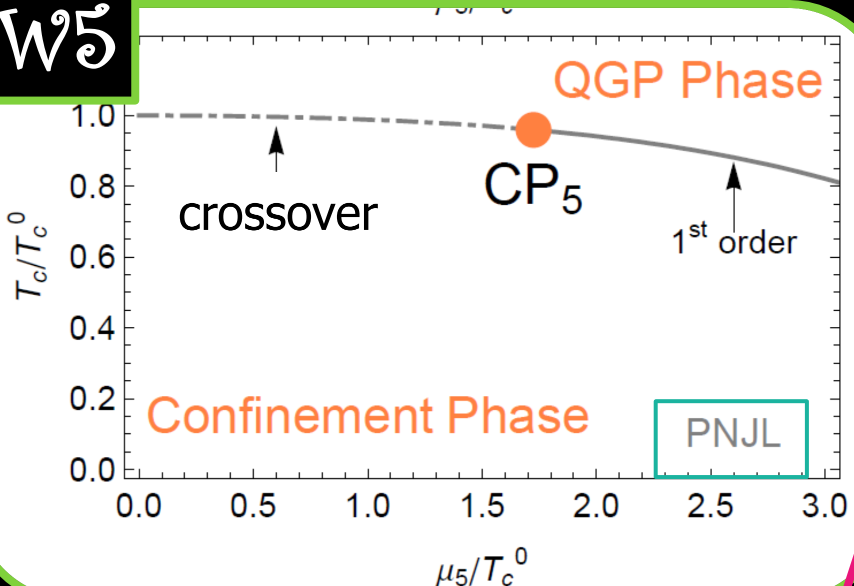
Statistically confining distribution functions

Phase Diagram of the Model

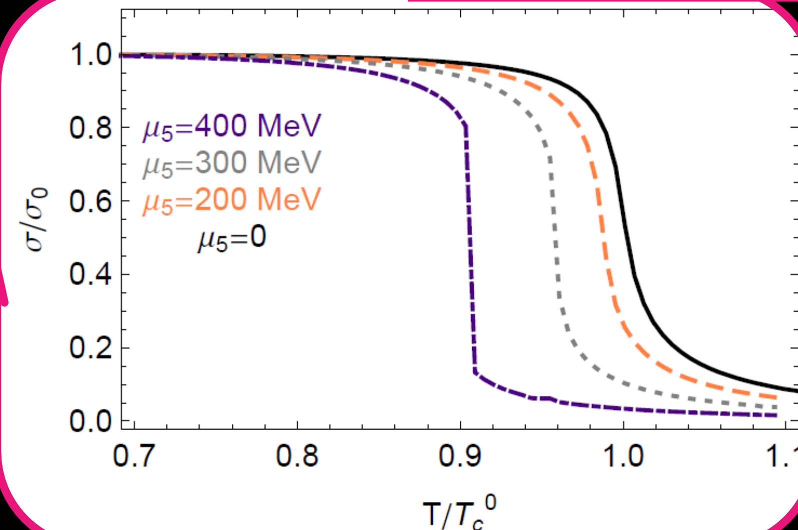
M.R., **arXiv:1103.6186**

Phase Diagram: Results

W5

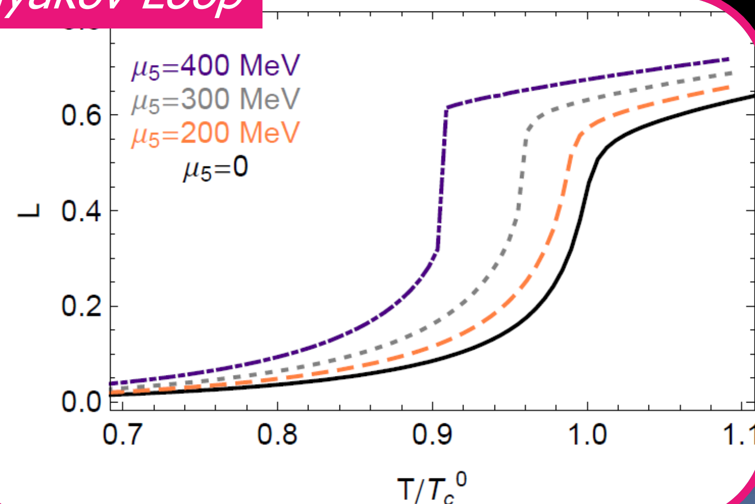


Chiral Condensate



Deconfinement and χS Restoration are entangled for any value of $\mu 5$

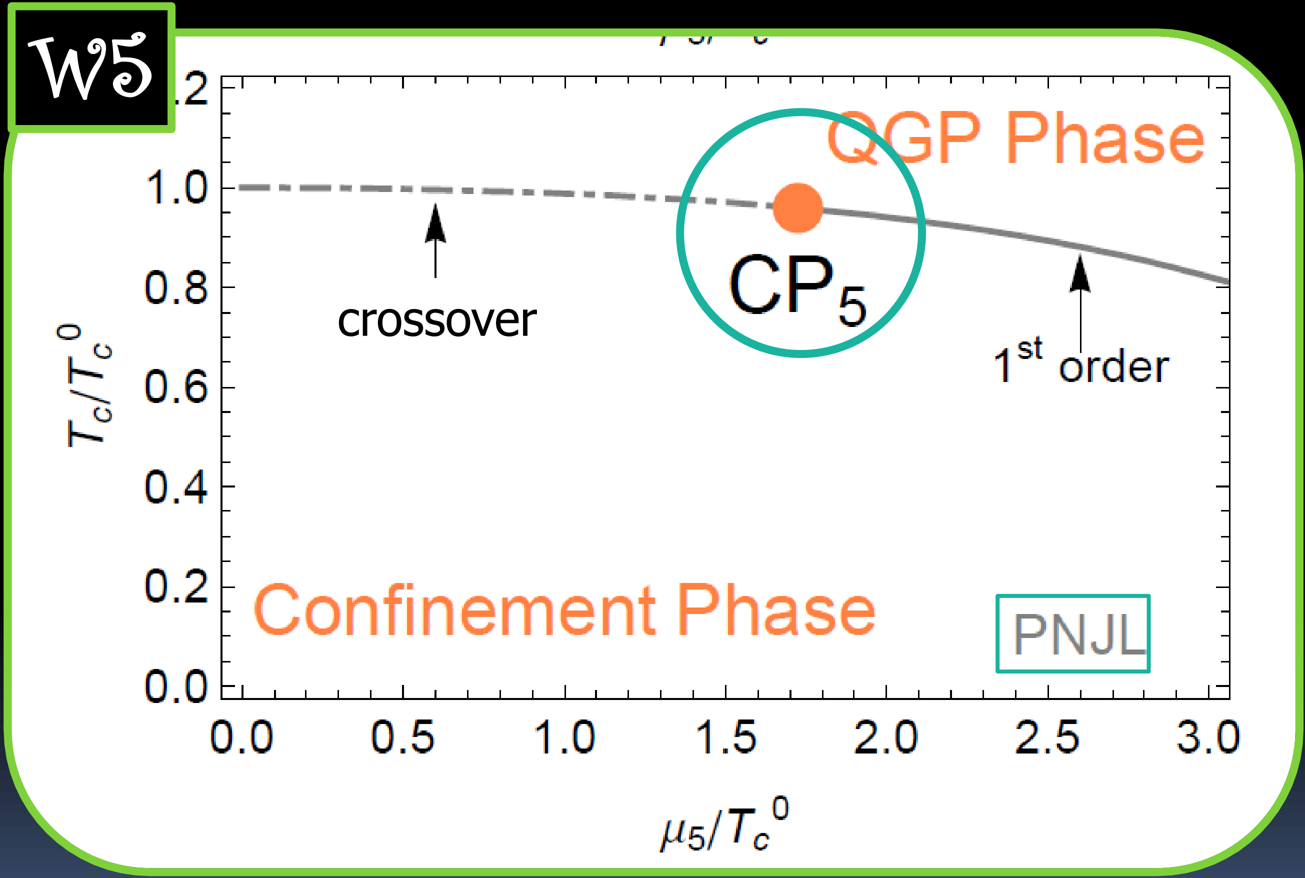
Polyakov Loop



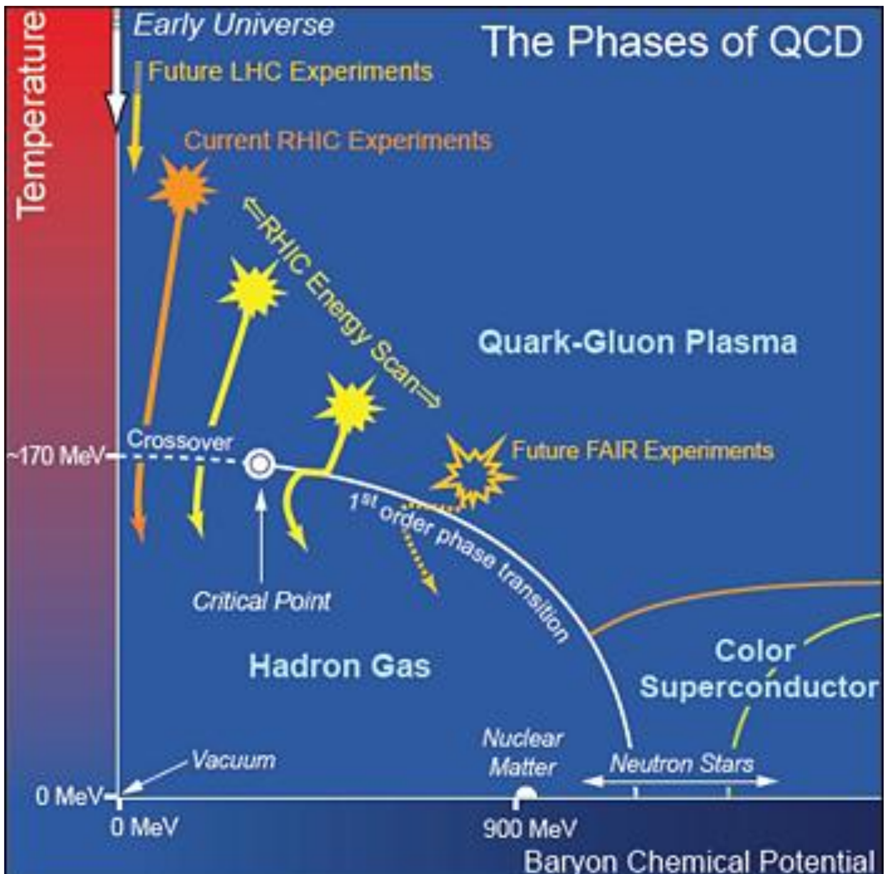
This is different from what is found at real chemical potential, see:
M. Yahiro *et al.*, **arXiv:1104.2394 [hep-ph]**

M.R., **arXiv:1103.6186**

Phase Diagram: Results

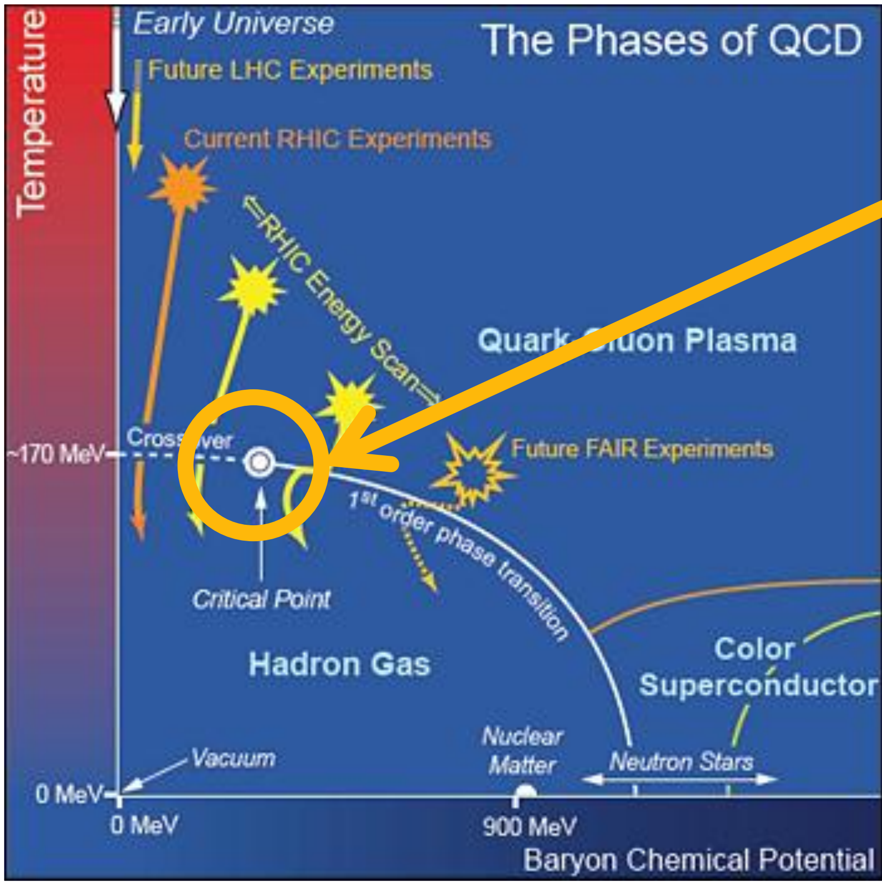


Critical Endpoint of QCD



Sophie Bushwick,
http://www.bnl.gov/today/story.asp?ITEM_NO=1870

Critical Endpoint of QCD



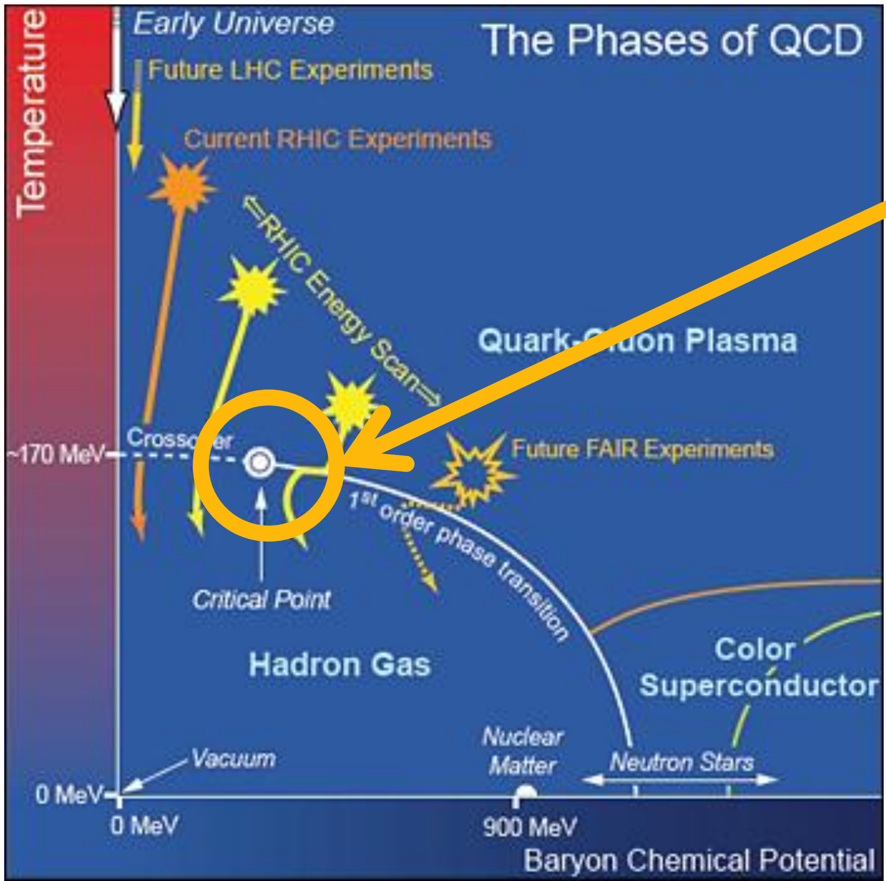
Sophie Bushwick,
http://www.bnl.gov/today/story.asp?ITEM_NO=1870

Critical Endpoint (CP)
First order and crossover lines intersect at CP

Asakawa and Yazaki,
Nucl.Phys. A504 (1989) 668-684

Based on NJL model

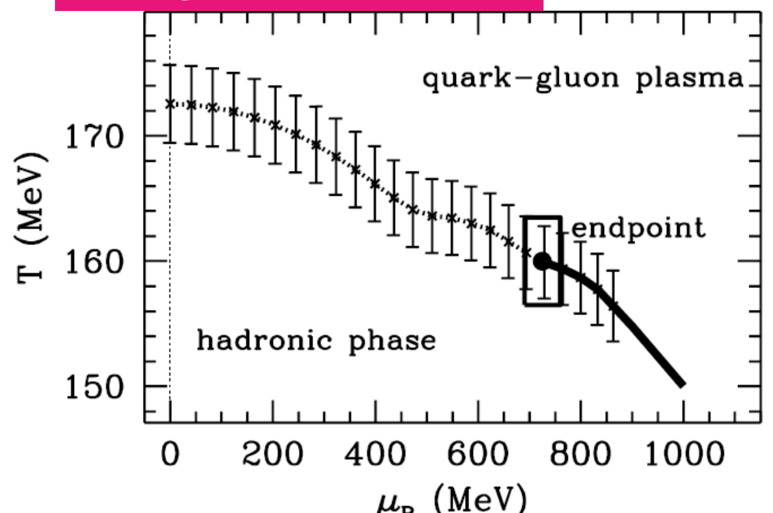
Critical Endpoint of QCD



Sophie Bushwick, http://www.bnl.gov/today/story.asp?ITEM_NO=1870

Critical Endpoint (CP)
First order and crossover lines intersect at CP

It might exist in QCD



Fodor and Katz, JHEP03(2002)014

For a (critical) review see De Forcrand in **PoS LAT2009 (2009) 010**

Critical Endpoint of QCD

Lattice

Fodor and Katz, **JHEP03(2002)014**
C. R. Allton *et al*., **Phys. Rev. D71 (2005) 054508**
Gavai and Gupta, **Phys. Rev. D78 (2008) 114503**
De Forcrand and Philipsen, **Nucl. Phys. B642 (2002) 290**
P. De Forcrand *et al*., **arXiv:0911.5682**
S. Eijiri, **Phys. Rev. D78 (2008) 074507**
A. Ohnishi *et al*., **Pos LAT2010 (2010) 202**

Models

K. Fukushima et al., **Phys. Rev. D80 (2009) 054012**
Bowman and Kapusta, **Phys. Rev. C79 (2009) 015202**
Zhang and Kunihiro, **Phys. Rev. D80 (2009) 290**
A. Ohnishi *et al*., **arXiv:1102.3753**
M. A. Stephanov, **PoS LAT2006 (2006) 024**
Abuki *et al*., **Phys. Rev. D81 (2010) 125010**
Basler and Buballa, **Phys. Rev. D82 (2010) 094004**
Hanada and Yamamoto, **arXiv:1103.5480 [hep-ph]**

Critical Endpoint of QCD

Nowadays, it has been hard to detect CP by means of Lattice simulations with ***Nc=3****, because of the sign problem.*

Critical Endpoint of QCD

Nowadays, it has been hard to detect CP by means of Lattice simulations with ***Nc=3****, because of the sign problem.*

Idea for (theoretical) detection:
Continue CP to another critical point, which can be detected on the Lattice

Besides progresses achieved with simulations at *finite isospin* or *imaginary chemical potential*:

De Forcrand and Philipsen, **JHEP0811 (2008) 012**
De Forcrand and Philipsen, **Phys. Rev. Lett. 105 (2010) 152001**
Kogut and Sinclair, **Phys. Rev. D66 (2002) 034505**
P. De Forcrand et al., **PoS LAT2007 (2007) 237**
P. Cea et al., **Phys. Rev. D80 (2009) 034501**

chiral chemical potential offers an interesting alternative of *continuation* of the critical point.

I implement the continuation idea within model (M.R., **arXiv:1103.6186**)

M.R., **arXiv:1103.6186**

Continuation of CP

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5\boldsymbol{\tau}q\right)^2\right]$$

$$+\ \mu\bar{q}\gamma^0 q$$

Baryon Chemical Potential
conjugated to baryon density

$$+\ \mu_5\bar{q}\gamma^0\gamma^5 q$$

Chiral Chemical Potential
conjugated to chiral density:
N5=nR - nL

M.R., **arXiv:1103.6186**

Continuation of CP

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5\boldsymbol{\tau}q\right)^2\right]$$

$$+ \mu_5 \bar{q}\gamma^0\gamma^5 q$$

Chiral Chemical Potential
conjugated to chiral density:
N5=nR - nL

$$+ \mu\bar{q}\gamma^0 q$$

Baryon Chemical Potential
conjugated to baryon density

I introduce two worlds:

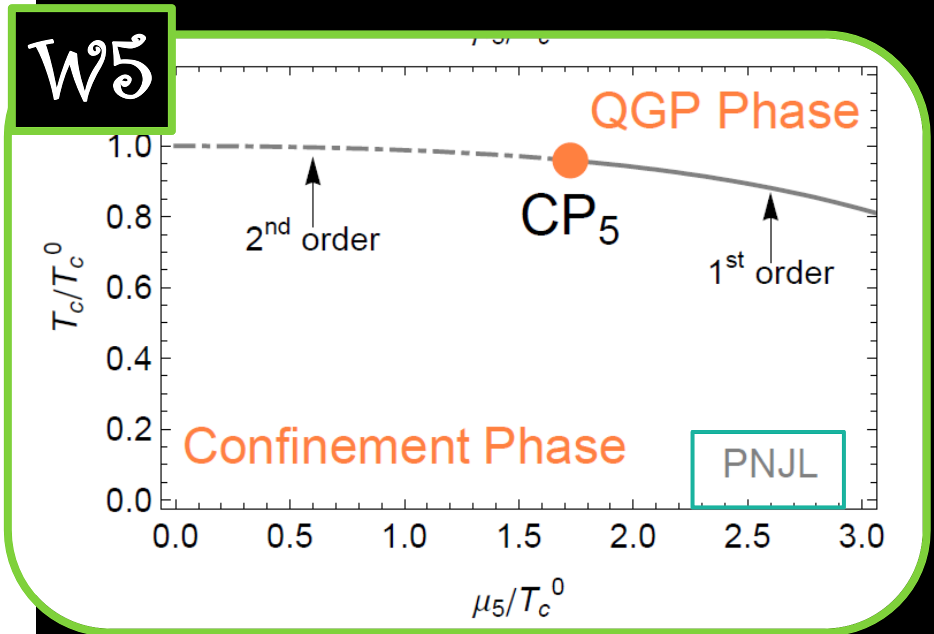
W5:
World with μ=0 and finite μ5

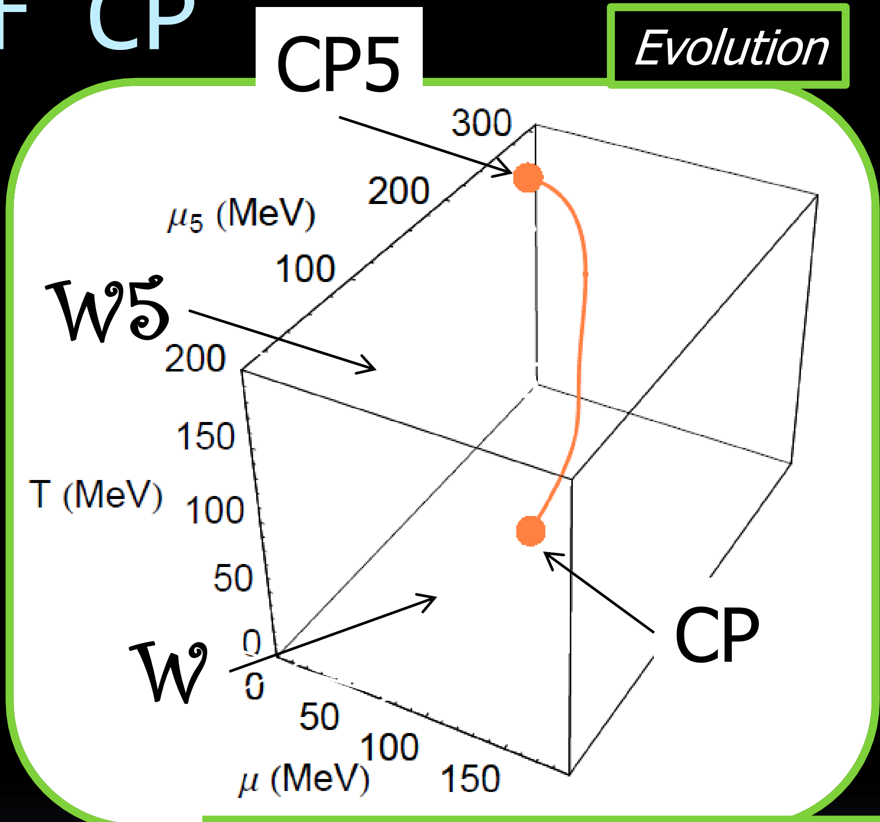
W:
World with μ5=0 and finite μ

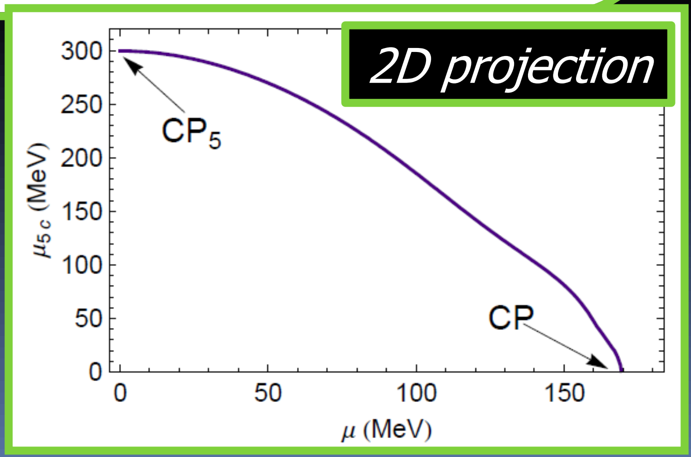
CP5 ⟷ *evolution* ⟷ **CP**

M.R., **arXiv:1103.6186**

Continuation of CP



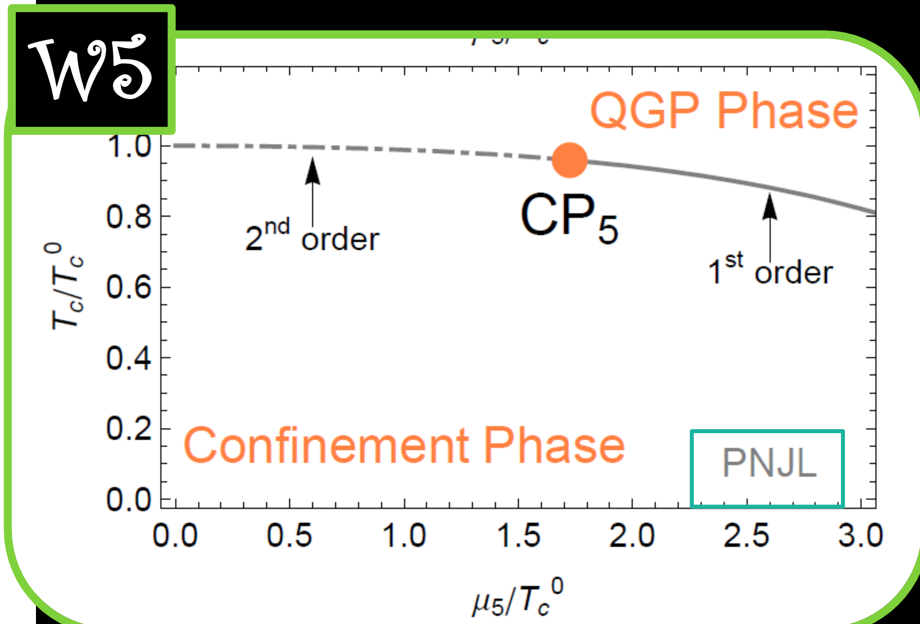


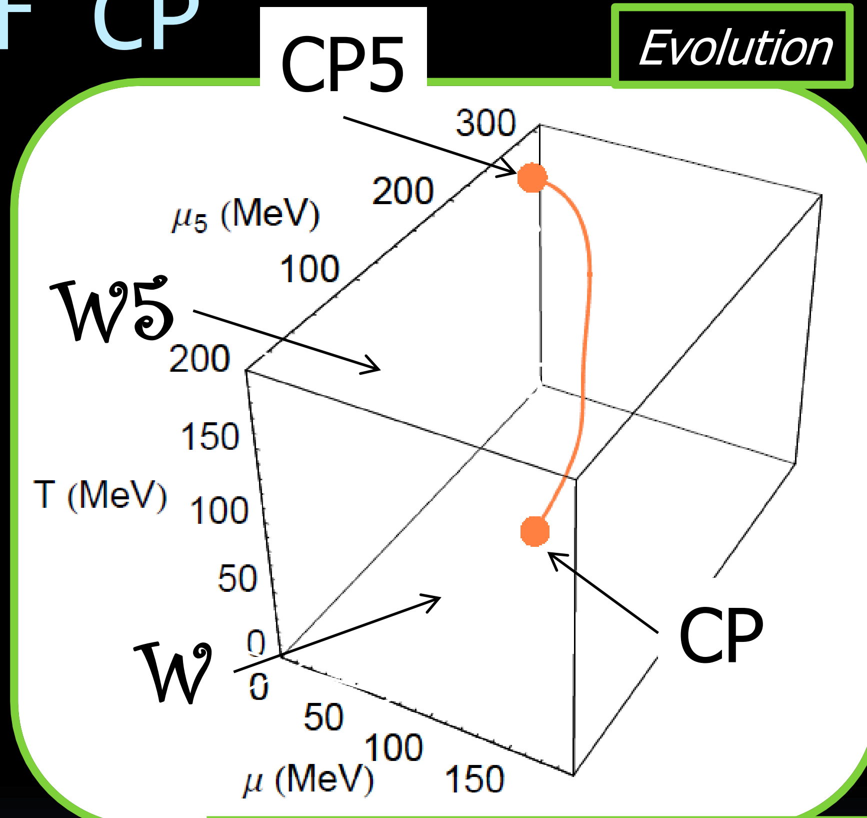


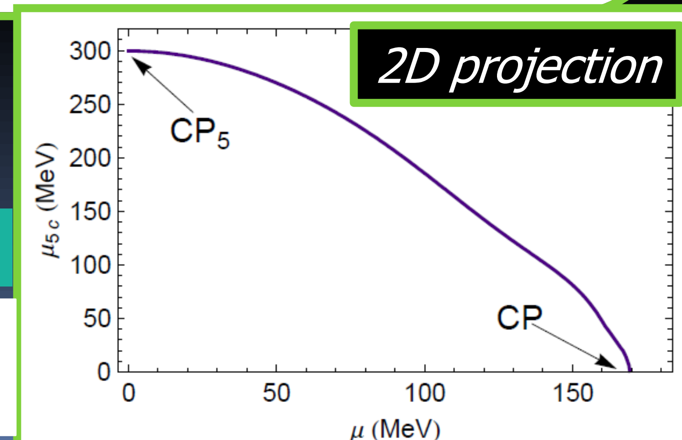
CP5 is not an accident, but
CP viewed by hot quark matter in W5.
Its detection (?) can be interpreted as a
theoretical signatureof the real world **CP**.

M.R., **arXiv:1103.6186**

Continuation of CP







Use the model to estimate:

$$\frac{\mu_c}{\mu_{5c}} \approx 0.53\ , \qquad \frac{T_c}{T_{5c}} \approx 0.97$$

How do the above ratios change when Nc is larger?

For a comparision with the framework of:
Hanada and Yamamoto, **arXiv:1103.5480 [hep-ph]**

Conclusions and Outlook

Conclusions

- Chiral chemical potential is introduced to mimic chirality-changing processes in hot QCD medium
- Phase Structure of Quark Matter (QM) with $\mu 5$ similar to that of QM of our Universe
- QM with $\mu 5$ can be simulated on Lattice (no sign problem)
- Critical Endpoint (CP) of QCD is continued to a new Critical Endpoint, CP5
- Detection of CP5, if found on the Lattice, can be interpreted as a signature of CP

Outlook

Interesting comparison with results from SS model,
C. A. Ballon Bayona et al., **arXiv:1104.2291[hep-th]**

- Study of inhomogeneous phases around the critical endpoint at finite $\mu 5$
- Compute quantitative dependence of the critical endpoint on the quark masses
- Study the Nc dependence of the mapping coordinates
- Mapping the critical endpoint within the Ginzburg-Landau effective potential approach
- From 2 to 2+1 flavors

I acknowledge:
K. Fukushima and **R. Gatto** for collaboration on some of the topics discussed here.
Moreover, I acknowledge:
H. Abuki, **P. De Forcrand**, **M. D'Elia**, **M. Frasca**, **T. Hatsuda**, **A. Ohnishi**, **M. Tachibana**, **A. Yamamoto** and **N. Yamamoto** for interesting discussions about the topics discussed in this talk.

Thanks for your attention.

Non pentirti di ciò che hai fatto, se quando l'hai fatto eri felice
(Do not regret the things you did, if when you did them you were happy)

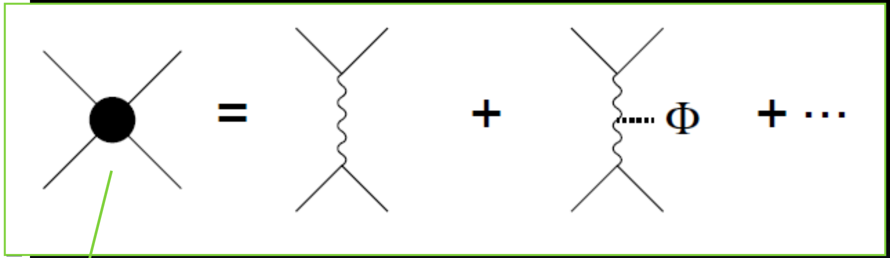
Appendix

M.R., **arXiv:1103.6186**

The L-dependent coupling

NJL Model with the Polyakov Loop

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5\boldsymbol{\tau}q\right)^2\right]$$



Interaction among background field and gluons leads to a tree-level coupling among G and L

$$G = g\left[1 - \alpha_1 L L^\dagger - \alpha_2 (L^3 + (L^\dagger)^3)\right]$$

M. Yahiro *et al.*, **Phys.Rev. D82 (2010) 076003**
K. Kondo, **Phys.Rev. D82 (2010) 065024**

M.R., **arXiv:1103.6186**

The 1-loop TP

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[(\bar{q}q)^2 + (i\bar{q}\gamma_5\boldsymbol{\tau}q)^2\right] + \mu_5\bar{q}\gamma^0\gamma^5 q$$

One-loop Thermodynamic Potential

$$V = \mathcal{U}(L, L^\dagger, T) + \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm1}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\omega_s - \frac{N_c N_f}{\beta}\sum_{s=\pm1}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\log\left(F_+F_-\right)$$

Minimization of V leads to physical values of
.) σ (chiral condensate)
.) L

$$F_- = 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$

$$F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s+\mu)} + 3Le^{-2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)}$$

Statistically confining distribution functions

M.R., **arXiv:1103.6186**

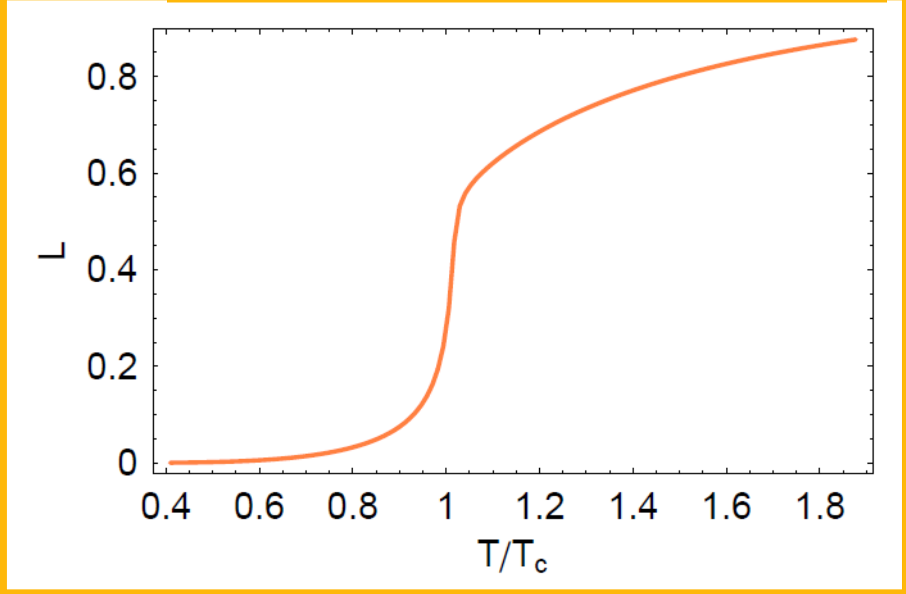
Statistical Confinement

$$F_- = 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$

$$F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s+\mu)} + 3Le^{-2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)}$$

Statistically confining distribution functions

Polyakov loop expectation value



M.R., **arXiv:1103.6186**

Statistical Confinement

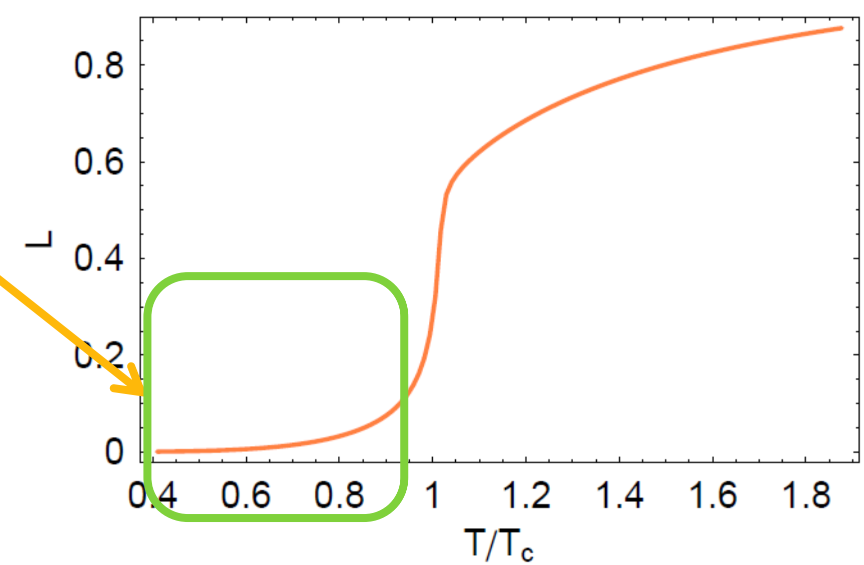
$$F_- = 1 + 3 \,[\text{covered}]\, {}^{\beta(\omega_s-\mu)} + 3\,[\text{covered}]\, {}^{2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$

$$F_+ = 1 + [\text{covered}]\, {}^{-\beta(\omega_s+\mu)} + 3\,[\text{covered}]\, {}^{2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)}$$

Statistically confining distribution functions

Confinement phase: L=0 (approximately)

Polyakov loop expectation value

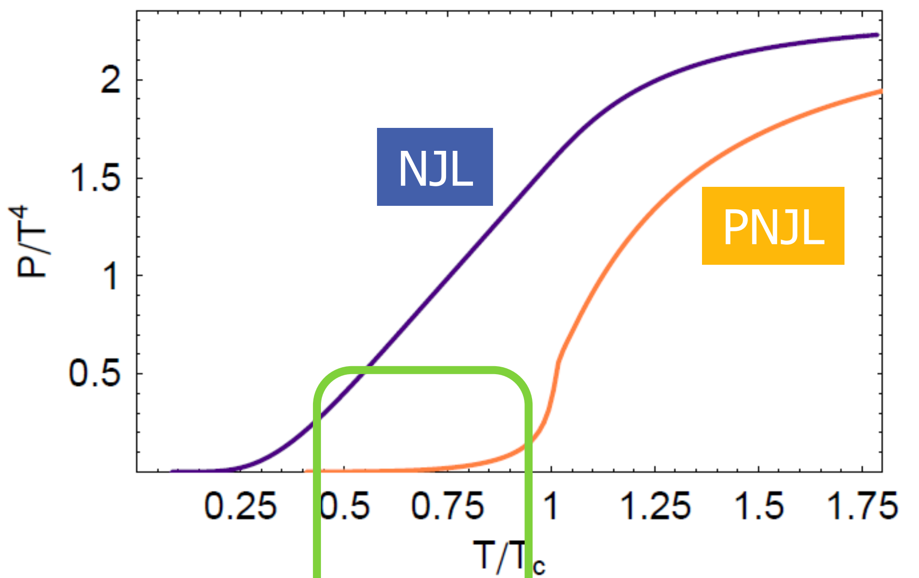


The colorless 3-quark states give the main contribution to the thermodynamic potential in the confinement phase.

M.R., **arXiv:1103.6186**

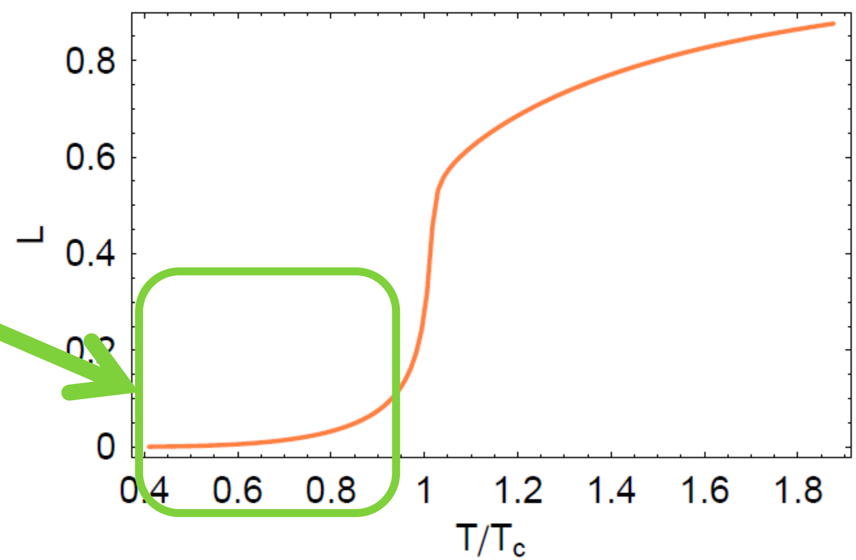
Statistical Confinement

Pressure



Thermal suppression of pressure

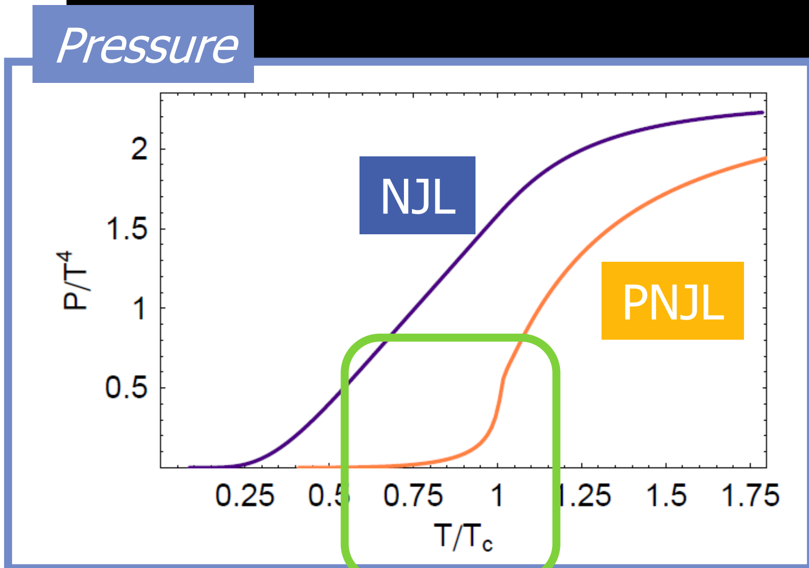
Polyakov loop expectation value

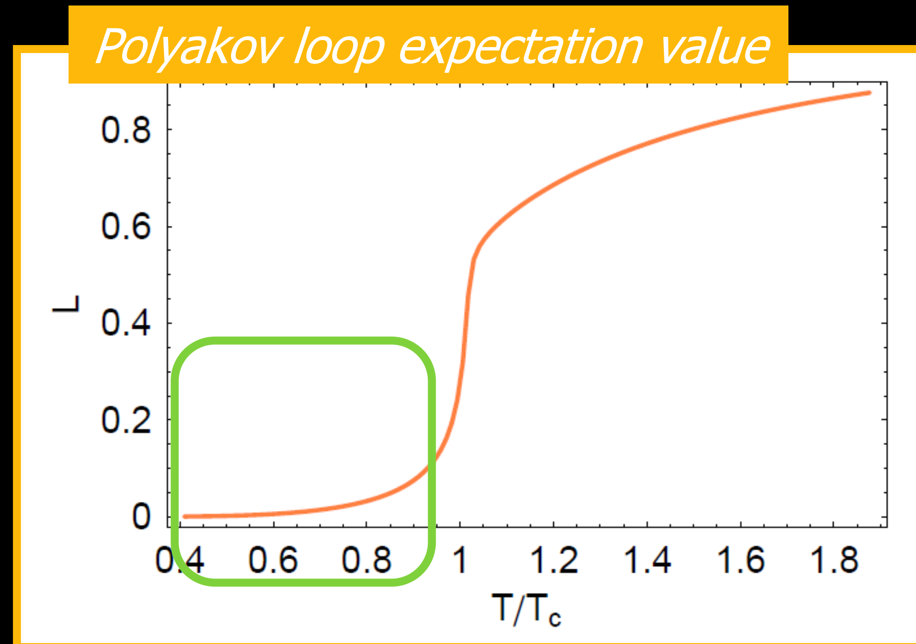


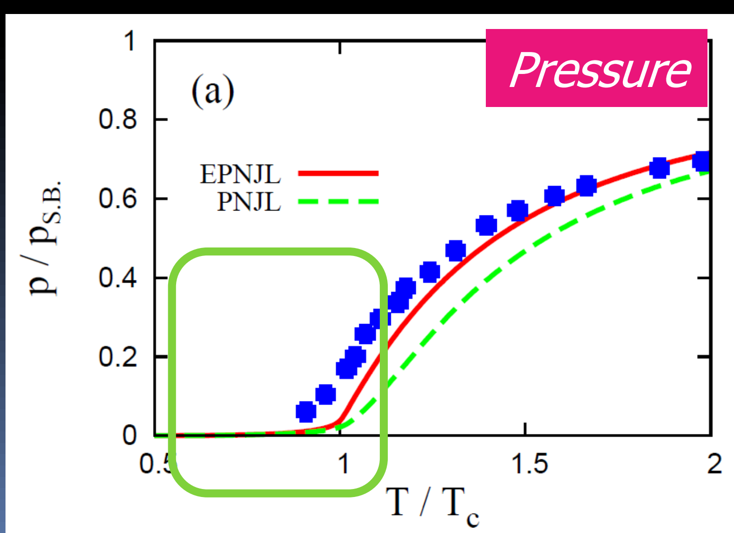
The colorless 3-quark states give the main contribution to the thermodynamic potential in the confinement phase.

M.R., **arXiv:1103.6186**

Statistical Confinement







Qualitative agreement with Lattice data

Picture from:
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Lattice data from:
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PNJL offers a better description of finite temperature QCD than NJL

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Statistical *De*-confinement

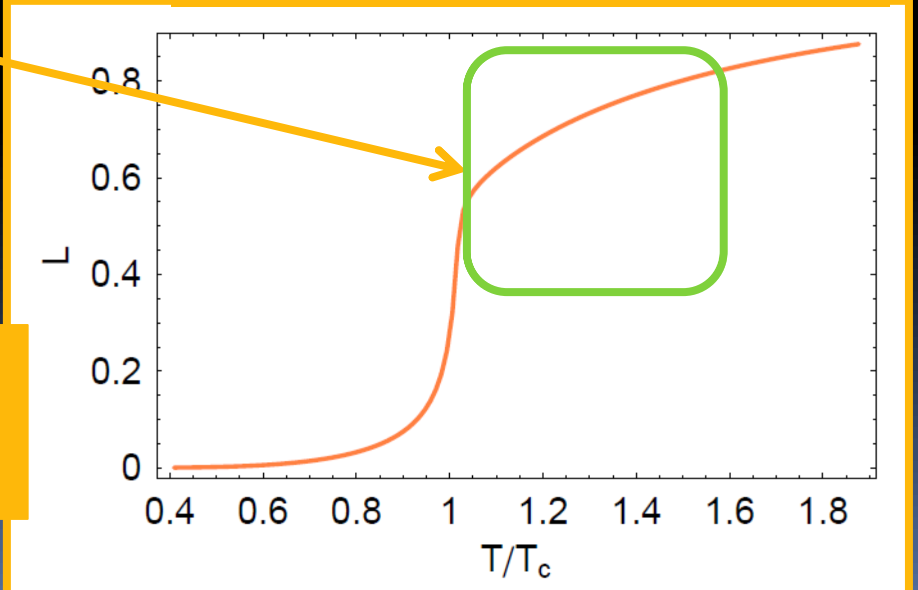
$$F_- = 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$

$$F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s+\mu)} + 3Le^{-2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)}$$

Statistically confining distribution functions

Deconfinement phase: L>0

Polyakov loop expectation value



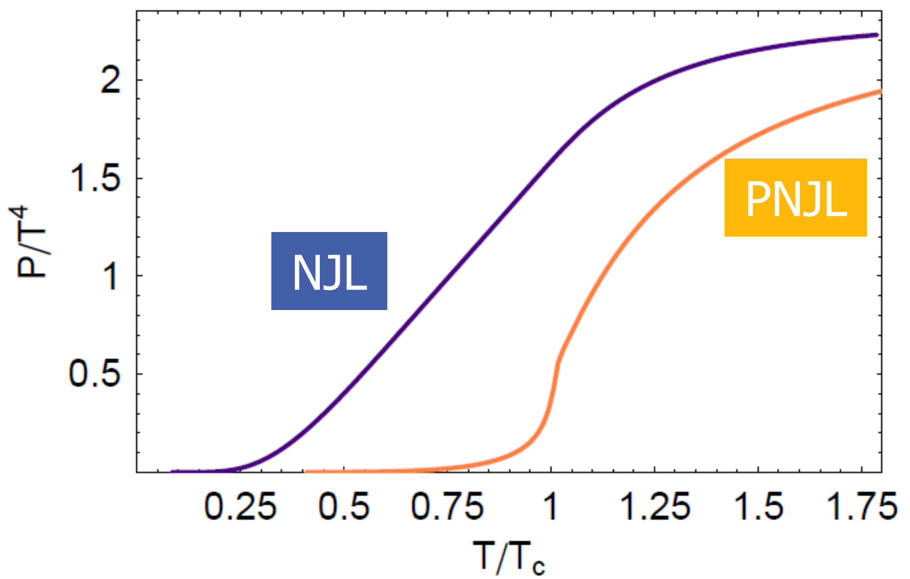
1- and 2-quark states are liberated

The colored 1-quark and 2-quark states give a finite contribution to the thermodynamic potential in the deconfinement phase.

M.R., **arXiv:1103.6186**

Statistical *De*-confinement

Pressure



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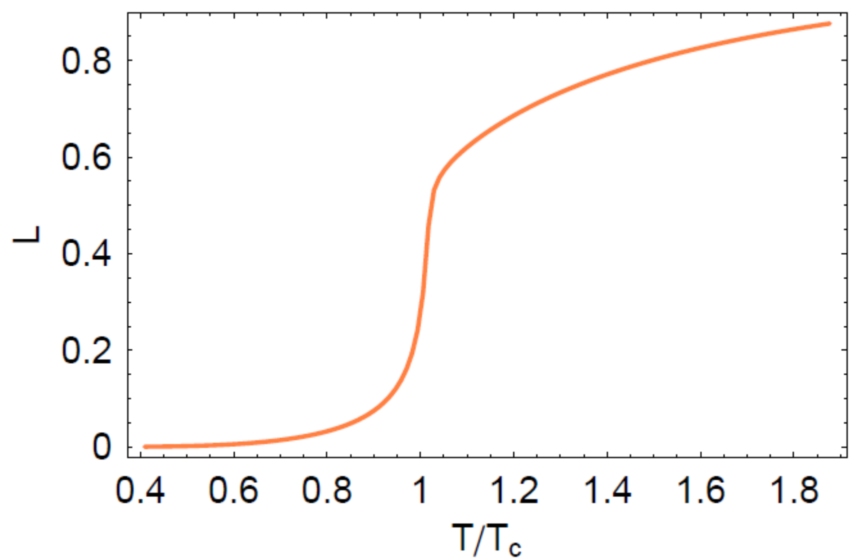

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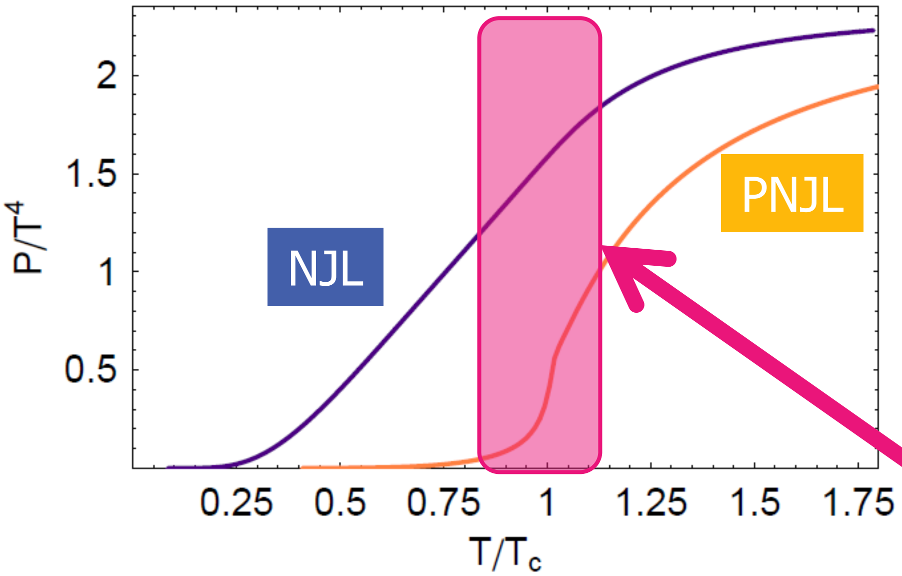

Thermal growht of pressure in correspondence of the crossover

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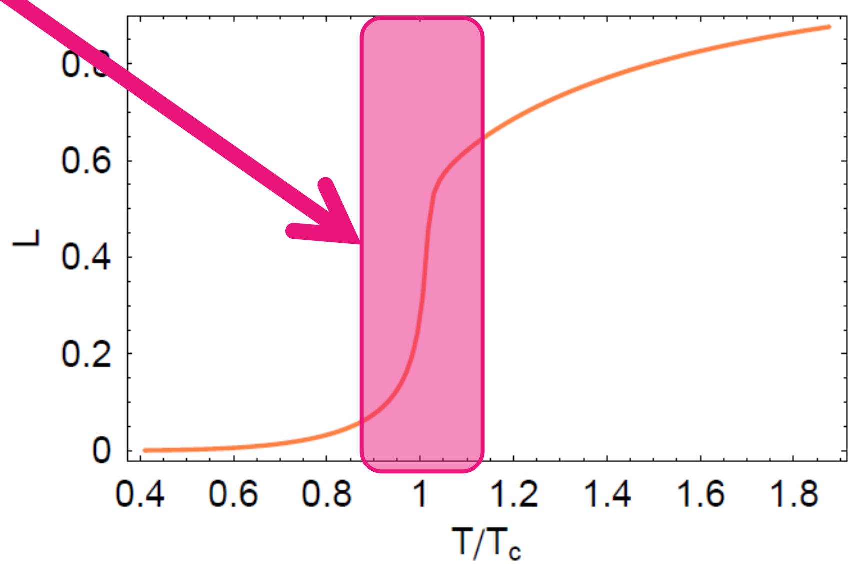

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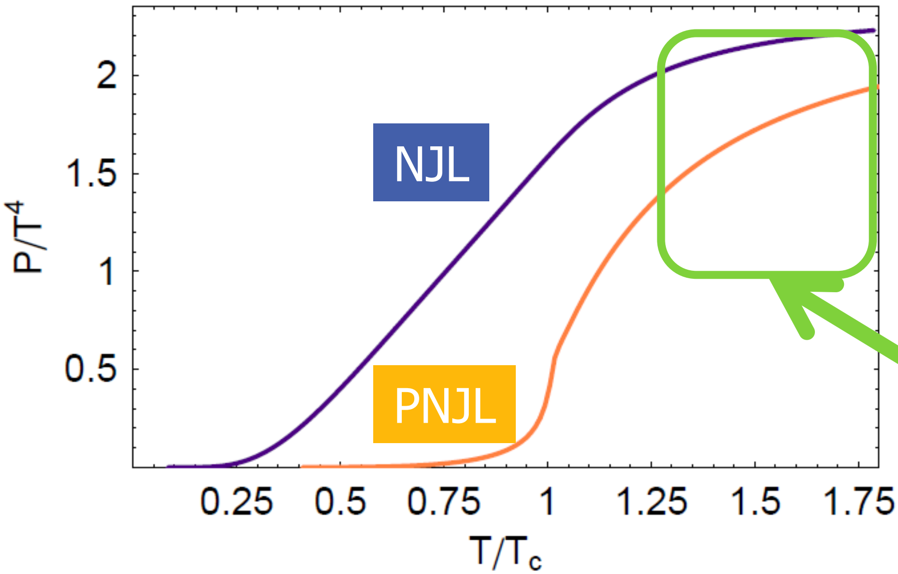

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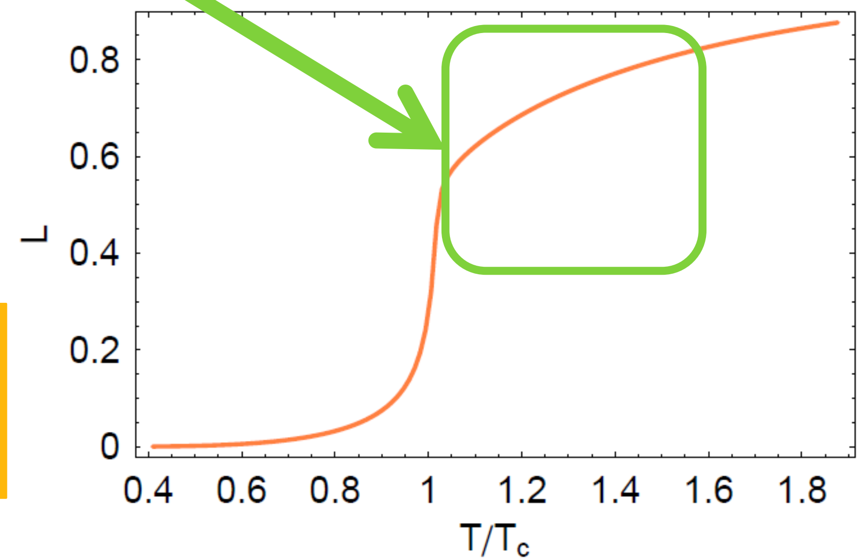

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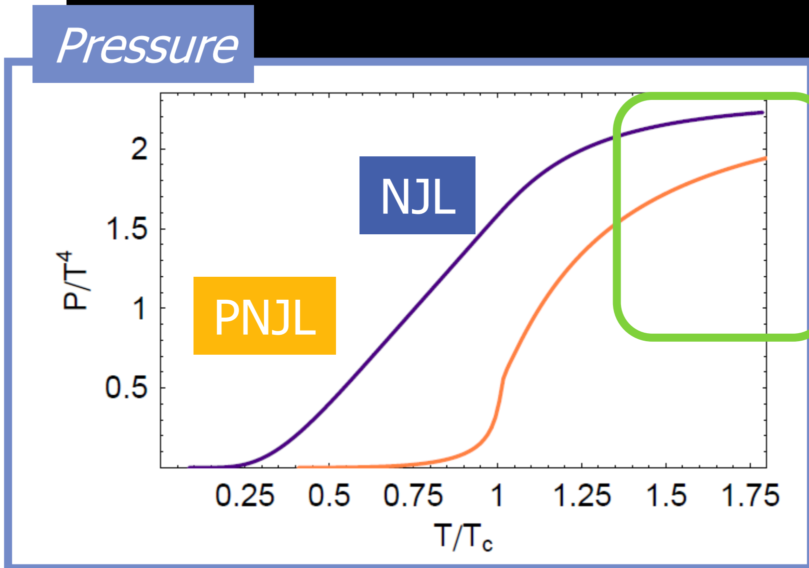

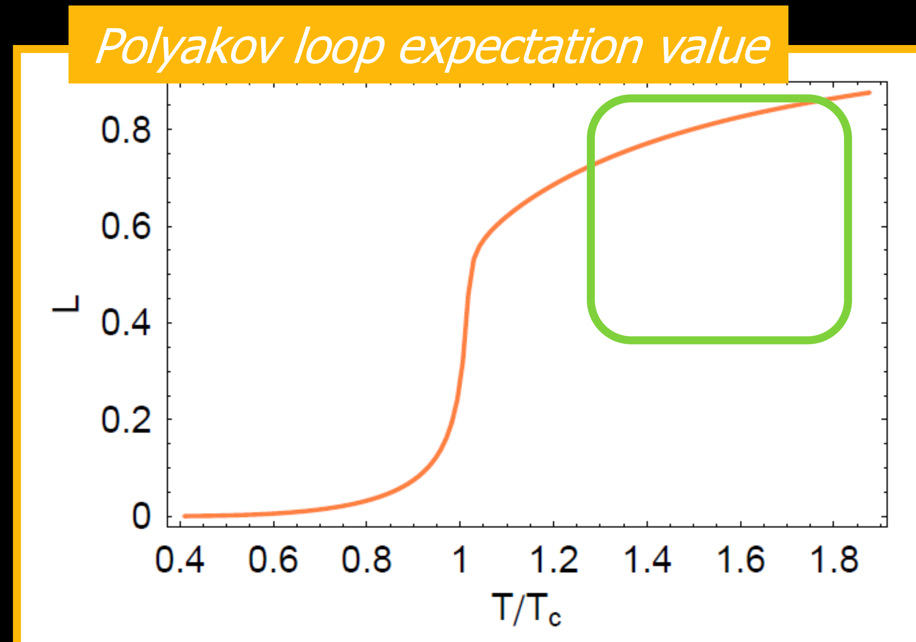

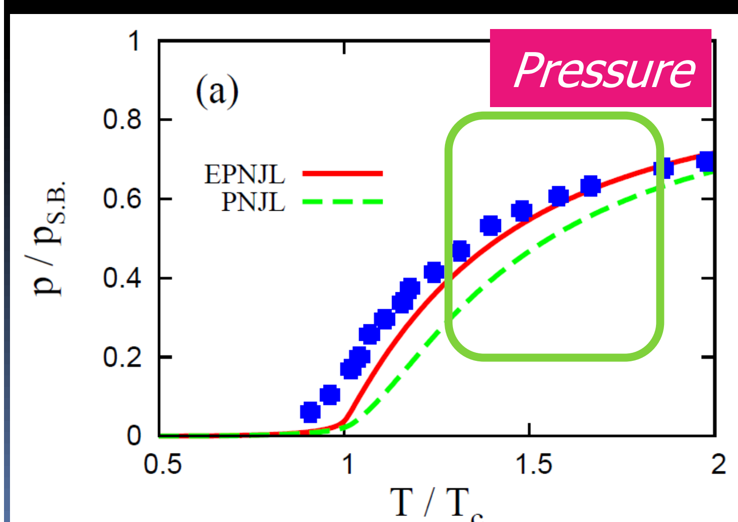

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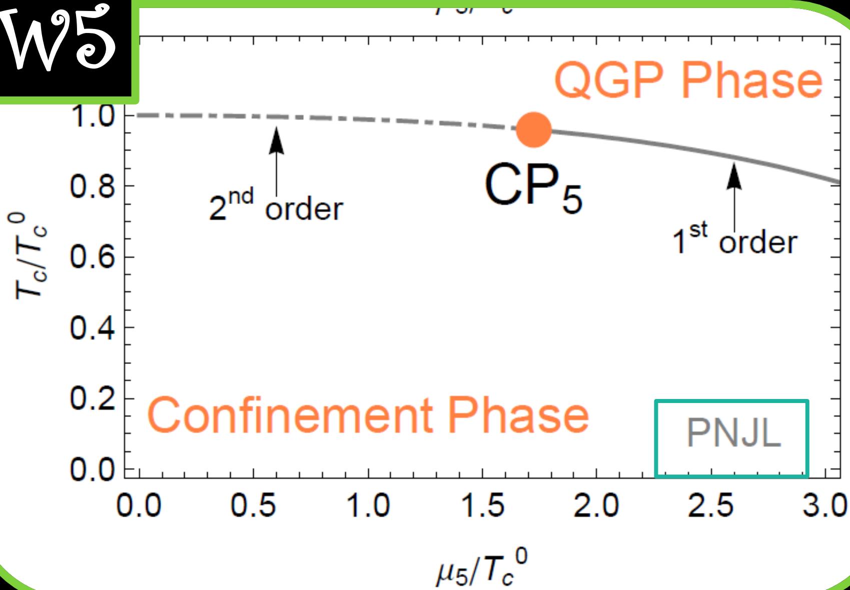
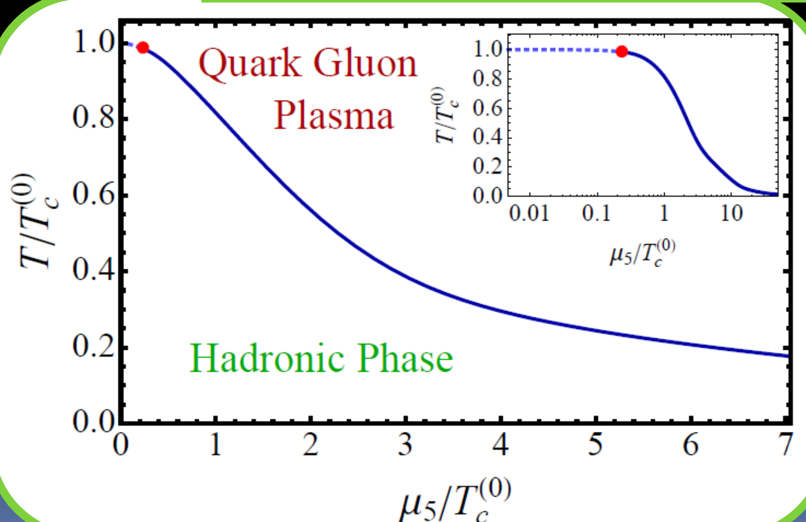
Phase Diagram

W5



PNJL vs PQM

M Chernodub and A. Nedelin,
arXiv:1102.0188 [hep-ph]

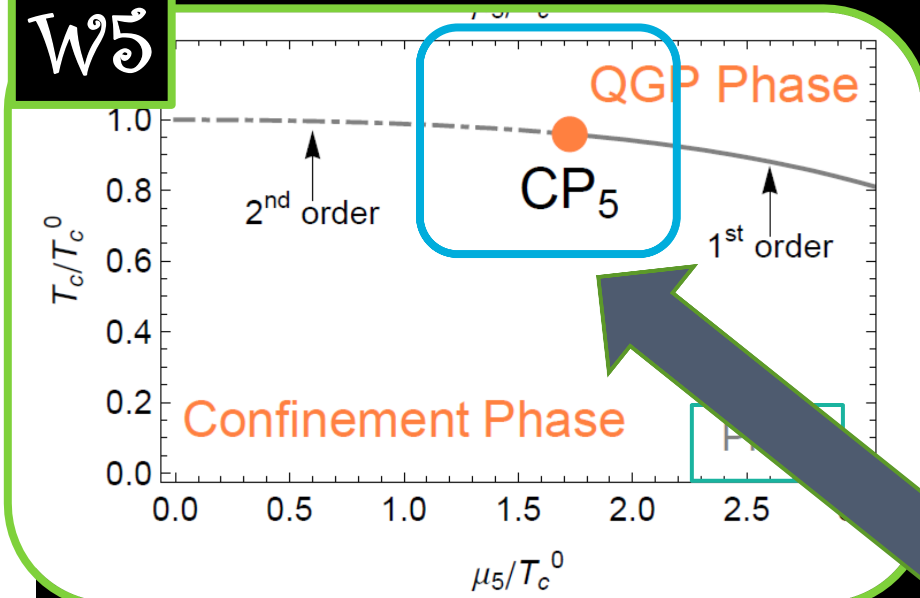


Comparison with previous results
QM model (without vacuum fluctuations)

M.R., **arXiv:1103.6186**

Phase Diagram

W5



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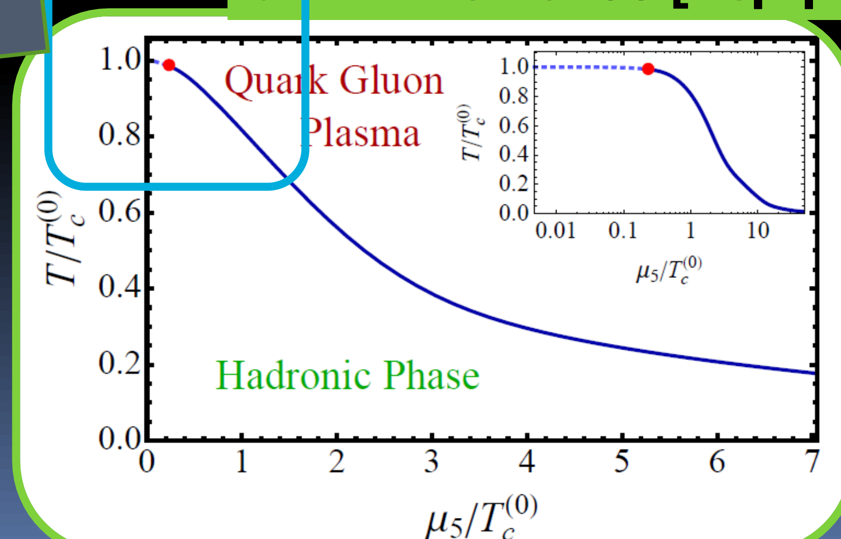

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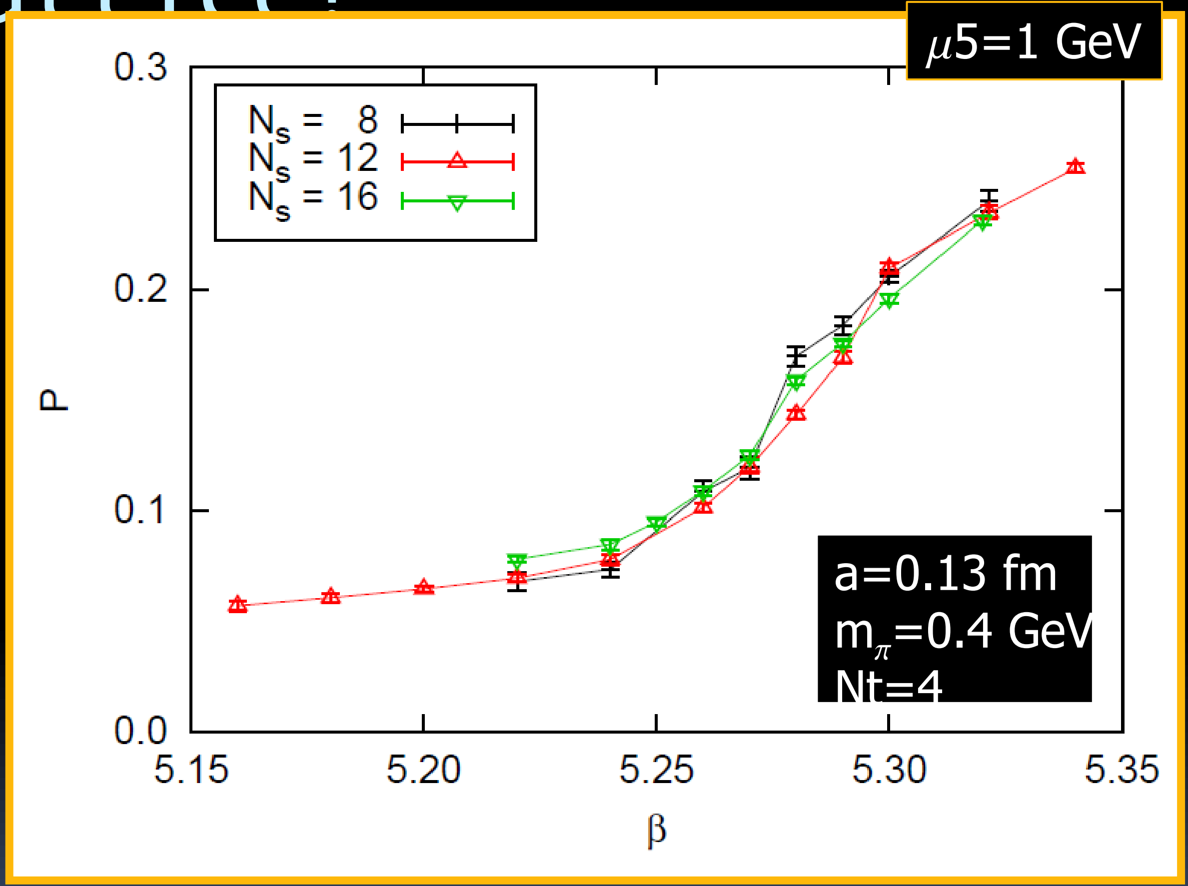
A. Yamamoto, **arXiv:1105.0385 [hep-lat]**

What do we know from Lattice?



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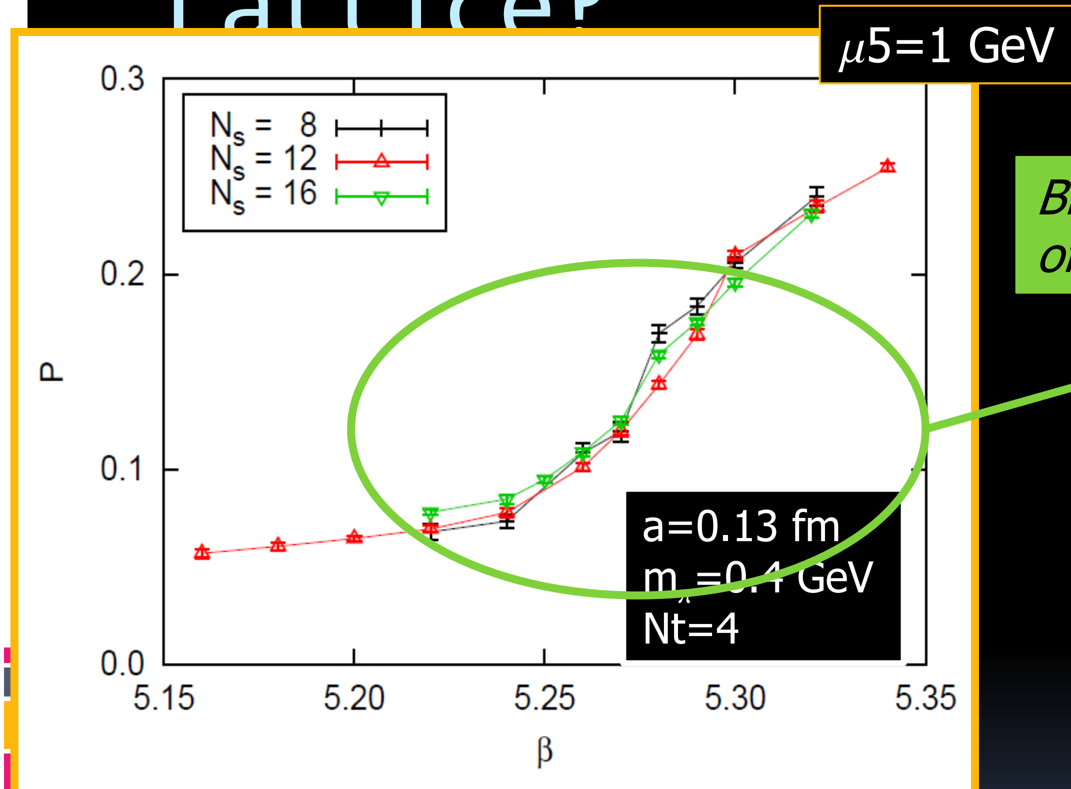

Broad crossover instead of the expected 1st order transition

Speculation

If the result is confirmed with finer lattices and with the physical pion, the crossover could be interpreted as the smoothed phase transition due to inhomogeneous phases.

Phase Diagram: Model Calculation

The effective potential for the Polyakov loop:

$$V = \mathcal{U}(L, L^\dagger, T) + \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm 1} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \omega_s - \frac{N_c N_f}{\beta} \sum_{s=\pm 1} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \log\left(F_+ F_-\right)$$

Thermodynamic Potential

$$\mathcal{U}[\Phi, \bar{\Phi}, T] = T^4 \left\{ -\frac{a(T)}{2} \bar{\Phi}\Phi + b(T) \ln\left[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2\right] \right\}$$

W. Weise *et al*, **Phys.Rev.D75:034007,2007**

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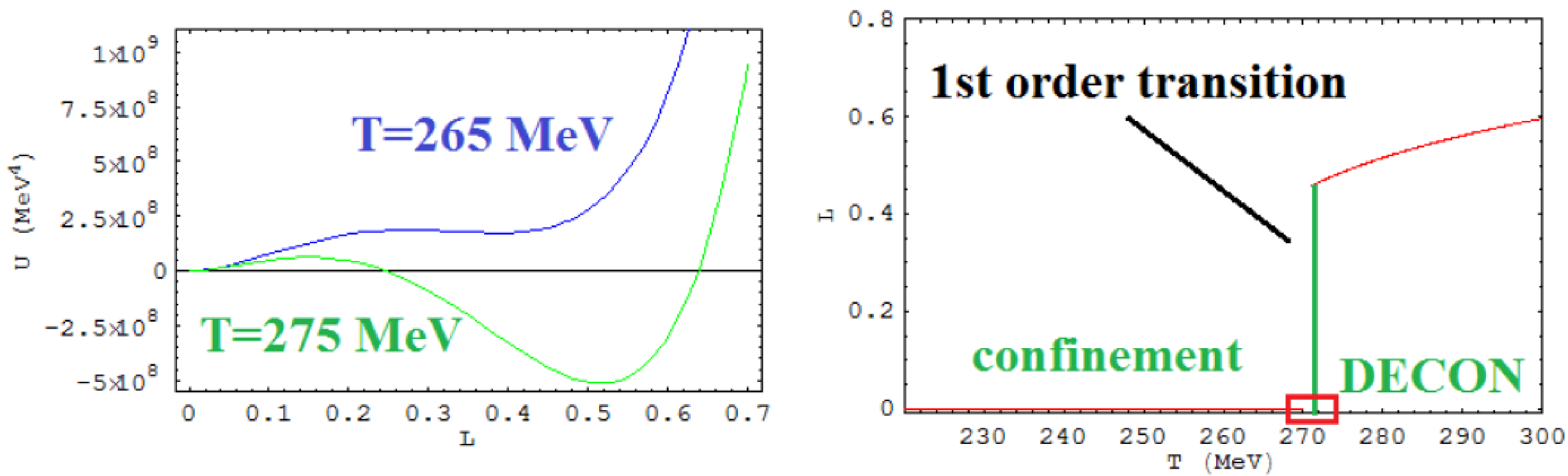
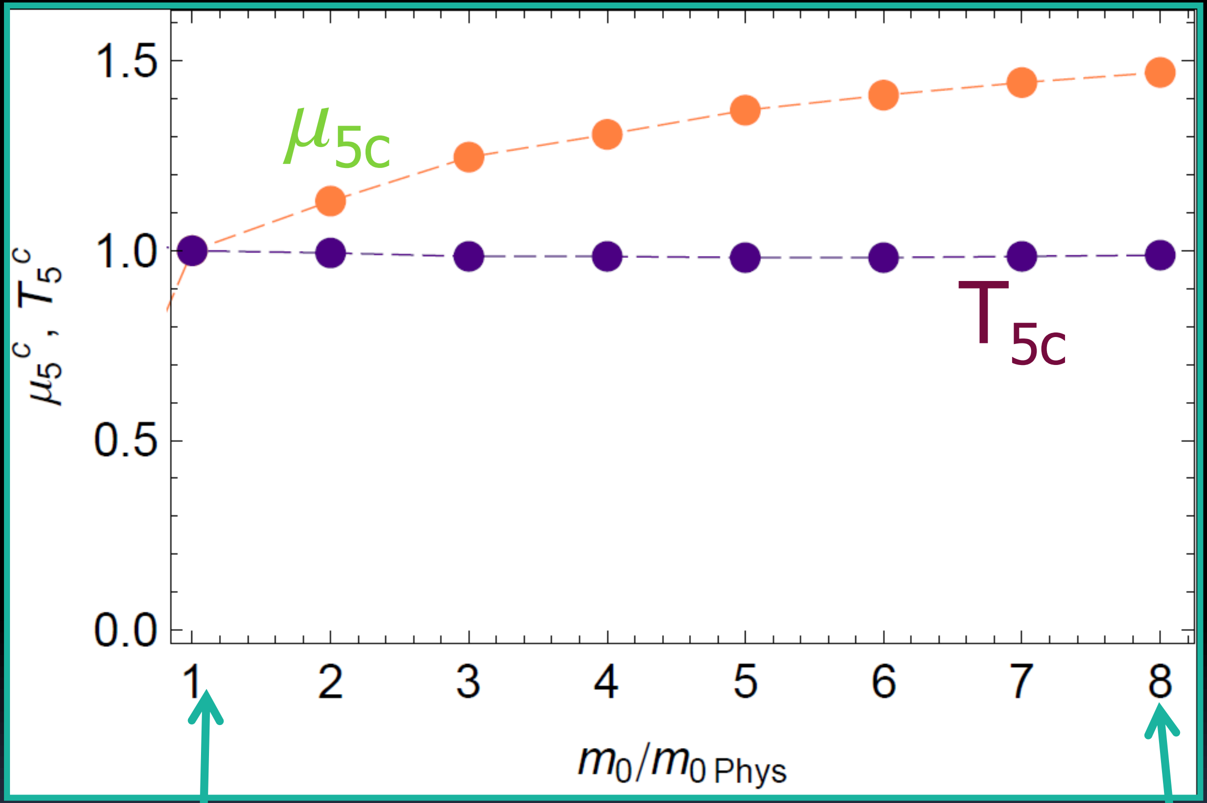
Polyakov loop effective potential

$$\mathcal{U}[\Phi,\bar{\Phi},T] = T^4\Big\{-\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln\big[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2\big]\Big\}$$

Expectation value of L: identified with the global minima of the effective potential.



Quark Mass Dependence of CP5



m_π=139 MeV

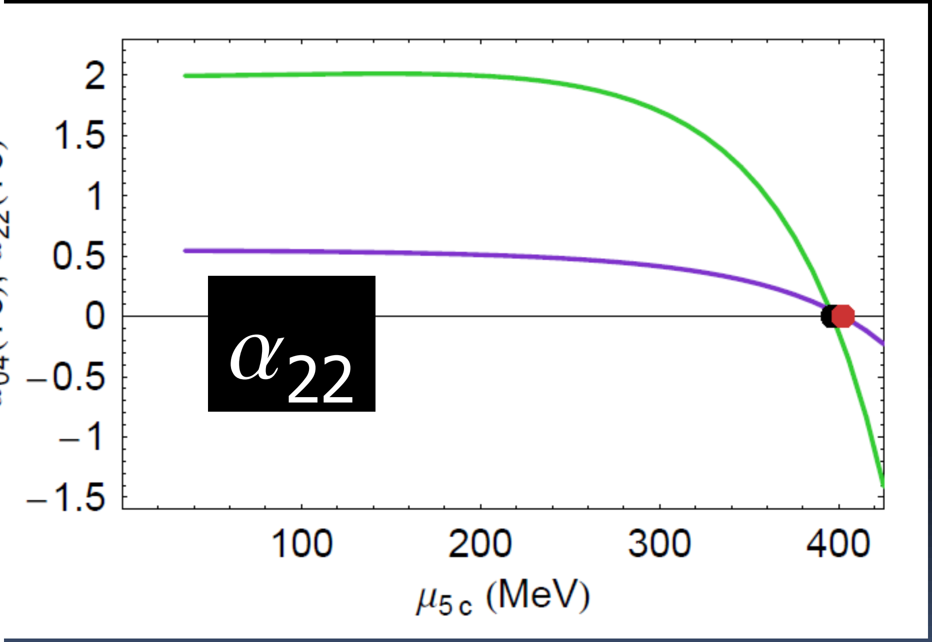
m_π=400 MeV

Outlook 2: work in progress

Inhomogeneous Phases?

Effective potential for the chiral condensate in vicinity of the critical point:

$$\Gamma = \frac{\alpha_{02}}{2}\sigma^2 + \frac{\alpha_{04}}{4}\sigma^4 + \frac{\alpha_{22}}{2}\boldsymbol{q}^2\sigma^2$$



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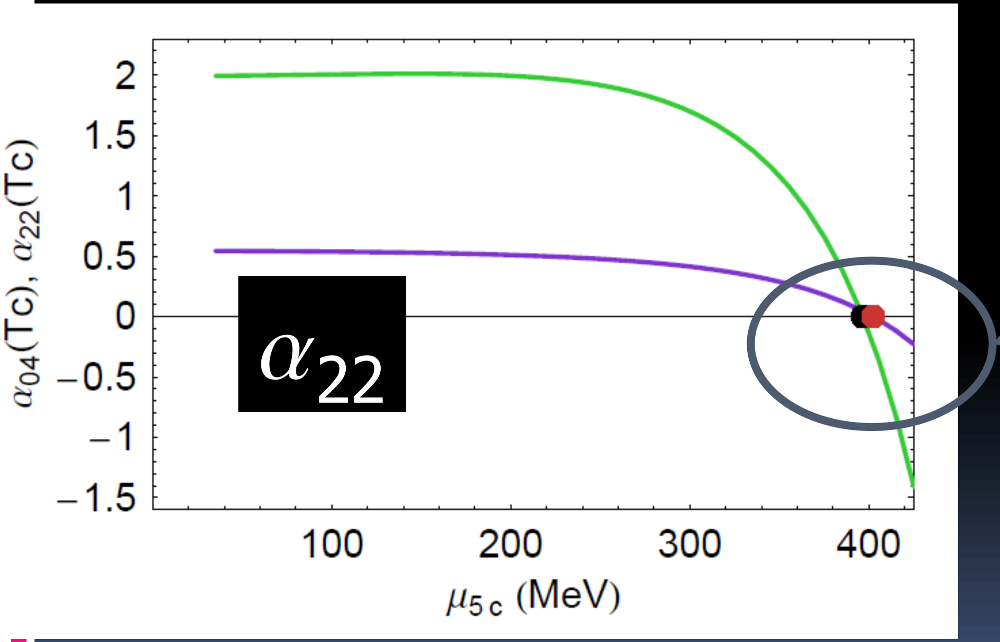
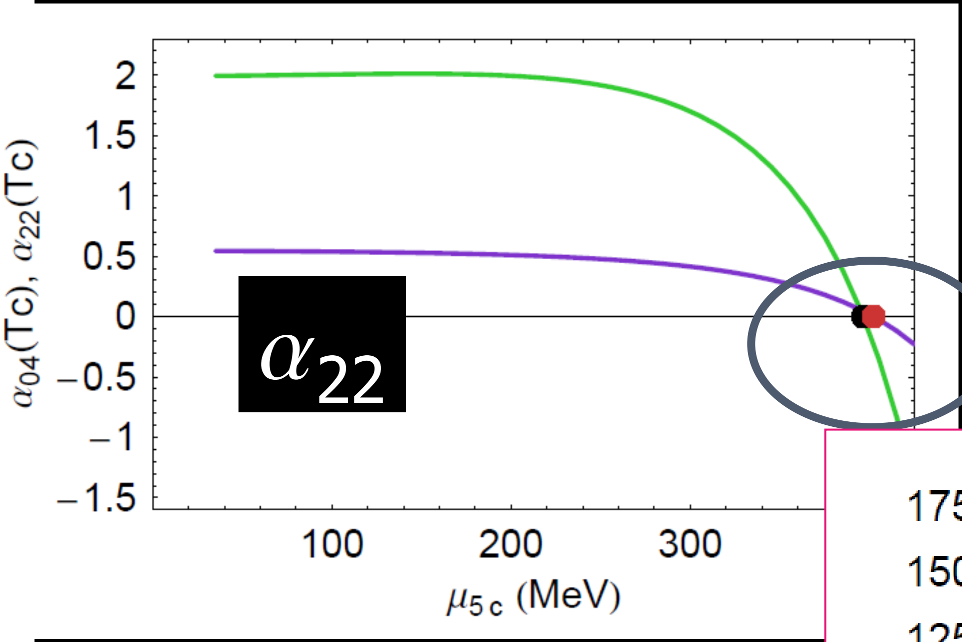


Onset of instability towards inhomogeneous phases

Outlook 2: work in progress

Inhomogeneous Phases?



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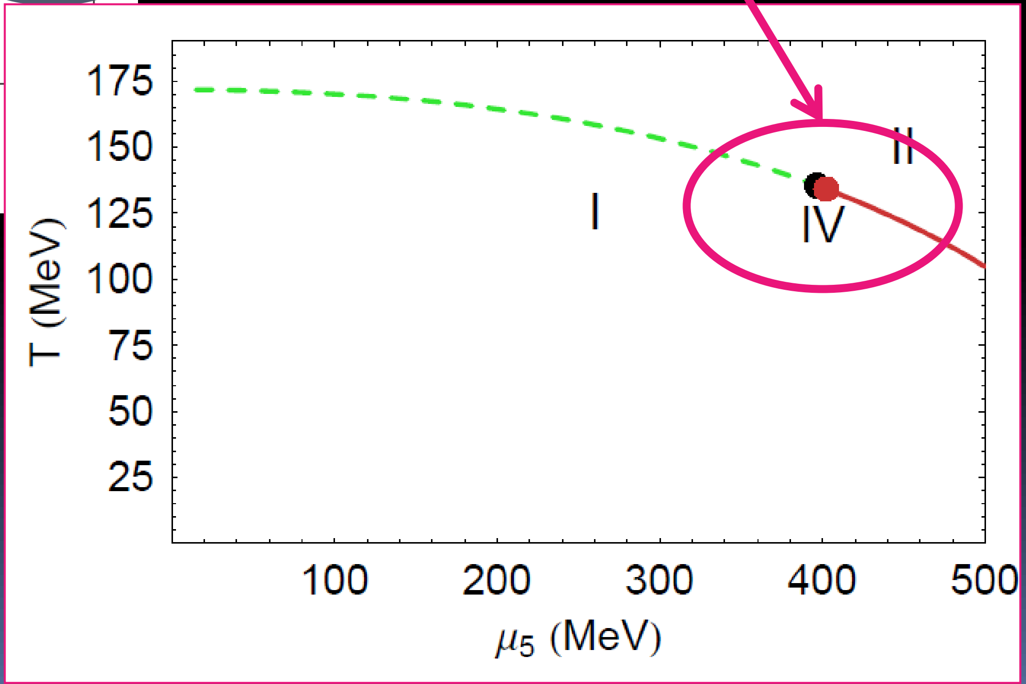
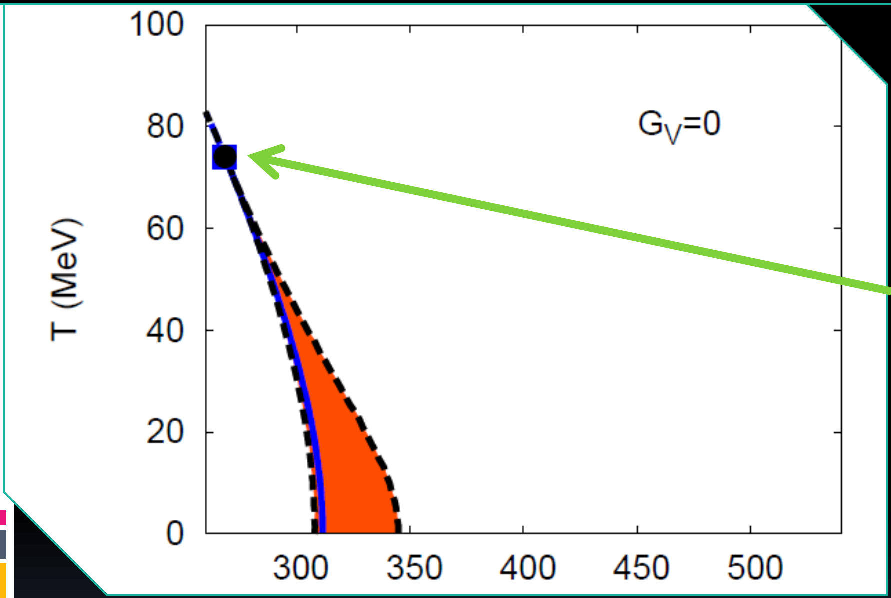


Inhomogeneous Phases?

S. Carignano et al, **Phys.Rev. D82 (2010) 054009**



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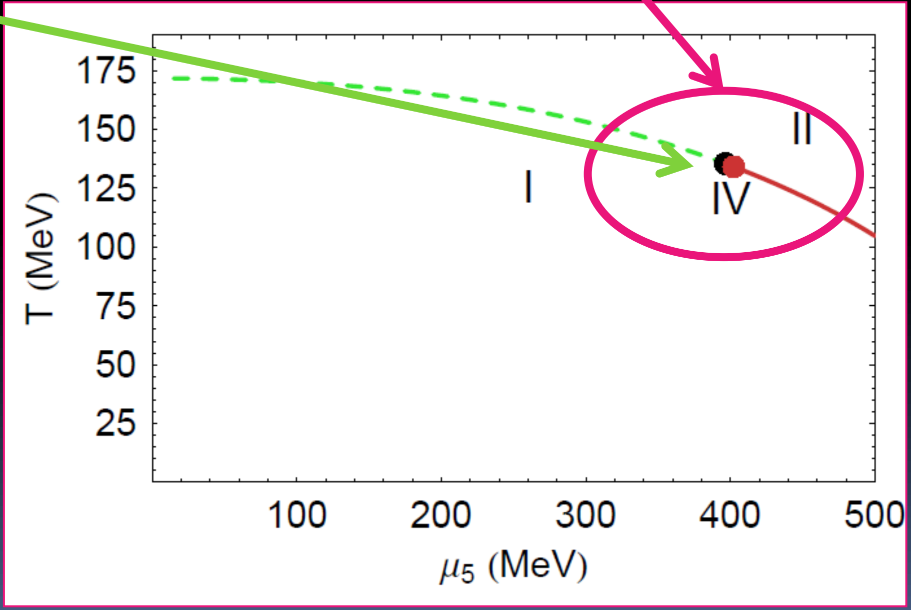

At finite *baryon* chemical potential, the 1st order transition is smoothed by inhomogeneous condensation

Does the same happen at finite *chiral* chemical potential?

Generating chirality in QCD

Gluon configurations with winding number

Ward identity in QCD:

$$(N_L - N_R)_{+\infty} - (N_L - N_R)_{-\infty} = 2Q_W$$

with $Q_W \equiv$ winding number of a background gluon configuration:

$$Q_W = \frac{g^2}{32\pi^2} \int d^4x F \cdot \tilde{F}$$

If in a region of space there is a gluon configuration with $Q_W \neq 0$, this will cause the chirality of quarks to change.

- Perturbative QCD: only $Q_W = 0 \rightarrow$ absence of chirality change
- Non-perturbative QCD: classical gluon configurations with $Q_W \neq 0$ can give contribution to physical quantities

Generating chirality in QCD

Connecting winding number to Chern-Simon number

- Pure gauge $SU(3)$ theory: energy minimized by pure gauge configurations
- In the gauge $A_0 = 0$: $A_i(\mathbf{x}) = ig^{-1}U(\mathbf{x})\partial_i U^\dagger(\mathbf{x})$, with $U(\mathbf{x}) \in SU(3)$
- Each vacuum configuration can be labelled by an integer number:

$$N_{CS} = \frac{1}{24\pi^2}\int d^3x\ \epsilon^{ijk}\mathrm{Tr}\left[(U^\dagger\partial_i U)(U^\dagger\partial_j U)(U^\dagger\partial_k U)\right]$$

- The different vacua are separated by energy barrier of order Λ_{QCD}
- Gauge field configuration with $Q_W \neq 0$ interpolates between two vacua:

$$Q_W = N_{CS}(t = +\infty) - N_{CS}(t = -\infty)$$

Generating chirality in QCD

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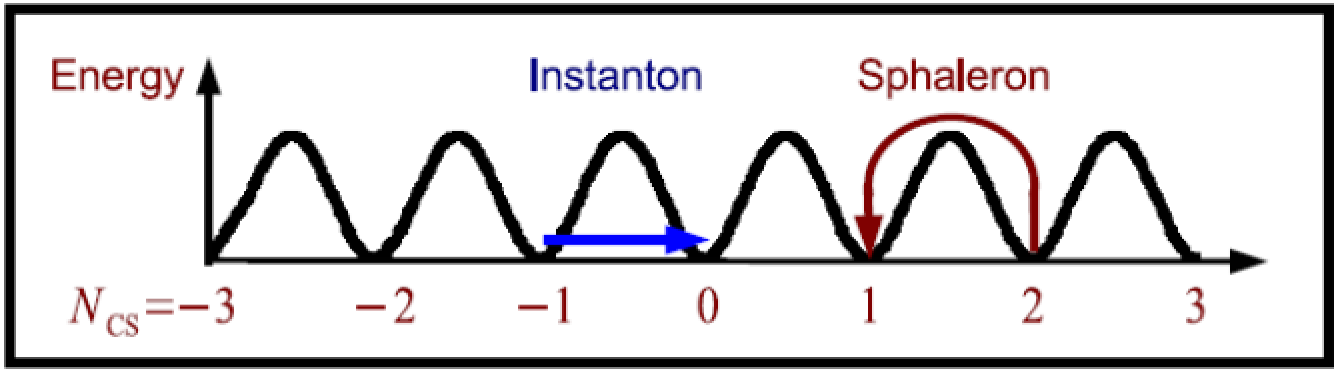
Generating chirality in QCD

Energy Landscape, Instantons and Sphalerons

$$Q_W = N_{CS}(t=+\infty) - N_{CS}(t=-\infty)$$



- Instantons: tunneling between two different vacua.
- Sphalerons: hopping over the barrier.

Transition rate via sphaleron: from Lattice (Moore, 2000):

$$\Gamma = \frac{dN}{d^3x dt} \propto \alpha_S^5 T^4$$

See also Moore and Tassler, **JHEP 1102 (2011) 105**