Pions and Strange Mesons in Soft-Wall AdS/QCD

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Overview

- What is AdS/QCD?
  - Previous Results for radial excitations of mesons
- Pseudoscalar Sector
- Adding a Third Flavor

AdS/CFT Correspondence

Duality between:

4D Conformal Field Theory	⟷	5D Gravity theory in AdS Space
Strongly Coupled CFT	⟷	Weakly coupled Gravity
Operators	⟷	Fields
Global Symmetries	⟷	Gauged Symmetries

“Bottom-Up” AdS/QCD

- Assume QCD has suitable 5D dual
- Use for strong coupling problems
  - Hadron structure
- QCD is not scale-invariant
  - Confinement sets a scale
- Model must break conformal symmetry
  - Cut off 5^{th} Dimension

Modified Soft-Wall Model

- Anti-de Sitter space metric:

$$ds^2 = \frac{L^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2), \qquad z \geq 0$$

- 5-D Action:

$$\int d^5x\sqrt{-g}e^{\underline{-\phi(z)}}Tr\left[|D\underline{X}|^2 + m_X^2|\underline{X}|^2 - \kappa|\underline{X}|^4 + \frac{1}{2g_5^2}(\underline{F_A^2} + \underline{F_V^2})\right]$$

Dilaton

Scalar Field

Vector and Axial Field Tensors

Gherghetta, Kapusta, Kelley, 2009

Modified Soft-Wall Model

- Vacuum expectation value (VEV) of *X*
  - Breaks chiral symmetry

$$\langle X \rangle \equiv \frac{v(z)}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

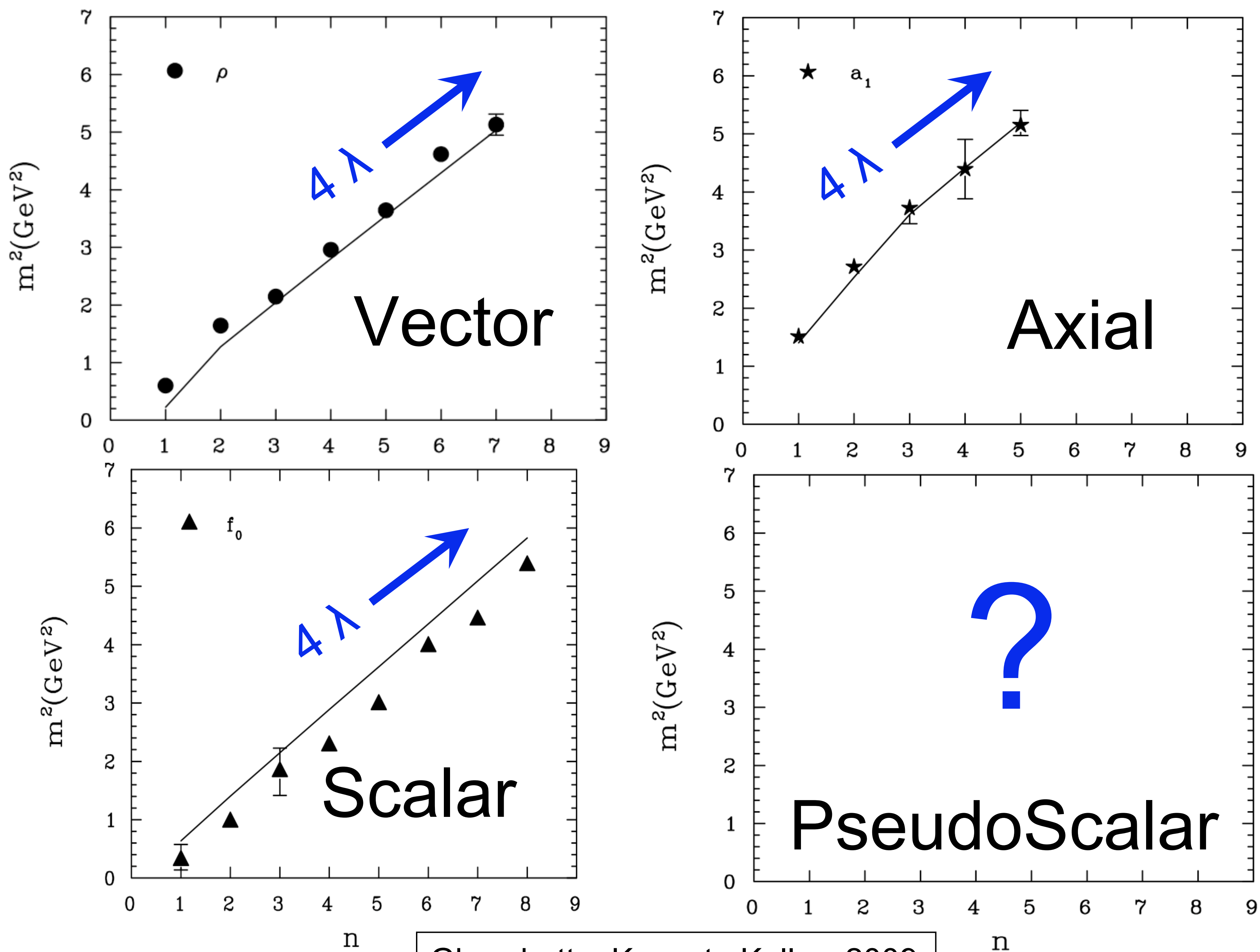
- Dilaton/VEV Equation of Motion:

$$\partial_z(a^3 e^{-\phi} \partial_z v(z)) - a^5 e^{-\phi}(m_X^2 v(z) - \frac{\kappa}{2} v^3(z)) = 0$$

$$v(z) = z\left(A + B \tanh Cz^2\right) \rightarrow A,B \sim m_q \ldots C \sim \sigma$$

$$v(z \rightarrow 0) = Az + BCz^3 \quad \text{independent } m_q \text{ and } \sigma$$

$$v(z \rightarrow \infty) = \left(A + B\right)z \quad \text{so that } \phi(z \rightarrow \infty) = \lambda z^2$$



Gherghetta, Kapusta,Kelley, 2009

Desired Features of Pseudoscalars

- *Kelley, S.B., Kapusta 2011*
- Linear trajectory
  - Large-n Modes
- Pseudo-Goldstone Boson
  - Low mass ground state
  - Large mass gap
  - Linear relationship of m_q and m_π^2
- Good Correspondence to Data

Pseudoscalar Representations

- Linear

$$X_l = (v(z)/2 + S(x,z))\, I + i\pi_l^a(x,z)t^a$$

- Exponential

$$X_e = (v(z)/2 + S(x,z))\, I\, \mathrm{e}^{2i\pi_e^a(x,z)t^a}$$

- Equivalence Relation:

$$\pi_e \to \pi_l / v(z)$$

Equations of Motion

- Use Linear Representation
- Separating $A_\mu = A_{\mu\perp} + \partial_\mu \varphi$
- Varying π and ϕ yields Equations of Motion (EOM)

$$z^3 e^{\phi} \partial_z \left(\frac{e^{-\phi}}{z^3} \partial_z \pi_n \right) - \left(\frac{m_X^2}{z^2} - \frac{\kappa L^2 v^2}{2z^2} \right) \pi_n + m_n^2 \pi_n = m_n^2 v \varphi_n$$

$$e^{\phi} \partial_z \left(\frac{e^{-\phi}}{z} \partial_z \varphi_n \right) + \frac{g_5^2 L^2 v}{z^3} (\pi_n - v \varphi_n) = 0$$

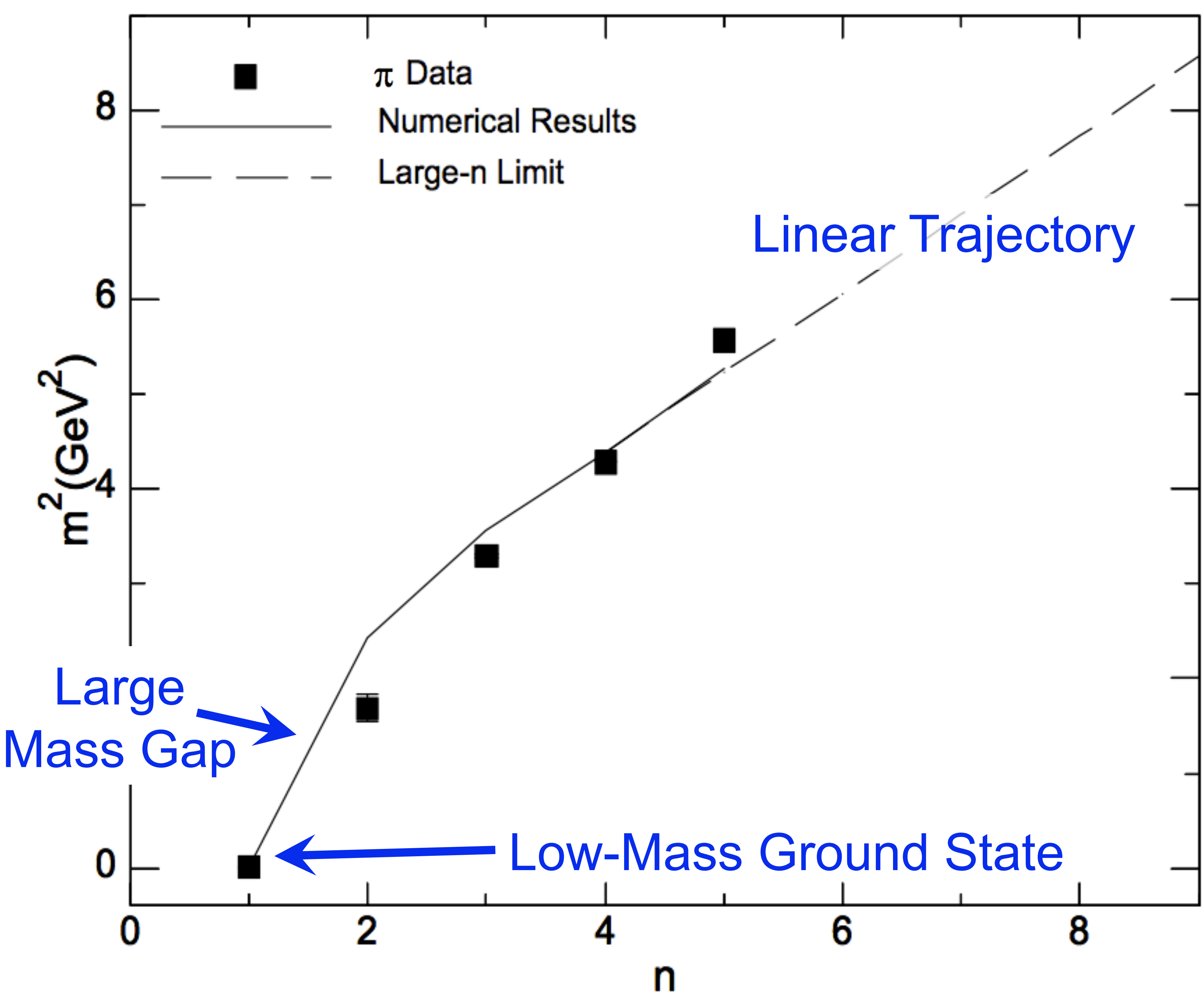
Calculating Mass Spectrum

- Numerical Solution
  - Good for small *n*
- For *n* >4, take large-z limit: $\phi = \lambda z^2, \; v = \Gamma z$
- Equations become harmonic oscillators:

$$
\begin{aligned}
-\pi_n'' + \xi^2 \pi_n &= \frac{m_n^2}{\lambda}\left(\pi_n - \Gamma \phi_n\right) \\
-\phi_n'' + \xi^2 \phi_n &= \frac{g_5^2}{\lambda}\left(\pi_n - \Gamma \phi_n\right).
\end{aligned}
$$

Where $\xi = \sqrt{\lambda} z$

$$m_n^2 = (4n-3)\lambda + g_5^2 \Gamma^2 \qquad n = 4, 5, \ldots$$



π Data
Numerical Results
Large-n Limit
Linear Trajectory
Large
Mass Gap
Low-Mass Ground State
m²(GeV²)
n
0
2
4
6
8

Pseudo-Goldstone Boson

- Spontaneously broken symmetry
  - Yields massless boson
- Chiral Symmetry Breaking ➔ Pion
- Explicit Chiral Symmetry Breaking ($m_q \neq 0$)
  - Pion not massless
- Masses related by Gell-Mann—Oakes—Renner relation:

$$2m_q\sigma = m_\pi^2 f_\pi^2$$

Gell-Mann—Oakes—Renner Relation

- Massless quark ➔ Massless pion
- Start with EOM: $$\frac{g_5^2 L^2 v^2}{z^2}\partial_z\left(\frac{\pi_l}{v}\right) = m_\pi^2 \partial_z \phi$$
- And let $\phi(z) = A(0,z) - 1$
- Integrate: $$\frac{\pi(z)}{v(z)} = m_\pi^2 \int_0^z du\, \frac{u^3}{v^2(u)} \frac{\partial_z A(0,u)}{g_5^2 u}$$
- Using $$f_\pi^2 = -\left.\frac{\partial_z A(0,z)}{g_5^2 z}\right|_{z\to 0}$$

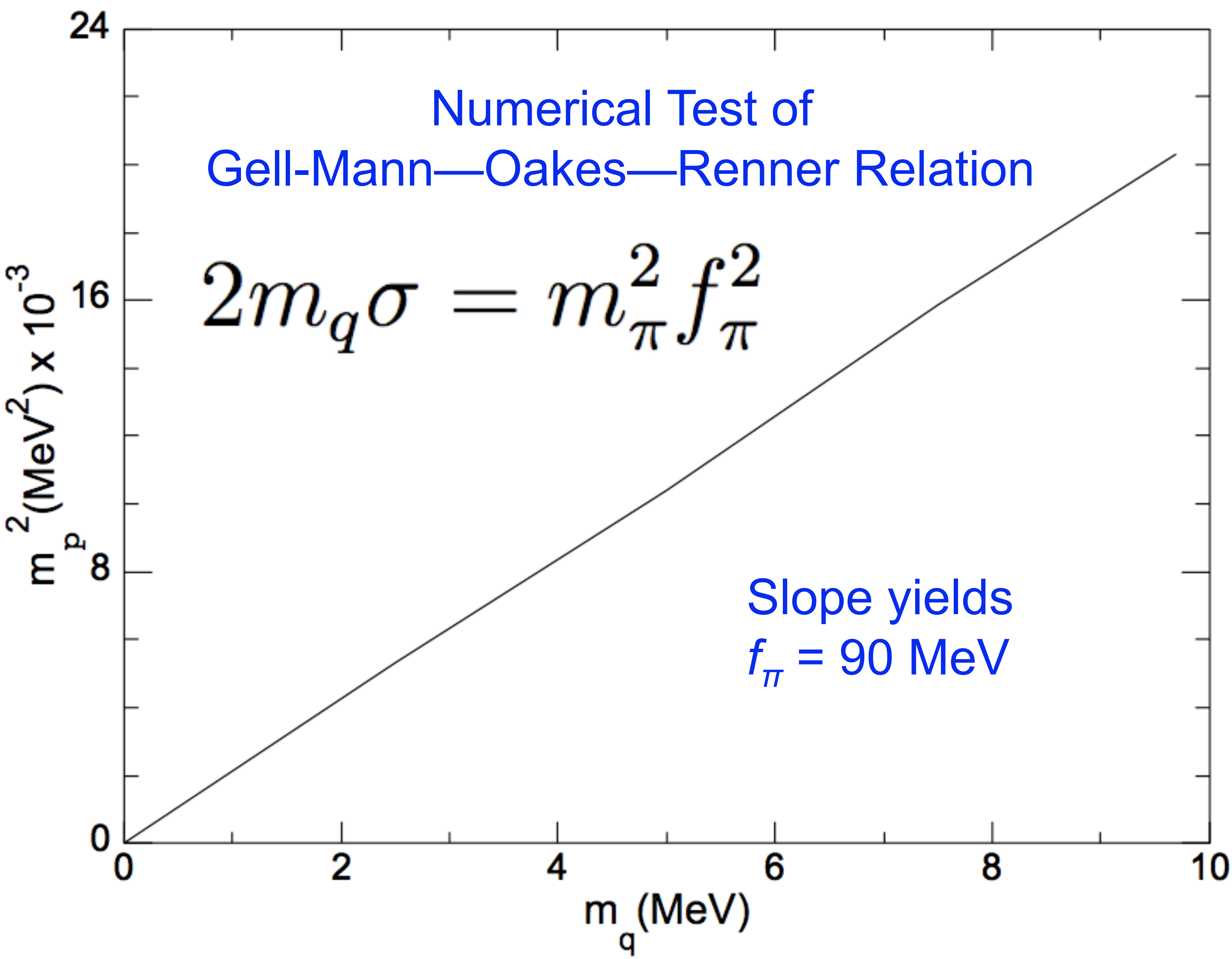
Gell-Mann—Oakes—Renner Relation

- Evaluate at small values $u \sim \sqrt{m_q/\sigma}$

$$\frac{\pi_l}{v} = -\frac{m_\pi^2 f_\pi^2}{2m_q\sigma}$$

- Pion satisfies Axial equation if $\pi_l = -v(z)$
- Giving the GOR Relation:

$$2m_q\sigma = m_\pi^2 f_\pi^2$$



Numerical Test of
Gell-Mann—Oakes—Renner Relation
$$2m_q\sigma = m_\pi^2 f_\pi^2$$
Slope yields
f_π = 90 MeV
m_p^2(MeV^2) x 10^{-3}
m_q(MeV)
0
2
4
6
8
10
0
8
16
24

Pion Results Recap

- ✓ Linear trajectory
- ✓ Pseudo-Goldstone Boson
- ✓ Good Correspondence to Measurement

Adding the Strange Quark

- Hard Wall Model:
  - *Shock, Wu 2006*   *Abidin, Carlson 2009*
- Non-flavor symmetric
  - $m_s \gg m_q \qquad \sigma_s \neq \sigma$
- Encoded in the VEV

$$X_0 = \frac{1}{2}\begin{pmatrix} v(z) & 0 & 0 \\ 0 & v(z) & 0 \\ 0 & 0 & v_s(z) \end{pmatrix}$$

  - Form for $v_s(z)$?

Strange VEV

- Same form, different parameters
  - *Sui, Wu, Yang 2010*
  - Good mass spectra
  - Does not satisfy dilaton equation of motion
- Solve dilaton/VEV EOM numerically
  - Mass spectra to be analyzed

Mass Matrices

- Covariant Derivative:

$$D_M X = \partial_M X - i\{A_M^a, X\} + i[V_M^a, X]$$

- Define $$\frac{1}{2} M_A^{a\,2} \delta^{ab} = Tr\{t^a, X_0\}\{t^b, X_0\}$$

$$\frac{1}{2} M_V^{a\,2} \delta^{ab} = -Tr[t^a, X_0][t^b, X_0]$$

- Vector and Axial EOMs:

$$-\Psi_n^{a\prime\prime} + \left(\frac{1}{4}\omega'^2 - \frac{1}{2}\omega'' + \frac{g_5^2 L^2}{z^2} M_\Psi^{a\,2}\right)\Psi_n^a = m_n^2 \Psi_n^a$$

Mass Matrices

a	Particle Type	${M_V^a}^2$	${M_A^a}^2$
$1,2,3$	isovector	→ 0	v^2
$4,5,6,7$	isodoublet	$\frac{1}{4}(v_s - v)^2$	$\frac{1}{4}(v + v_s)^2$
8	isosinglet	→ 0	$\frac{1}{3}(v^2 + 2v_s^2)$

- ✓ isovector same as previous work (ρ, a_1)
- Vector Sector
  - isovector and isosinglet are degenerate

Strange Pseudoscalars

- Equations of Motion:

$$e^{\phi}\partial_z\left(\frac{e^{-\phi}}{z}\partial_z\varphi_n\right)+\frac{g_5^2L^2}{z^3}(\xi^a(z)\pi_n-M_A^{a\,2}\varphi_n)=0$$

$$z^3e^{\phi}\partial_z\left(\frac{e^{-\phi}}{z^3}\partial_z\pi_n\right)-\left(\frac{m_X^2}{z^2}-\frac{\kappa L^2(M_A^{a\,2}-M_V^{a\,2})}{2z^2}\right)\pi_n+m_n^2\pi_n=m_n^2\xi^a(z)\varphi_n$$

- Where $2\xi^a(z)\delta_{ab}=\mathrm{Tr}\{t^a,\{t^b,X_0\}\}$

$$\xi^a(z)=\begin{cases} v; & a=1,2,3\\ v+v_s; & a=4,5,6,7\\ \frac{1}{3}(v+2v_s); & a=8.\end{cases}$$

Future Work

- Strange Pseudoscalars
  - Incompatible Representations?

- Mass Spectra

- Derive dilaton and scalar field from potential

References

- Kelley, Bartz, Kapusta: arXiv:1009.3009
- Gherghetta, Kapusta, Kelley: arXiv:0902.1998
- Sui, Wu, Yang: arXiv:1012.3518
- Shock, Wu: arXiv:hep-ph/0603142
- Abidin, Carlson: arXiv:0908.2452