QUARK MATTER WITH CHIRAL CHEMICAL POTENTIAL

Paris, 2011年6月6日
Outline

- Chiral Chemical Potential: motivations
- The Model
- Phase Diagram with a Chiral Chemical Potential
  .) Chiral Symmetry Restoration
  .) Deconfinement
  .) Critical Endpoint
- Conclusions and Outlook
Motivations

Why am I interested in QCD with a chiral chemical potential?
Chirality in QGP

Chiral density (*chirality*):

\[
N_5 = N_R - N_L
\]

*Imbalance between left- and right-handed quarks*
Chirality in QGP

Chiral density (chirality):

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*Imbalance between left- and right-handed quarks*

Integrated Ward Identity: connecting chirality to winding number

\[ \Delta N_5 = 2Q_W \]

\[ Q_W = \frac{g^2}{32\pi^2} \int d^4x \ F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a = \Delta N_{CS} \]

*Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number*
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\]

*Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number*

Moore and Tassler, *JHEP 1102 (2011) 105*

At high temperature, we expect copious production of *gluon* configurations with nonvanishing winding number (*strong* –i.e. QCD- sphaleron)

Chirality can be produced in the high temperature phase of QCD
Chirality in QGP

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*Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number*

Chirality can be produced in the high temperature phase of QCD

Simplest way to treat quark matter with chirality in effective models:

*Chiral chemical potential, conjugated to chiral density*

\[ \mu_5 \leftrightarrow N_5 \]

*Chiral density operator added to the Lagrangian density*

\[ \mu_5 \bar{q} \gamma^0 \gamma^5 q \]
Chirality in QGP

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Chirality can be produced in the high temperature phase of QCD

\[ \mu_5 \Leftrightarrow N_5 \]
\[ + \mu_5 \bar{q} \gamma^0 \gamma^5 q \]

*Chiral chemical potential*

\[ \mu \Leftrightarrow N \]
\[ + \mu \bar{q} \gamma^0 q \]

*Baryon chemical potential*
Chirality in QGP

Chiral density (chirality):

\[ N_5 = N_R - N_L \]

\textit{Imbalance between left- and right-handed quarks}

Integrated Ward Identity: connecting chirality to winding number

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\textit{Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number}

Chirality can be produced in the high temperature phase of QCD

\[ \mu_5 \Leftrightarrow N_5 \]

\[ + \mu_5 \bar{q} \gamma^0 \gamma^5 q \]

Chiral chemical potential

Remarks

We are aware that $\mu_5$ is not a true chemical potential: chiral condensate mixes L and R components, thus making N5 a non-conserved quantity.

Treat $\mu_5$ as a mere mathematical artifact
Remarks

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Remark 2

Another Motivation

Grand Canonical Ensembles with a chiral chemical potential can be simulated on the Lattice with Nc=3, see for example: A. Yamamoto, arXiv:1105.0385 [hep-lat]
Remarks

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Treat $\mu_5$ as a mere mathematical artifact

Remark 2

Grand Canonical Ensembles with a chiral chemical potential can be simulated on the Lattice with Nc=3, see for example: A. Yamamoto, arXiv:1105.0385 [hep-lat]

Remark 3
Relevant for the HICs phenomenology (Chiral Magnetic Effect):
In **QGP context**, to mimic *chirality change* induced by *instantons* and *strong sphalerons* in Quark-Gluon-Plasma:


C. A. Ballon Bayona et al., arXiv:1104.2291[hep-th]

In **several non-QGP contexts**:


A. N. Sisakian *et al*, hep-th/9806047

The Microscopic Model

Description of the model I use in my concrete calculations.
The Model


NJL Model with the Polyakov Loop

\[ \mathcal{L} = \bar{q} (i \gamma^\mu D_\mu - m) q + G \left[ (\bar{q}q)^2 + (i \bar{q}\gamma_5 \tau q)^2 \right] \]

\[ G = g \left[ 1 - \alpha_1 LL^\dagger - \alpha_2 (L^3 + (L^\dagger)^3) \right] \]

Polyakov Loop: sensitive to confinement – deconfinement transition

\[ L = \frac{1}{3} \text{Tr}_c \exp \left( i \beta \lambda_a A_4^a \right) \]

In the model: \( A_4 \) is background field

The Model

NJL Model with the Polyakov Loop

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Polyakov Loop:
sensitive to confinement – deconfinement transition

$$L = \frac{1}{3} \text{Tr}_c \exp \left( i\beta \lambda_a A_4^a \right)$$

Coupling to quarks via:
. Coupling constant
. Covariant derivative


Coupling dependent on $L$
inspired by:
The 1-loop TP

\[ \mathcal{L} = \bar{q} (i \gamma^\mu D_\mu - m) q + G \left[ (\bar{q}q)^2 + (i \bar{q} \gamma_5 \tau q)^2 \right] \]
The 1-loop TP

\[ \mathcal{L} = \bar{q} (i \gamma^\mu D_\mu - m) q + G \left[ (\bar{q}q)^2 + (i\bar{q}\gamma_5 \tau q)^2 \right] + \mu_5 \bar{q} \gamma^0 \gamma^5 q \]

Add a chiral chemical potential
The 1-loop TP

\[ \mathcal{L} = \bar{q} (i\gamma^\mu D_\mu - m) q + G \left[ (\bar{q}q)^2 + (i\bar{q}\gamma_5 \tau q)^2 \right] + \mu_5 \bar{q} \gamma^0 \gamma^5 q \]

One-loop Thermodynamic Potential

\[ V = U(L, L^\dagger, T) + \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} \omega_s \left( -\frac{N_c N_f}{\beta} \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} \log (F_+ + F_-) \right) \]

Minimization of \( V \) leads to physical values of

\( \sigma \) (chiral condensate)

\( L \)

1- and 2-quark states suppression in the confinement phase

\[ F_- = 1 + 3Le^{-\beta(\omega_s - \mu)} + 3L^\dagger e^{-2\beta(\omega_s - \mu)} + e^{-3\beta(\omega_s - \mu)} \]

\[ F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s + \mu)} + 3Le^{-2\beta(\omega_s + \mu)} + e^{-3\beta(\omega_s + \mu)} \]

Statistically confining distribution functions
Phase Diagram of the Model
Deconfinement and $\Sigma S$ Restoration are entangled for any value of $\mu_5$

This is different from what is found at real chemical potential, see: M. Yahiro et al., arXiv:1104.2394 [hep-ph]
Phase Diagram: Results

\[ W_5 \]

\[ \frac{T_c}{T_c^0} \]

\[ \frac{\mu_5}{T_c^0} \]

QGP Phase

Confinement Phase

\textbf{CP}_5

crossover

1\textsuperscript{st} order

PNJL

M.R., arXiv:1103.6186
Critical Endpoint of QCD

Sophie Bushwick,
Critical Endpoint of QCD

Critical Endpoint (CP)
First order and crossover lines intersect at CP


Based on NJL model

Sophie Bushwick,
Critical Endpoint of QCD

First order and crossover lines intersect at CP

It might exist in QCD


Fodor and Katz, JHEP03(2002)014

For a (critical) review see De Forcrand in PoS LAT2009 (2009) 010
Critical Endpoint of QCD

**Lattice**

- P. De Forcrand *et al.*, *arXiv:0911.5682*

**Models**

- A. Ohnishi *et al.*, *arXiv:1102.3753*
**Critical Endpoint of QCD**

| Nowadays, it has been hard to detect CP by means of Lattice simulations with $N_c=3$, because of the sign problem. |
Critical Endpoint of QCD

Nowadays, it has been hard to detect CP by means of Lattice simulations with $N_c=3$, because of the sign problem.

Idea for (theoretical) detection: Continue CP to another critical point, which can be detected on the Lattice

Besides progresses achieved with simulations at finite isospin or imaginary chemical potential:

De Forcrand and Philipsen, *JHEP*0811 (2008) 012

*Chiral chemical potential* offers an interesting alternative of continuation of the critical point.

I implement the continuation idea within model (M.R., *arXiv:1103.6186*)
Continuation of CP

\[ \mathcal{L} = \bar{q} (i\gamma^\mu D_\mu - m) q + G \left[ (\bar{q}q)^2 + (i\bar{q}\gamma_5 \tau q)^2 \right] \]

\[ + \mu_5 \bar{q} \gamma^0 \gamma^5 q \]

Baryon Chemical Potential conjugated to baryon density

Chiral Chemical Potential conjugated to chiral density: \( N5 = nR - nL \)
I introduce two worlds:

**W5:** World with $\mu=0$ and finite $\mu_5$

**W:** World with $\mu_5=0$ and finite $\mu$

Continuation of CP

$$\mathcal{L} = \bar{q} (i\gamma^\mu D_\mu - m) q + G \left[ (\bar{q}q)^2 + (i\bar{q}\gamma_5 \tau q)^2 \right] + \mu_5 \bar{q} \gamma^0 \gamma^5 q + \mu \bar{q} \gamma^0 q$$

Chiral Chemical Potential conjugated to chiral density: $N5=nR-nL$
CP5 is not an accident, but CP viewed by hot quark matter in W5. Its detection (?) can be interpreted as a theoretical signature of the real world CP.
Continuation of CP

How do the above ratios change when $N_c$ is larger?

For a comparison with the framework of: Hanada and Yamamoto, arXiv:1103.5480 [hep-ph]
Conclusions and Outlook
Conclusions

- Chiral chemical potential is introduced to mimic chirality-changing processes in hot QCD medium
- Phase Structure of Quark Matter (QM) with $\mu_5$ similar to that of QM of our Universe
- QM with $\mu_5$ can be simulated on Lattice (no sign problem)
- Critical Endpoint (CP) of QCD is continued to a new Critical Endpoint, CP$_5$
- Detection of CP$_5$, if found on the Lattice, can be interpreted as a signature of CP
Outlook

- Study of inhomogeneous phases around the critical endpoint at finite $\mu_5$
- Compute quantitative dependence of the critical endpoint on the quark masses
- Study the $N_c$ dependence of the mapping coordinates
- Mapping the critical endpoint within the Ginzburg-Landau effective potential approach
- From 2 to 2+1 flavors

Interesting comparison with results from SS model, C. A. Ballon Bayona et al., arXiv:1104.2291[hep-th]

Thanks for your attention.

Non pentirti di ciò che hai fatto, se quando l'hai fatto eri felice
(Do not regret the things you did, if when you did them you were happy)
The \( L \)-dependent coupling

\[
\mathcal{L} = \bar{q} (i \gamma^\mu D_\mu - m) q + G \left[ (\bar{q}q)^2 + (i\bar{q}\gamma_5 \tau q)^2 \right]
\]

Interaction among background field and gluons leads to a tree-level coupling among \( G \) and \( L \)

\[
G = g \left[ 1 - \alpha_1 LL^\dagger - \alpha_2 (L^3 + (L^\dagger)^3) \right]
\]

The 1-loop TP

\[ \mathcal{L} = \bar{q} (i \gamma^\mu D_\mu - m) q + G \left[ (\bar{q}q)^2 + (i\bar{q}\gamma_5 \tau q)^2 \right] + \mu_5 \bar{q} \gamma^0 \gamma^5 q \]

One-loop Thermodynamic Potential

\[ V = \mathcal{U}(L, L^\dagger, T) + \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} \omega_s \left[ -\frac{N_c N_f}{\beta} \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} \log (F_+ F_-) \right] \]

\[ F_- = 1 + 3L e^{-\beta(\omega_s - \mu)} + 3L^\dagger e^{-2\beta(\omega_s - \mu)} + e^{-3\beta(\omega_s - \mu)} \]

\[ F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s + \mu)} + 3L e^{-2\beta(\omega_s + \mu)} + e^{-3\beta(\omega_s + \mu)} \]

Minimization of \( V \) leads to physical values of .) \( \sigma \) (chiral condensate).

.\) \( L \)

Statistically confining distribution functions.
Statistical Confinement

\[ F_-= 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)} \]

\[ F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s+\mu)} + 3Le^{-2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)} \]

Statistically confining distribution functions

Polyakov loop expectation value
Statistical Confinement

Confinement phase: $L=0$ (approximately)

The colorless 3-quark states give the main contribution to the thermodynamic potential in the confinement phase.
The colorless 3-quark states give the main contribution to the thermodynamic potential in the confinement phase.
Statistical Confinement

Polyakov loop expectation value

Pressure

Qualitative agreement with Lattice data

Picture from: M. Yhiro et al., arXiv:1104.2394 [hep-ph]

PNJL offers a better description of finite temperature QCD than NJL
Statistical De-confinement

Deconfinement phase: $L > 0$

1- and 2-quark states are liberated

The colored 1-quark and 2-quark states give a finite contribution to the thermodynamic potential in the deconfinement phase.

Statistically confining distribution functions

Polyakov loop expectation value

$$F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s+\mu)} + 3Le^{-2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)}$$

$$F_- = 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$
Statistical De-confinement

Pressure

The colored 1-quark and 2-quark states give a finite contribution to the thermodynamic potential in the deconfinement phase.
Statistical De-confinement

The colored 1-quark and 2-quark states give a finite contribution to the thermodynamic potential in the deconfinement phase.

Thermal growth of pressure in correspondence of the crossover

1- and 2-quark states are liberated

Polyakov loop expectation value
The colored 1-quark and 2-quark states give a finite contribution to the thermodynamic potential in the deconfinement phase.
Statistical De-confinement

**Pressure**

- **NJL**
- **PNJL**

**Polyakov loop expectation value**

Qualitative agreement with Lattice data

Picture from: M. Yahirot et al., arXiv:1104.2394 [hep-ph]


*PNJL offers a better description of finite temperature QCD than NJL*
Phase Diagram

Comparison with previous results
QM model (without vacuum fluctuations)

M.R., arXiv:1103.6186

W5

QGP Phase
2nd order
CP5

Confinement Phase

PNJL vs PQM


PNJL

Quark Gluon Plasma

Hadronic Phase
Phase Diagram

Comparison with previous results
QM model (without vacuum fluctuations)

M Chernodub and A. Nedelin,
arXiv:1102.0188 [hep-ph]
What do we know from Lattice?


$N_s = 8$
$N_s = 12$
$N_s = 16$

$\mu_5 = 1 \text{ GeV}$

$a = 0.13 \text{ fm}$

$m_\pi = 0.4 \text{ GeV}$

$N_t = 4$
What do we know from Lattice?


\[ \mu_5 = 1 \text{ GeV} \]

Broad crossover instead of the expected 1st order transition

\[ \begin{align*}
N_s &= 8, \\
N_s &= 12, \\
N_s &= 16
\end{align*} \]

\[ \begin{align*}
a &= 0.13 \text{ fm,} \\
m_\pi &= 0.4 \text{ GeV,} \\
N_t &= 4
\end{align*} \]

Speculation

If the result is confirmed with finer lattices and with the physical pion, the crossover could be interpreted as the smoothed phase transition due to inhomogeneous phases.
Phase Diagram: Model Calculation

The effective potential for the Polyakov loop:

\[ V = \mathcal{U}(L, L^\dagger, T) - \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} \omega_s \]

\[ - \frac{N_c N_f}{\beta} \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} \log (F_+ F_-) \]

\[ \mathcal{U}[\Phi, \bar{\Phi}, T] = T^4 \left\{ - \frac{a(T)}{2} \bar{\Phi} \Phi \right. \]

\[ + b(T) \ln \left[ 1 - 6 \bar{\Phi} \Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi} \Phi)^2 \right] \]

Polyakov loop effective potential

\[ U[\Phi, \bar{\Phi}, T] = T^4 \left\{ -\frac{a(T)}{2} \bar{\Phi}\Phi \\
+ b(T) \ln \left[ 1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2 \right] \right\} \]

Expectation value of \( L \): identified with the global minima of the effective potential.
Quark Mass Dependence of CP5

Outlook 1: work in progress

$m_\pi = 139$ MeV

$m_\pi = 400$ MeV
Inhomogeneous Phases?

Effective potential for the chiral condensate in vicinity of the critical point:

$$\Gamma = \frac{\alpha_{02}}{2} \sigma^2 + \frac{\alpha_{04}}{4} \sigma^4 + \frac{\alpha_{22}}{2} q^2 \sigma^2$$
Inhomogeneous Phases?

Effective potential for the chiral condensate in vicinity of the critical point:

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Onset of instability towards inhomogeneous phases

Outlook 2: work in progress
Inhomogeneous Phases?

Onset of instability towards inhomogeneous phases

Outlook 2: work in progress
Inhomogeneous Phases?

At finite baryon chemical potential, the 1st order transition is smoothed by inhomogeneous condensation.

Does the same happen at finite chiral chemical potential?

Outlook 2: work in progress

Generating chirality in QCD

Gluon configurations with winding number

Ward identity in QCD:

\[(N_L - N_R)_{+\infty} - (N_L - N_R)_{-\infty} = 2Q_W\]

with \(Q_W \equiv \text{winding number}\) of a background gluon configuration:

\[Q_W = \frac{g^2}{32\pi^2} \int d^4xF \cdot \tilde{F}\]

If in a region of space there is a gluon configuration with \(Q_W \neq 0\), this will cause the chirality of quarks to change.

- Perturbative QCD: only \(Q_W = 0\) \(\rightarrow\) absence of chirality change
- Non-perturbative QCD: classical gluon configurations with \(Q_W \neq 0\) can give contribution to physical quantities
Generating chirality in QCD

Connecting winding number to Chern-Simon number

- Pure gauge $SU(3)$ theory: energy minimized by pure gauge configurations
- In the gauge $A_0 = 0$: $A_i(x) = ig^{-1}U(x)\partial_i U^\dagger(x)$, with $U(x) \in SU(3)$
- Each vacuum configuration can be labelled by an integer number:
  \[ N_{CS} = \frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} \text{Tr} \left[ (U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U) \right] \]
- The different vacua are separated by energy barrier of order $\Lambda_{QCD}$
- Gauge field configuration with $Q_W \neq 0$ interpolates between two vacua:
  \[ Q_W = N_{CS}(t = +\infty) - N_{CS}(t = -\infty) \]
Generating chirality in QCD

Connecting winding number to Chern-Simon number

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- The different vacua are separated by energy barrier of order $\Lambda_{QCD}$
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Generating chirality in QCD

Energy Landscape, Instantons and Sphalerons

\[ Q_W = N_{CS}(t = +\infty) - N_{CS}(t = -\infty) \]

- Instantons: tunneling between two different vacua.
- Sphalerons: hopping over the barrier.

Transition rate via sphaleron: from Lattice (Moore, 2000):

\[ \Gamma = \frac{dN}{d^3xdt} \propto \alpha_5^5 T^4 \]

See also Moore and Tassler, JHEP 1102 (2011) 105