INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

QCD and String Theory

(Review)

Shigeki Sugimoto (IPMU)

(The main part is based on works with
T.Sakai, H.Hata, S.Yamato, K. Hashimoto, T.Imoto)

Eleventh Workshop on Non-Perturbative QCD, June 6, 2011 @ l'Institut d'Astrophysique de Paris

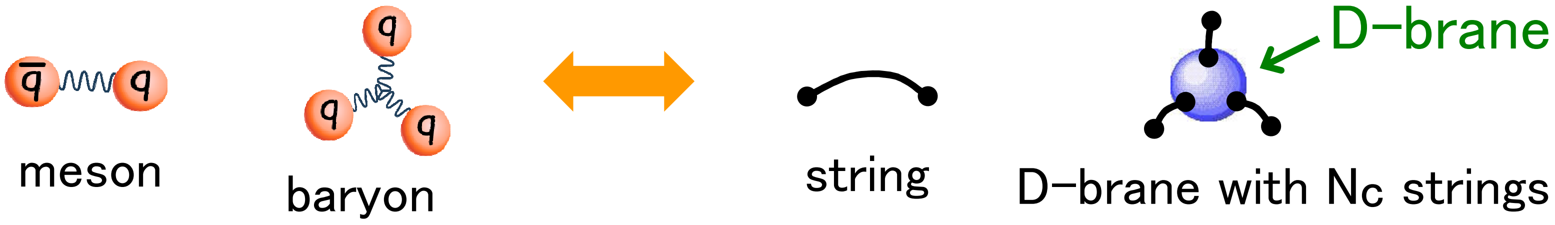
① Introduction

Claim :

Hadrons can be described by ***string theory***
without using quarks and gluons!

QCD ⟷ **dual !** (at low energy) ⟷ String theory (in a certain curved background)

“ Holographic QCD ”



★ Gauge / String duality

[Maldacena 1997, …]

Gauge theory
4 dim

dual ↔

String theory
10 dim curved space−time

"holographic description"

<u>example</u>

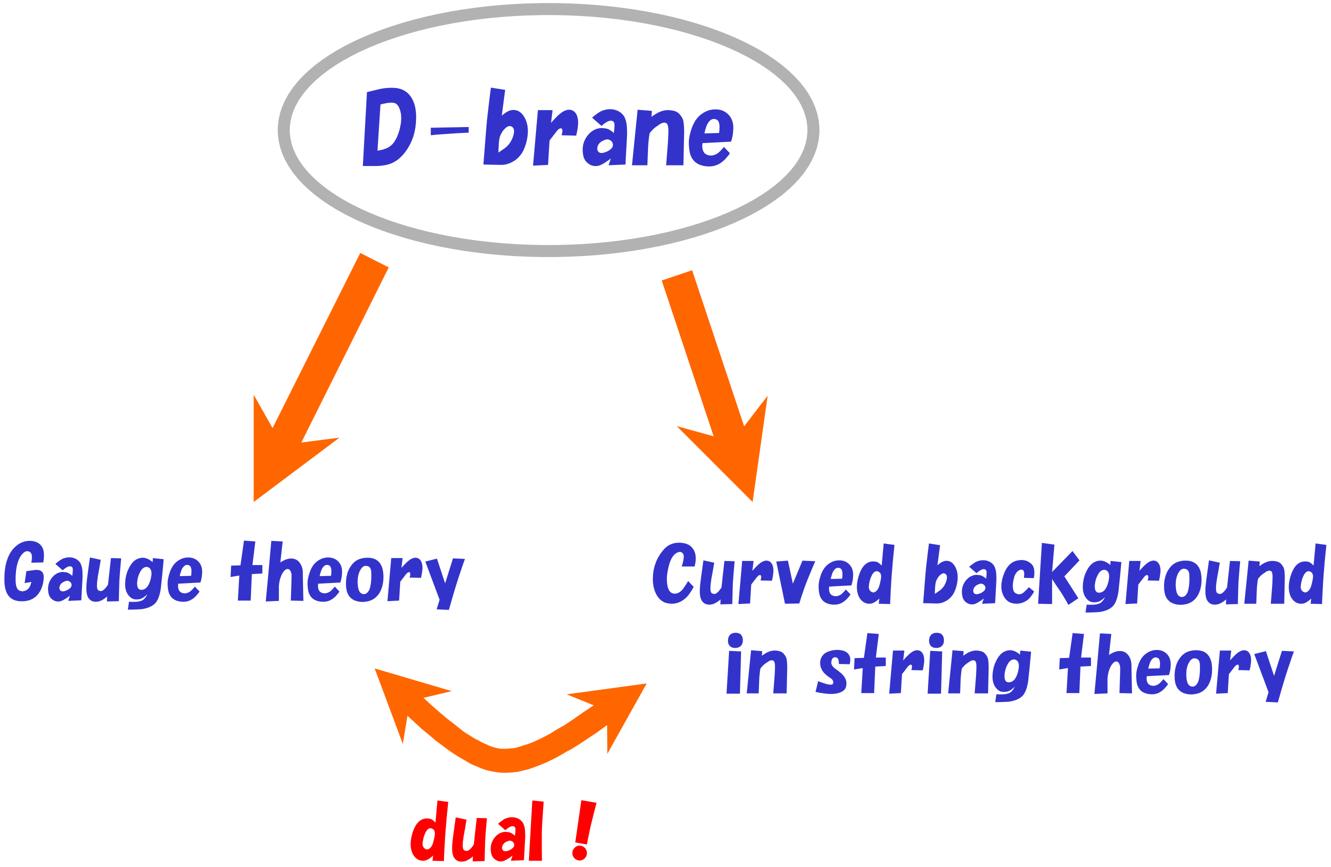
N=4 Super Yang-Mills

dual ↔

String theory in $\mathrm{AdS}_5 \times S^5$

These two look completely different.
But, they are conjectured to be equivalent!

★ Key idea



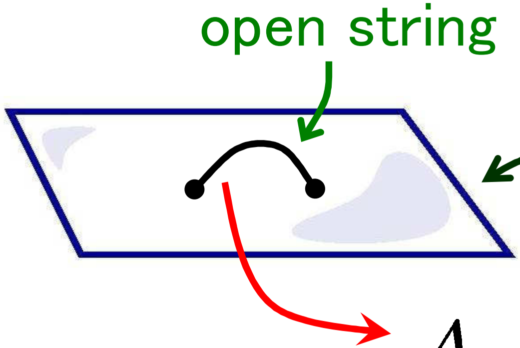
D-brane

Gauge theory

Curved background in string theory

dual !

★ D-brane and gauge theory



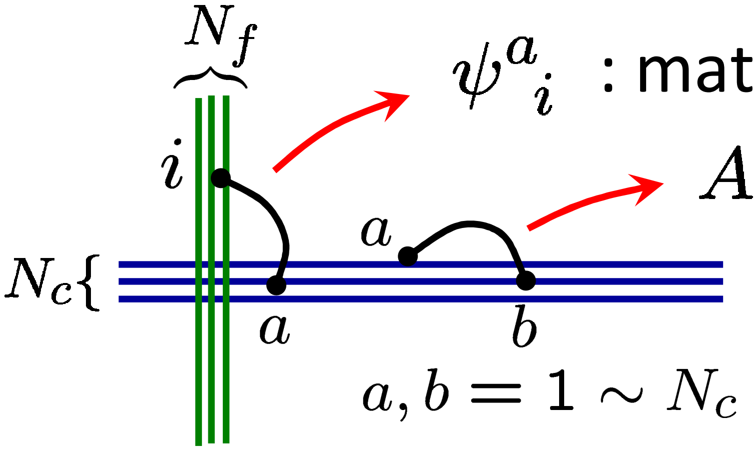
D*p*-brane

= (p+1) dim. plane, on which
open strings can end.

A_μ ➡ gauge theory realized on the brane

example

D3-brane ➡ **N=4 Super Yang-Mills**



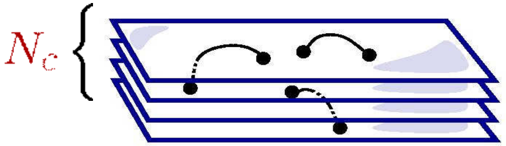
$\psi^a{}_i$: matters in fundamental rep.

$A_\mu{}^a{}_b$: $U(N_c)$ gauge field

➡ various gauge theory

★ D-brane as curved background

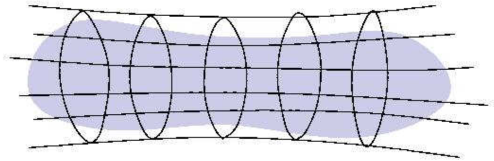
D−brane



$U(N_c)$ gauge theory

→

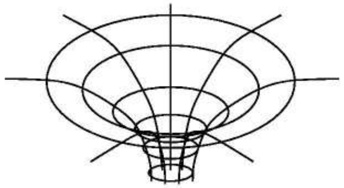
curved background
corresponding to the D-brane



(cf) particle

→

Solution of Einstein eq.



)

example

D3-brane → $AdS_5 \times S^5$

AdS/CFT

[Maldacena 1997]

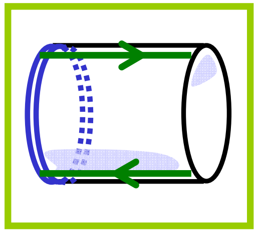
D3-brane

N=4 Super Yang-Mills

String theory in $AdS_5 \times S^5$

dual

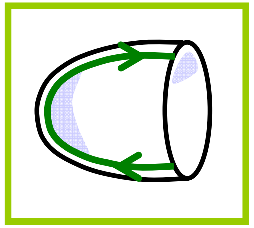
holographic QCD



Some brane config.

QCD

dual



String theory in some curved b.g.

➡ “holographic QCD”

Plan of Talk

✓ ① Introduction

② Construction of QCD

③ Applications

④ Conclusion

② Construction of QCD

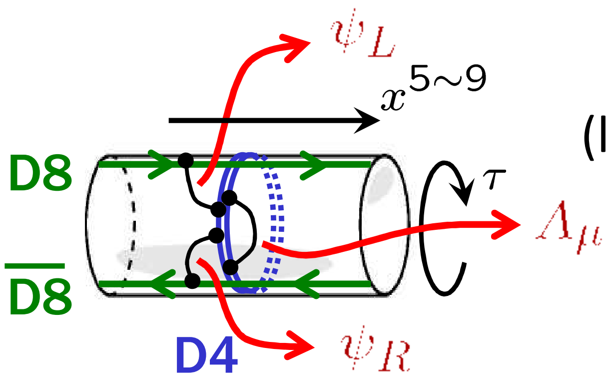
★ QCD in string theory

	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 $\times N_c$	○	○	○	○	○	–	–	–	–	–
D8-$\overline{\text{D8}}$ $\times N_f$	○	○	○	○	–	○	○	○	○	○

4 dim

S^1

[Sakai-S.S. 2004]



(low energy)

A_μ : gluon

$\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$: quark $\times N_f$

$SU(N_c)$ **QCD**
with N_f **massless quarks**

★ Holographic QCD

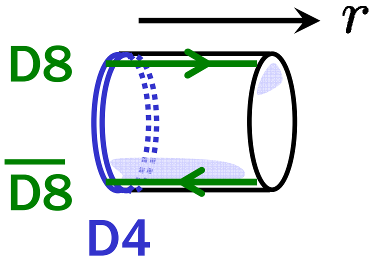
N_c **D4-brane on** S^1

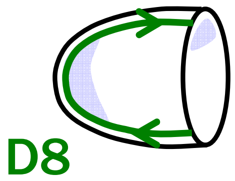
+ N_f **D8-$\overline{\text{D8}}$ pairs**

replace D4
with curved
background

String theory in
the D4 background

+ N_f **D8-branes**





(low energy)

$SU(N_c)$ **QCD**
with N_f **massless quarks**

dual

Open + closed string theory
in this background

" Holographic QCD "

(low energy)

$SU(N_c)$ **QCD**
with N_f **massless quarks**

dual

Open + closed string theory
in this background

$$\begin{cases} \text{string coupling } \propto 1/N_c \\ \text{string length } \propto \lambda^{-1/2} \end{cases}$$

parameters :

$$\begin{cases} M_{\mathrm{KK}} \sim \text{ cut off scale} \\ \lambda = g_{\mathrm{YM}}^2 N_c \end{cases}$$

('t Hooft coupling)

good description for

$$\begin{cases} \text{large } N_c \\ \text{large } \lambda \leftrightarrow \text{ low energy} \end{cases}$$

③ Applications

Now we are ready to discuss the applications

But, don't trust too much !

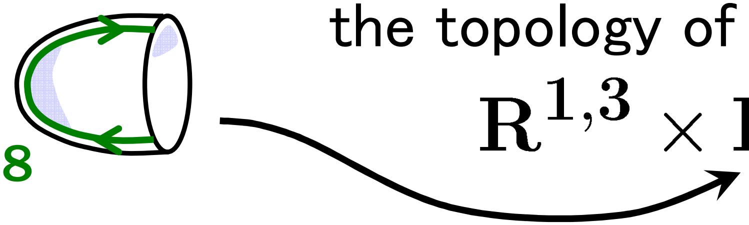
- We have only estimated the leading terms in $1/N_c$, $1/\lambda$ expansions
- The model deviates from real QCD at high energy～M_{KK}～1GeV
- quark masses are neglected

But, don't be too pessimistic.

- The effect of "cut off" at M_{KK} is milder than lattice cut off.
- Remember "quench approximation" works in lattice QCD
- At least, we should not give up before trying.

★ Hadrons in the model

Recall:

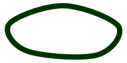


the topology of the space−time is

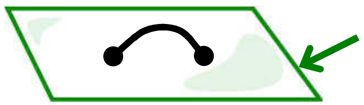
$$\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$$

Particles in this system

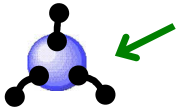
- Closed strings ↔ glueballs



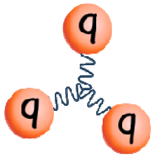
- Open strings on D8 ↔ mesons



- D4 wrapped on S^4 ↔ baryons



Nc strings are attached



[Sakai-S.S. 2004, 2005]

Meson effective theory is written as a 5 dim $U(N_f)$ YM-CS theory in a curved background.

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$k(z) = 1 + z^2$

CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$\mu, \nu = 0 \sim 3$

$h(z) = (1 + z^2)^{-1/3}$

$$A_\mu(x^\mu, z) = \sum_{n \geq 1} B_\mu^{(n)}(x^\mu) \psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n \geq 0} \varphi^{(n)}(x^\mu) \phi_n(z)$$

complete sets of functions of z

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

$$\Rightarrow \quad \boxed{S_{\text{5dim}}(A) = S_{\text{4dim}}(\pi, \rho, a_1, \rho', a_1', \cdots)}$$

traditional meson effective action

- A lot of old models are reproduced without making any phenomenological assumptions!

Skyrme model	[Skyrme 1961]
Vector meson dominance	[Sakurai 1960, Gell-Mann-Zachriasen 1961]
Gell-Mann Sharp Wagner model	[Gell-Mann –Sharpe-Wagner 1962]
Hidden local symmetry	[Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]

next slide

- masses and couplings roughly agree with experimental data.

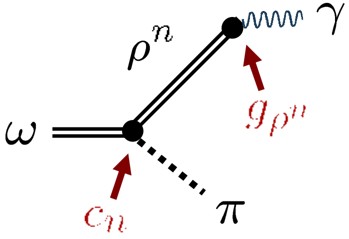
mass	ρ	a_1	ρ'
exp.(MeV)	776	1230	1465
our model	[776]	1189	1607
ratio	[1]	1.03	0.911

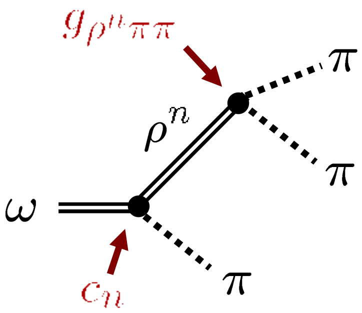
input

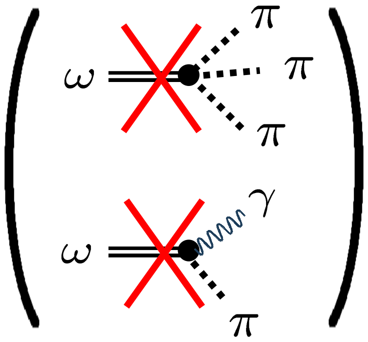
coupling	our model	experiment
f_π	[92.4 MeV]	92.4 MeV
L_1	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	4.81	5.99
g_ρ	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	4.63 GeV	$2.8 \sim 4.2$ GeV

$\boldsymbol{\omega}$ meson decay ($\omega \to \pi^0\gamma$ and $\omega \to \pi^0\pi^+\pi^-$)

- Our model predicts that the relevant diagrams for $\omega \to \pi^0\gamma$ and $\omega \to \pi^0\pi^+\pi^-$ are







➡ Exactly the same as the ***GSW model*** !

[Gell–Mann –Sharp–Wagner 1962]

- Furthermore, we find

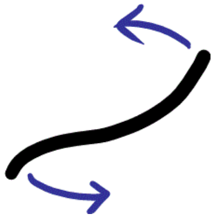
$$\Gamma(\omega \to \pi^0\gamma) = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2}\left(\sum_{n=1}^{\infty}\frac{c_n\, g_{\rho^n}}{m_{\rho^n}^2}\right)^2 |\mathbf{p}_\pi|^3 = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2} {g_{\rho\pi\pi}}^2 |\mathbf{p}_\pi|^3$$

➡ reproduces the proposal given by Fujiwara et al !

[Fujiwara–Kugo–Terao–Uehara–Yamawaki 1985]

- Other mesons, including higher spin mesons, are obtained as excited string states.

[Imoto-Sakai-S.S. 2010]



rotating
open string → higher spin
meson

- 1st excited states

→ $a_2(1320),\ b_1(1235),\ \pi(1300),\ a_0(1450),\ \cdots$

- 2nd excited states

→ $\rho_3(1690),\ \pi_2(1670),\ \cdots$

- 3rd excited states

→ $a_4(2040),\ \cdots$

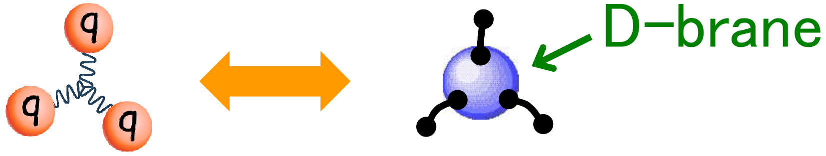
mass: $m_N \simeq \sqrt{\frac{N}{\alpha'}} + \frac{M_{\rm KK}}{2\sqrt{2}}$

$(N = 1, 2, 3, \cdots)$

(See arXiv:1005.0655 for more details)

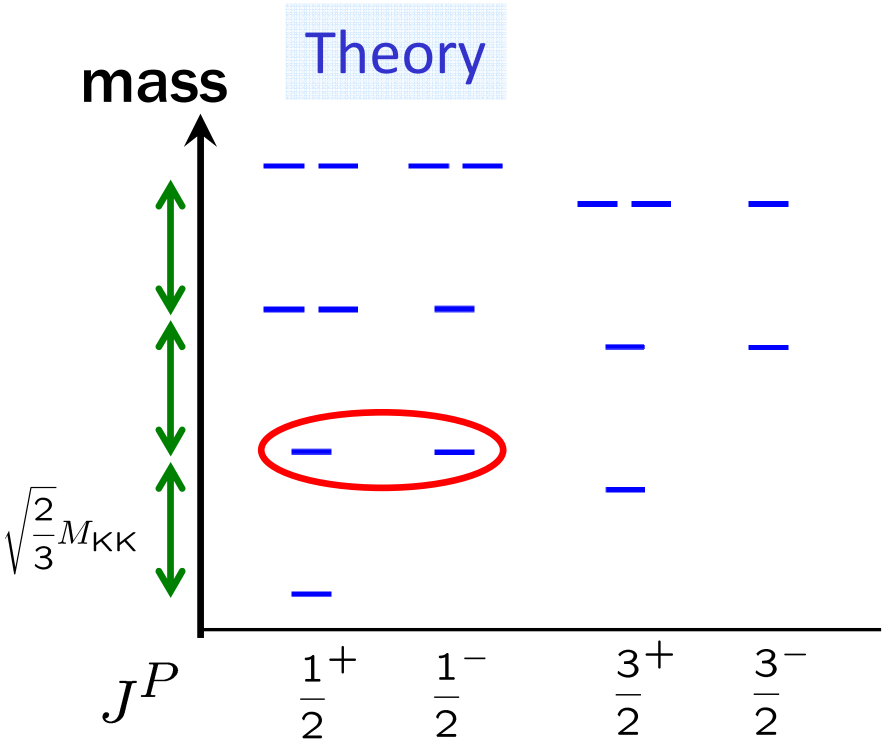
Baryon spectrum

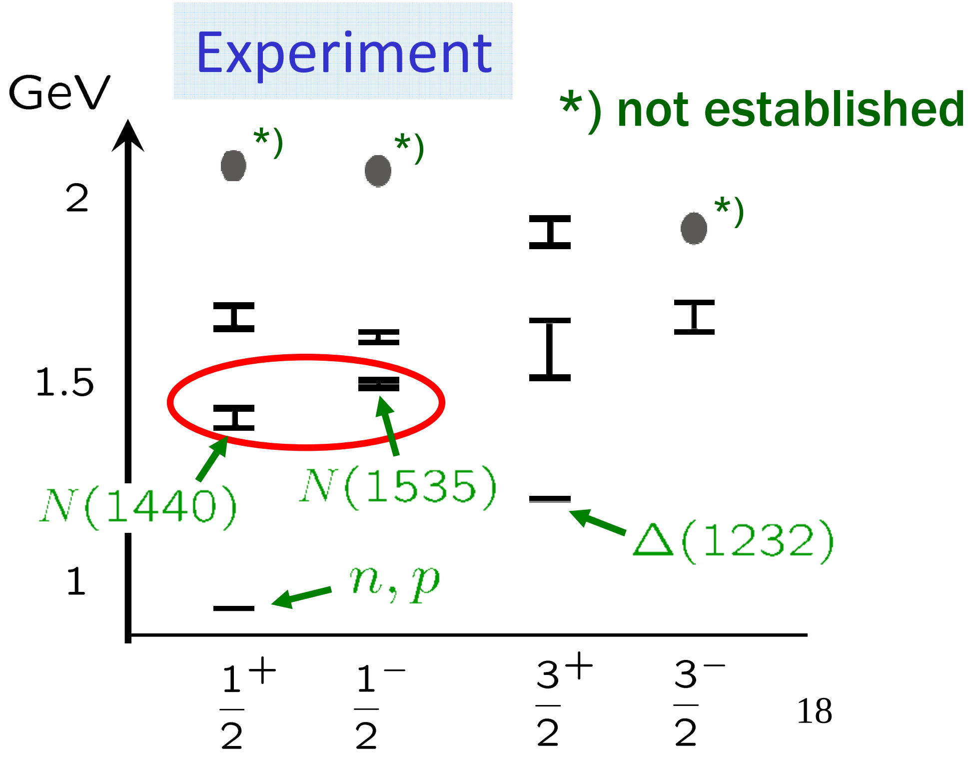
[Hata-Yamato-Sakai-S.S. 2007]



= soliton in the 5dim theory

We can analyze the spectrum, magnetic moments, charge radii by quantizing this soliton. cf) Skyrme model





***) not established**

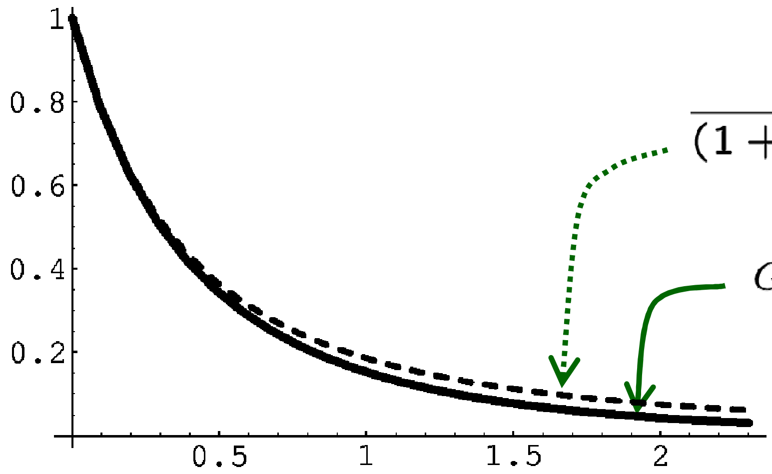
Properties of nucleons

[Hashimoto-Sakai-S.S. 2008]

[See also,
Hong-Rho-Yee-Yi 2007,
Hata-Murata-Yamato 2008,
Kim-Zahed 2008]

	our result	exp.
$\langle r^2 \rangle^{1/2}_{I=0}$	0.74 fm	0.81 fm
$\langle r^2 \rangle^{1/2}_{I=1}$	0.74 fm	0.94 fm
$\langle r^2 \rangle^{1/2}_{A}$	0.54 fm	0.67 fm
$g_{I=0}$	1.7	1.8
$g_{I=1}$	7.0	9.4
g_A	0.73	1.3

nucleon ele-mag form factor



$\frac{1}{(1+Q^2/\Lambda^2)^2}$ dipole ($\simeq$ experiment)

$G_E^p(Q^2) = \sum_{n\geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2}$ our result

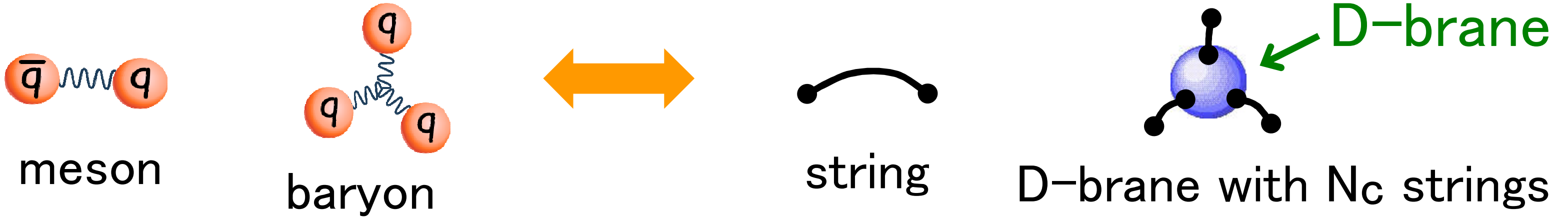
Q^2 GeV^2

④ Conclusion

Hadrons can be described by ***string theory***
without using quarks and gluons!

QCD ↔ **dual !** (at low energy) ↔ String theory (in a certain curved background)

“ Holographic QCD ”



meson

baryon

string

D-brane

D-brane with Nc strings