Latest results on *PT* quantum theory

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Assumptions of quantum mechanics:

- causality
- locality
- relativistic invariance
- existence of a ground state
- conservation of probability (unitarity)
- positive real energies
- **Hermitian Hamiltonian**

The point of this talk:

Dirac Hermiticity is too strong an axiom of quantum mechanics!

$$H = H^\dagger$$

$\dagger$ means ***transpose* + *complex conjugate***

- guarantees real energy and conserved probability
- but … is a **mathematical** axiom and not a **physical** axiom of quantum mechanics

$$H = p^2 + ix^3$$

Many people looked at this model...

(1) C.-I Tan, R. Brower, M. Moshe, M. Furman, ...
(Reggeon field theory, the Pomeron, and all that)

(2) J. Cardy, G. Mussardo, M. Fisher, A. Zamolodchikov, ...
(Lee-Yang edge singularity)

$$H = p^2 + ix^3$$

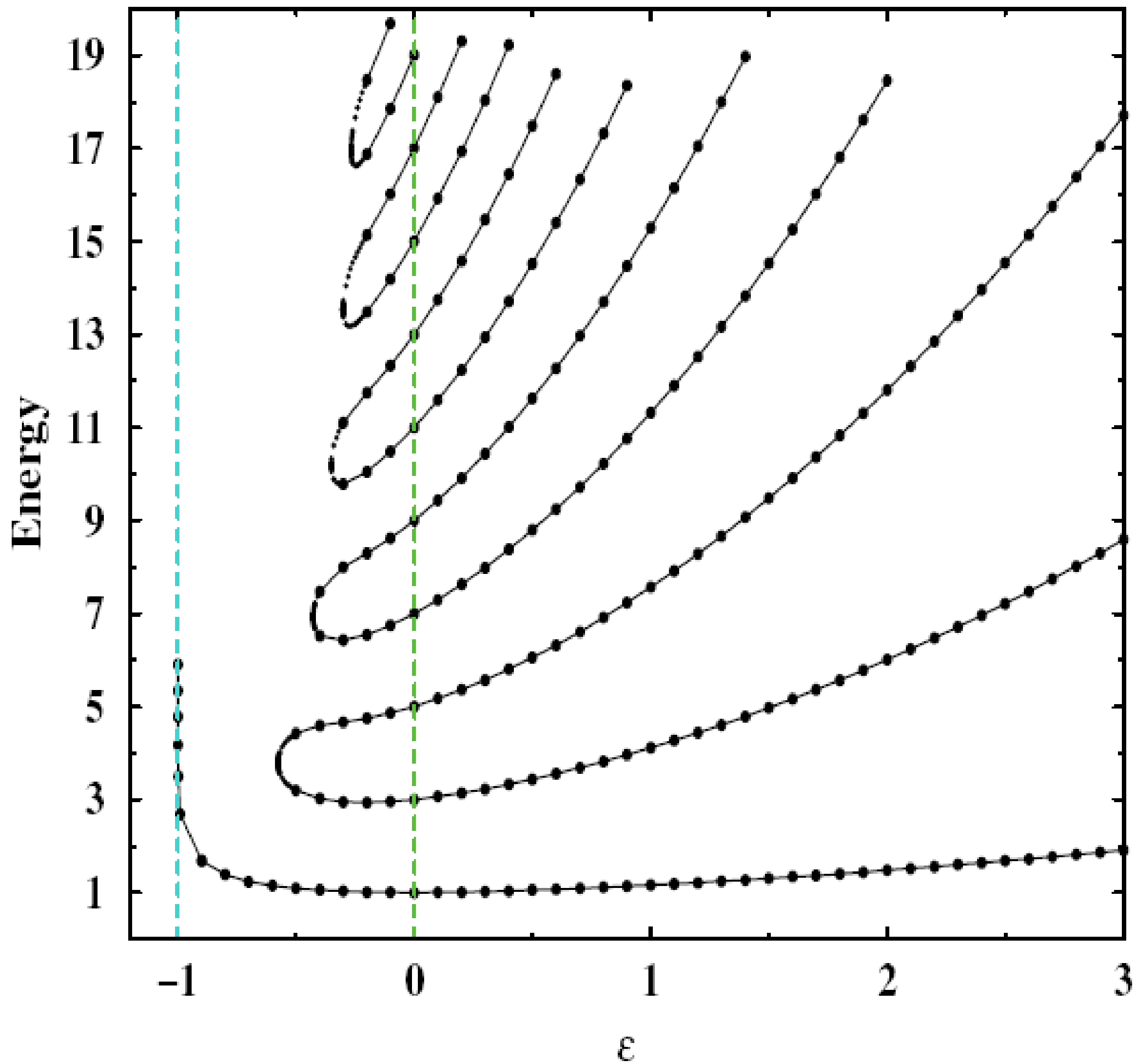
Wait a minute…
this Hamiltonian has
PT **symmetry!**

P = parity

T = time reversal

Perturbative solution: $H = p^2 + x^2(ix)^\epsilon \quad (\epsilon \text{ real})$

$$H = p^2 + x^2(ix)^\epsilon \quad (\epsilon \text{ real})$$



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How to prove that the eigenvalues are real

The proof is difficult! It uses techniques from conformal field theory and statistical mechanics:

(1) Bethe ansatz
(2) Monodromy group
(3) Baxter T-Q relation
(4) Functional Determinants

Other recent *PT* papers …

- K. Makris, R. El-Ganainy, D. Christodoulides, and Z. Musslimani, *Phyical Review Letters* **100**, 103904 (2008)
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- O. Bendix, R. Fleischmann, T. Kottos, and B. Shapiro, *Physical Review Letters* **103**, 030402 (2009)
- S. Longhi, *Physical Review Letters* **103**, 123601 (2009)
- A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides, *Physical Review Letters* **103**, 093902 (2009)

- H. Schomerus, *Physical Review Letters* **104**, 233601 (2010)
- S. Longhi, *Physical Review Letters* **105**, 013903 (2010)
- C. West, T. Kottos, T. Prosen, *Physical Review Letters* **104**, 054102 (2010)
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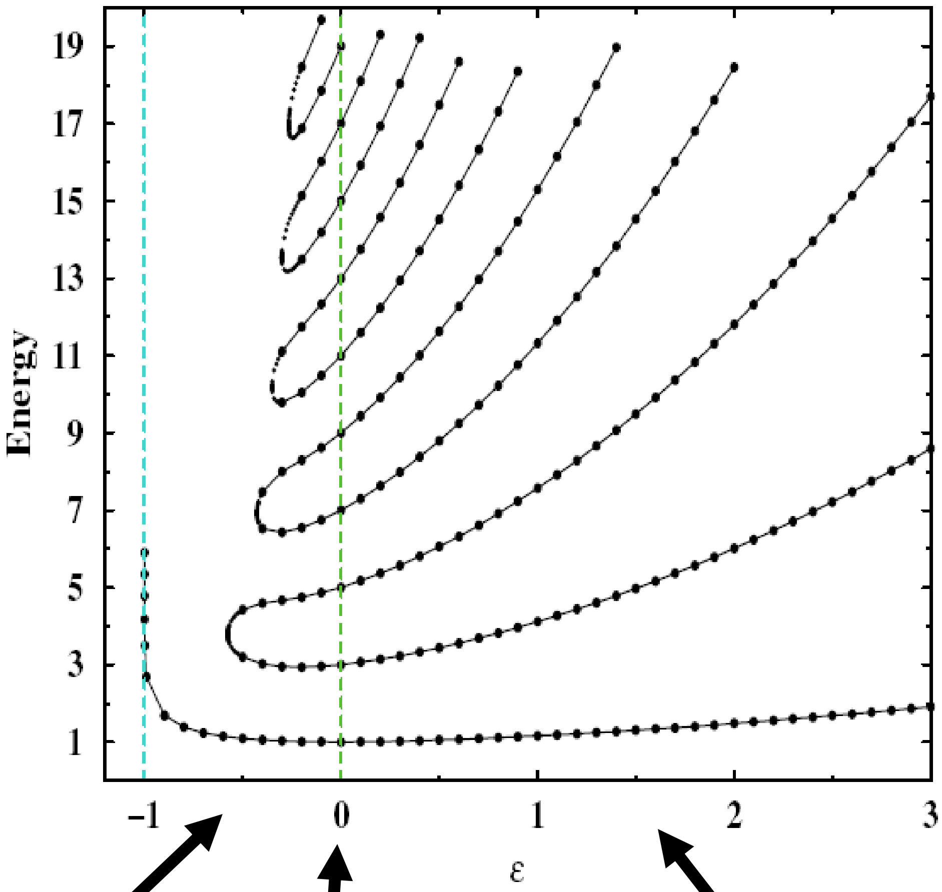
- C. Ruter, K. Makris, R. El-Ganainy, D. Christodoulides, M. Segev, and D. Kip, *Nature Physics* **6**, 192 (2010)

- Y. D. Chong, L. Ge, and A. D. Stone, *Physical Review Letters* **106**, 093902 (2011)
- Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, and D. N. Christodoulides, *Physical Review Letters* **106**, 213901 (2011)

$$H = p^2 + x^2(ix)^\epsilon \quad (\epsilon \text{ real})$$



Region of ***broken***
PT symmetry

***PT* Boundary**

Region of ***unbroken***
PT symmetry

Broken ***P***arro***T*** Unbroken ***P***arro***T***

Broken ***PT*** **symmetry in Paris**

The *PT* Boundary is a phase transition – at the *classical* level

(Ask me after the talk if you're interested!)

OK, so the eigenvalues are real …
But is this quantum mechanics??

- Probabilistic interpretation??
- Hilbert space with a positive metric??
- Unitarity??

The Hamiltonian determines its own adjoint

$$[\mathcal{C}, \mathcal{PT}] = 0,$$

$$[\mathcal{C}^2 = 1],$$

$$[\mathcal{C}, H] = 0$$

Replace $\dagger$ by $\mathcal{CPT}$

Unitarity

With respect to the ***CPT*** adjoint the theory has UNITARY time evolution.

Norms are strictly positive!
Probability is conserved!

OK, so we have unitarity…
But is *PT* quantum mechanics useful??

- Revives quantum theories that were thought to be dead
- Observed experimentally

Lee Model

$$V \to N + \theta, \qquad N + \theta \to V.$$

$$H = H_0 + g_0 H_1,$$

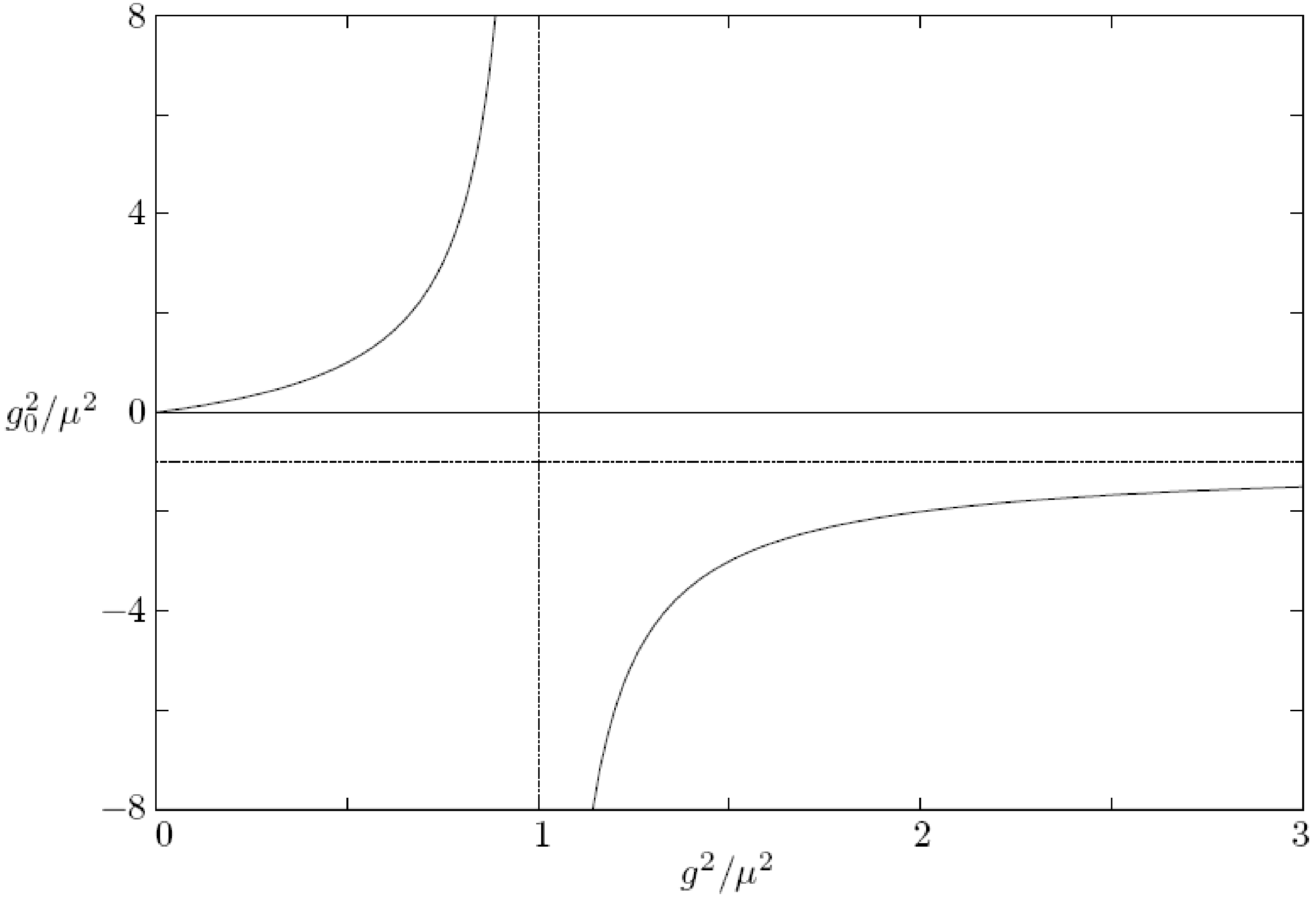
$$H_0 = m_{V_0} V^\dagger V + m_N N^\dagger N + m_\theta a^\dagger a,$$

$$H_1 = V^\dagger N a + a^\dagger N^\dagger V.$$

T. D. Lee, Phys. Rev. **95**, 1329 (1954)

G. Källén and W. Pauli, Dan. Mat. Fys. Medd. **30**, No. 7 (1955)

The problem with the Lee Model:



$$g_0^2 = g^2 / \left(1 - g^2/\mu^2\right)$$

MR0076639 (17,927d) 81.0X
Källén, G.; Pauli, W.
On the mathematical structure of T. D. Lee's model of a renormalizable field theory.
Danske Vid. Selsk. Mat.-Fys. Medd. **30** (1955), *no. 7,* 23 *pp.*

Lee [Phys. Rev. (2) **95** (1954), 1329–1334; MR0064658 (16,317b)] has recently suggested perhaps the first non-trivial model of a field-theory which can be explicitly solved. Three particles (V, N and θ) are coupled, the explicit solution being secured by allowing reactions $V \rightleftarrows N+\theta$ but forbidding $N \rightleftarrows V+\theta$. The theory needs conventional mass and charge renormalizations which likewise can be explicitly calculated. The renormalized coupling constant g is connected to the unrenormalized constant g_0 by the relation $g^2/{g_0}^2 = 1 - Ag^2$, where A is a divergent integral. This can be made finite by a introducing a cut-off.

The importance of Lee's result lies in the fact that Schwinger (unpublished) had already proved on very general principles, that the ratio $g^2/{g_0}^2$ should lie between zero and one. [For published proofs of Schwinger's result, see Umezawa and Kamefuchi, Progr. Theoret. Phys. **6** (1951), 543–558; MR0046306 (13,713d); Källén, Helv. Phys. Acta **25** (1952), 417–434; MR0051156 (14,435l); Lehmann, Nuovo Cimento (9) **11** (1954), 342–357; MR0072756 (17,332e); Gell-Mann and Low, Phys. Rev. (2) **95** (1954), 1300–1312; MR0064652 (16,315e)]. The results of Lee and Schwinger can be reconciled only if (i) there is a cut-off in Lee's theory and (ii) if g lies below a critical value $g_{\rm crit}$. The present paper is devoted to investigation of physical consequences if these two conditions are not satisfied.

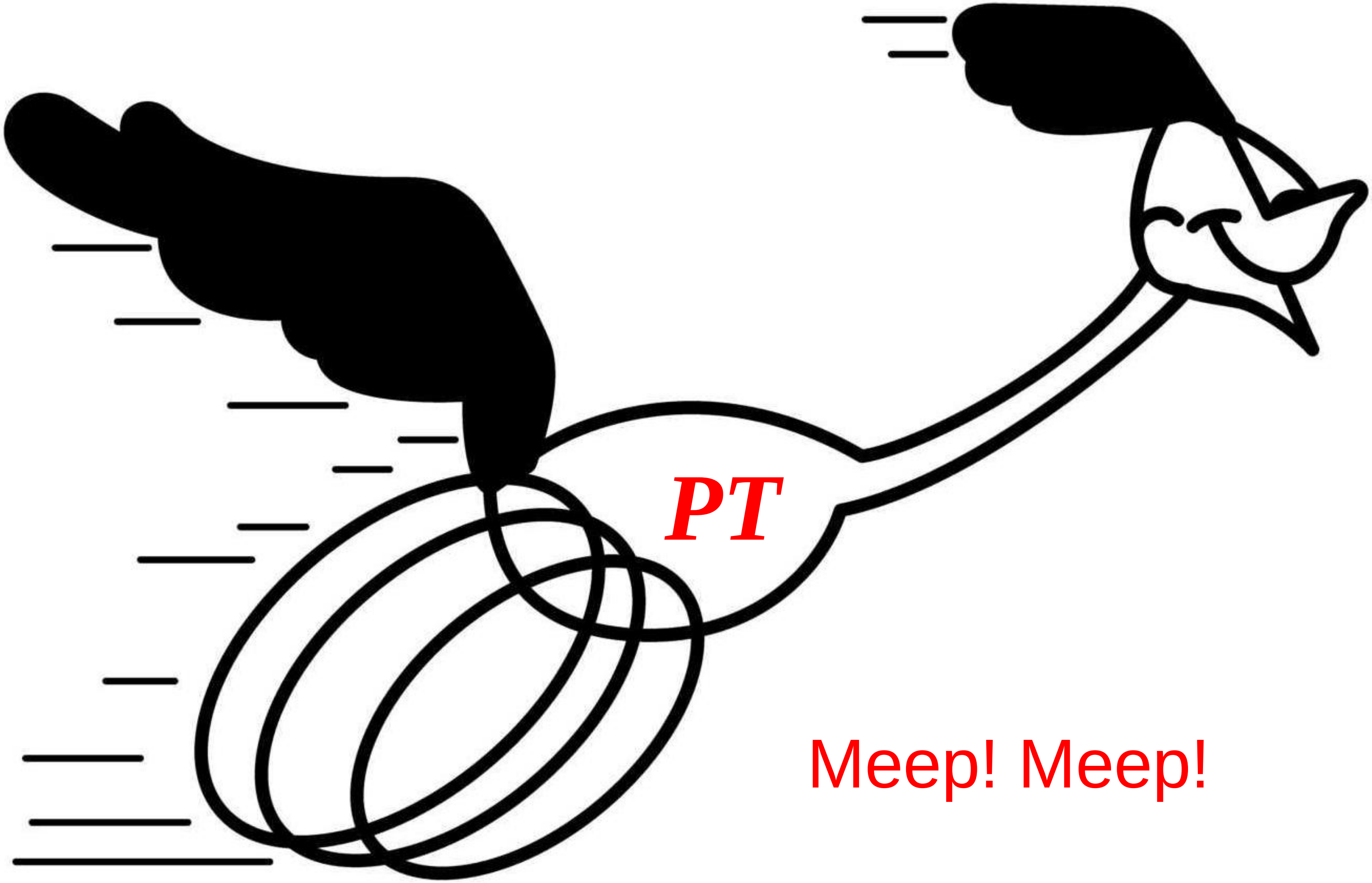
The authors discover the remarkable result that if $g > g_{\rm crit}$ there is exactly one new eigenstate for the physical V-particle having an energy that is below the mass of the normal V-particle. It is further shown that the S-matrix for Lee's theory is not unitary when $g > g_{\rm crit}$ and that the probability for an incoming V-particle in the normal state and a θ-meson, to make a transition to an outgoing V-particle in the new ("ghost") state, must be negative if the sum of all transition probabilities for the in-coming state shall add up to one. The possible implication of Källén and Pauli's results for quantum-electrodynamics, where in perturbation theory $(e/e_0)^2$ has a behaviour similar to $(g/g_0)^2$ in Lee's theory, need not be stressed.

Reviewed by *A. Salam*

“A non-Hermitian Hamiltonian is unacceptable partly because it may lead to complex energy eigenvalues, but chiefly because it implies a non-unitary S matrix, which fails to conserve probability and makes a hash of the physical interpretation.”

G. Barton, *Introduction to Advanced Field Theory* (John Wiley & Sons, New York, 1963)

PT quantum mechanics to the rescue…



Meep! Meep!

Pais-Uhlenbeck action

$$I = \frac{\gamma}{2}\int dt\,\left[\ddot{z}^2 - \left(\omega_1^2+\omega_2^2\right)\dot{z}^2 + \omega_1^2\omega_2^2 z^2\right]$$

Gives a fourth-order field equation:

$$z''''(t) + (\omega_1^2+\omega_2^2)z''(t) + \omega_1^2\omega_2^2 z(t) = 0$$

The problem: A fourth-order field equation gives a propagator like

$$G(E) = \frac{1}{(E^2 + m_1^2)(E^2 + m_2^2)}$$

$$G(E) = \frac{1}{m_2^2 - m_1^2}\left(\frac{1}{E^2 + m_1^2} - \frac{1}{E^2 + m_2^2}\right)$$

GHOST!

Two possible realizations…

(I) If a_1 and a_2 annihilate the 0-particle state $|\Omega\rangle$,

$$a_1|\Omega\rangle = 0, \qquad a_2|\Omega\rangle = 0,$$

then the energy spectrum is real and bounded below. The state $|\Omega\rangle$ is the ground state of the theory and it has zero-point energy $\frac{1}{2}(\omega_1 + \omega_2)$. The problem with this realization is that the excited state $a_2^\dagger|\Omega\rangle$, whose energy is ω_2 above ground state, has a *negative Dirac norm* given by $\langle\Omega|a_2 a_2^\dagger|\Omega\rangle$.

(II) If a_1 and $a_2^\dagger$ annihilate the 0-particle state $|\Omega\rangle$,

$$a_1|\Omega\rangle = 0, \qquad a_2^\dagger|\Omega\rangle = 0,$$

then the theory is free of negative-norm states. However, this realization has a different and equally serious problem; namely, that the energy spectrum is unbounded below.

There is another realization as well!

The Hamiltonian is not Dirac Hermitian, but it is ***PT*** symmetric. We can calculate the ***C*** operator exactly. Norm is positive and the spectrum is bounded below. This suggests how Pauli-Villars works.

No-ghost theorem for the fourth-order derivative Pais-Uhlenbeck model, CMB and P. Mannheim, *Physical Review Letters* **100**, 110402 (2008)

CMB and P. Mannheim, *Physical Review D* **78**, 025002 (2008)

Laboratory verification using table-top optics experiments!

Observing ***PT*** symmetry using optical wave guides:

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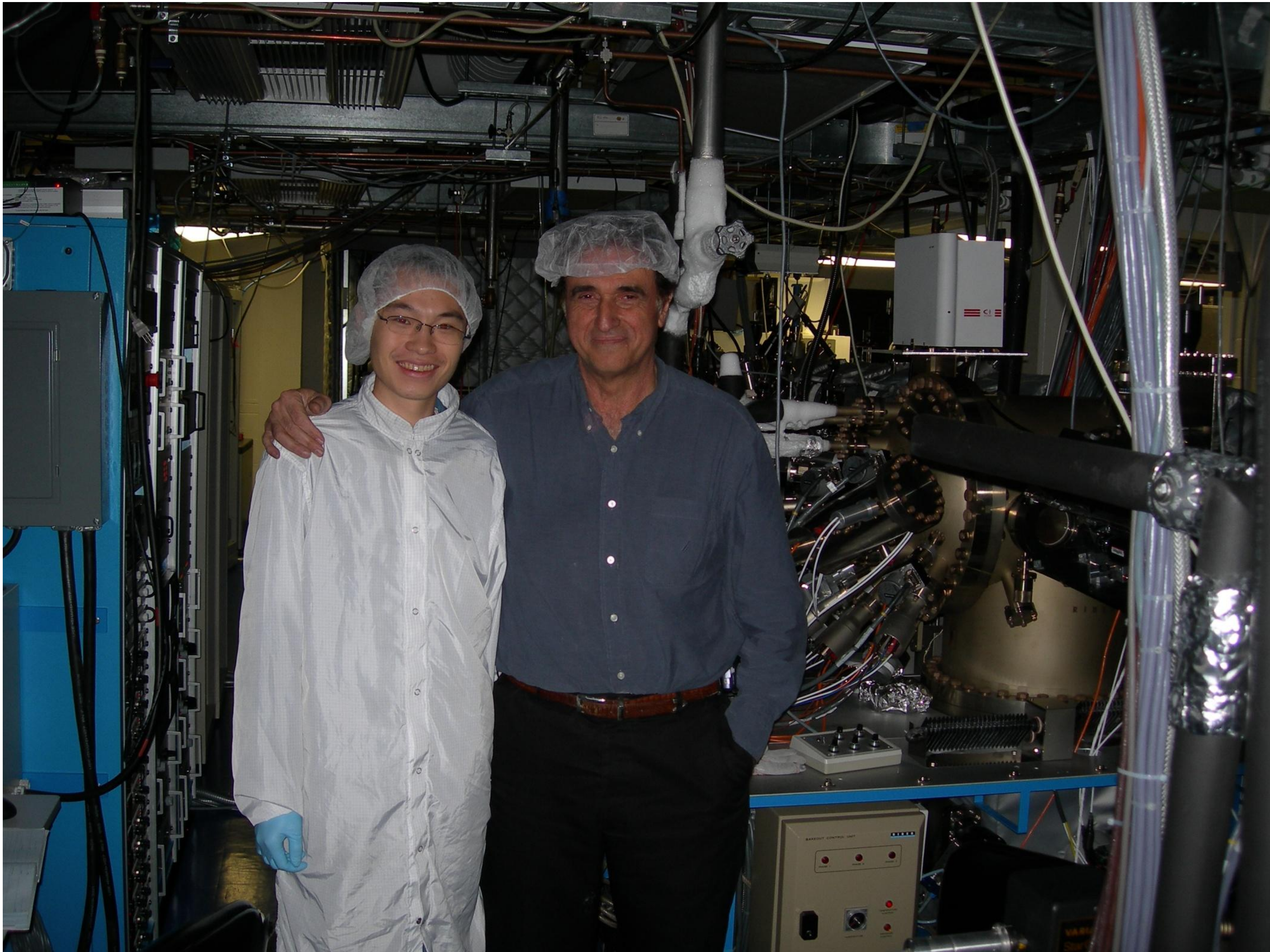
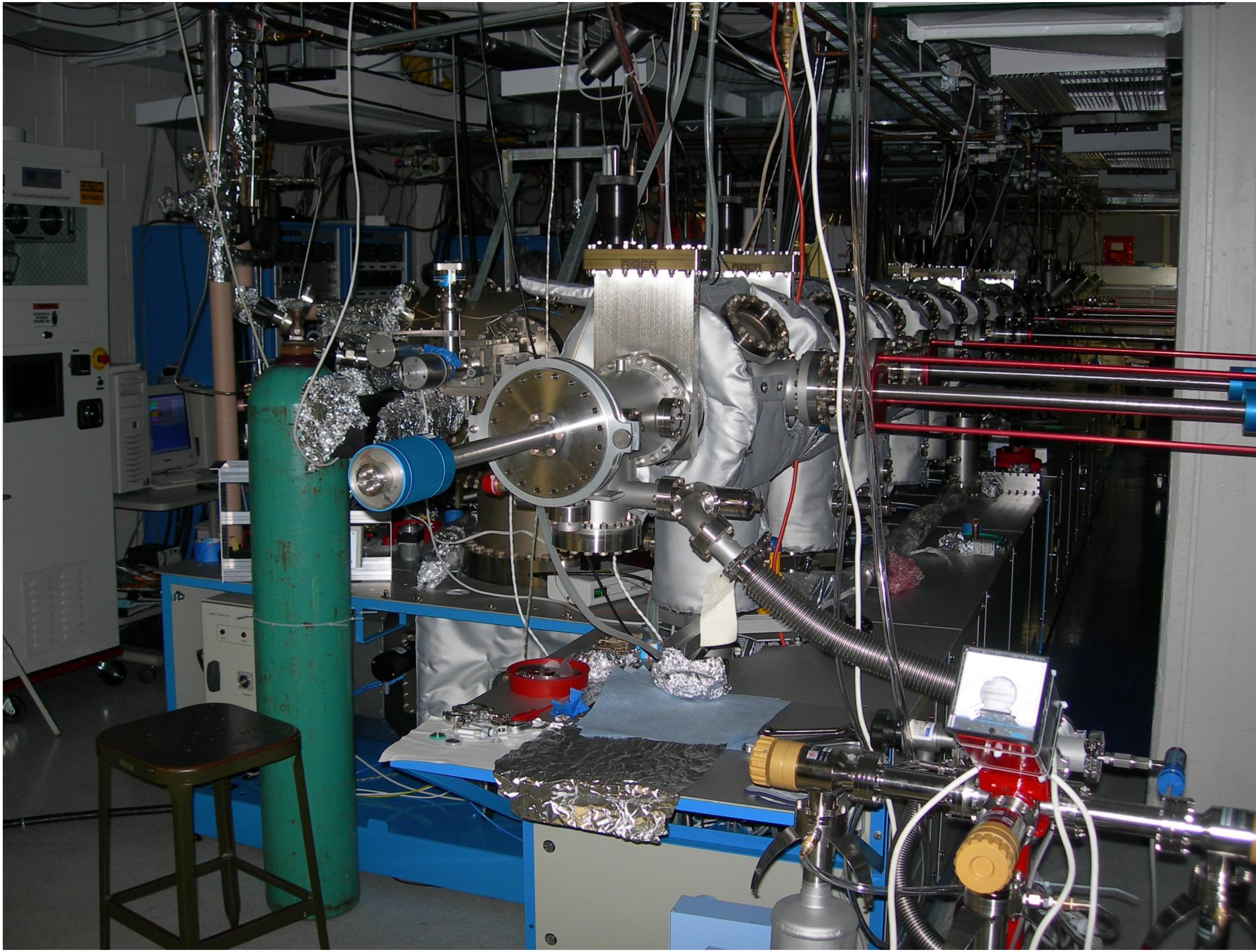
- K. Makris, R. El-Ganainy, D. Christodoulides, and Z. Musslimani, *Physical Review Letters* **100**, 103904 (2008)
- A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides, *Physical Review Letters* **103**, 093902 (2009)
- C. E. Ruter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, *Nature Physics* **6**, 192 (2010)

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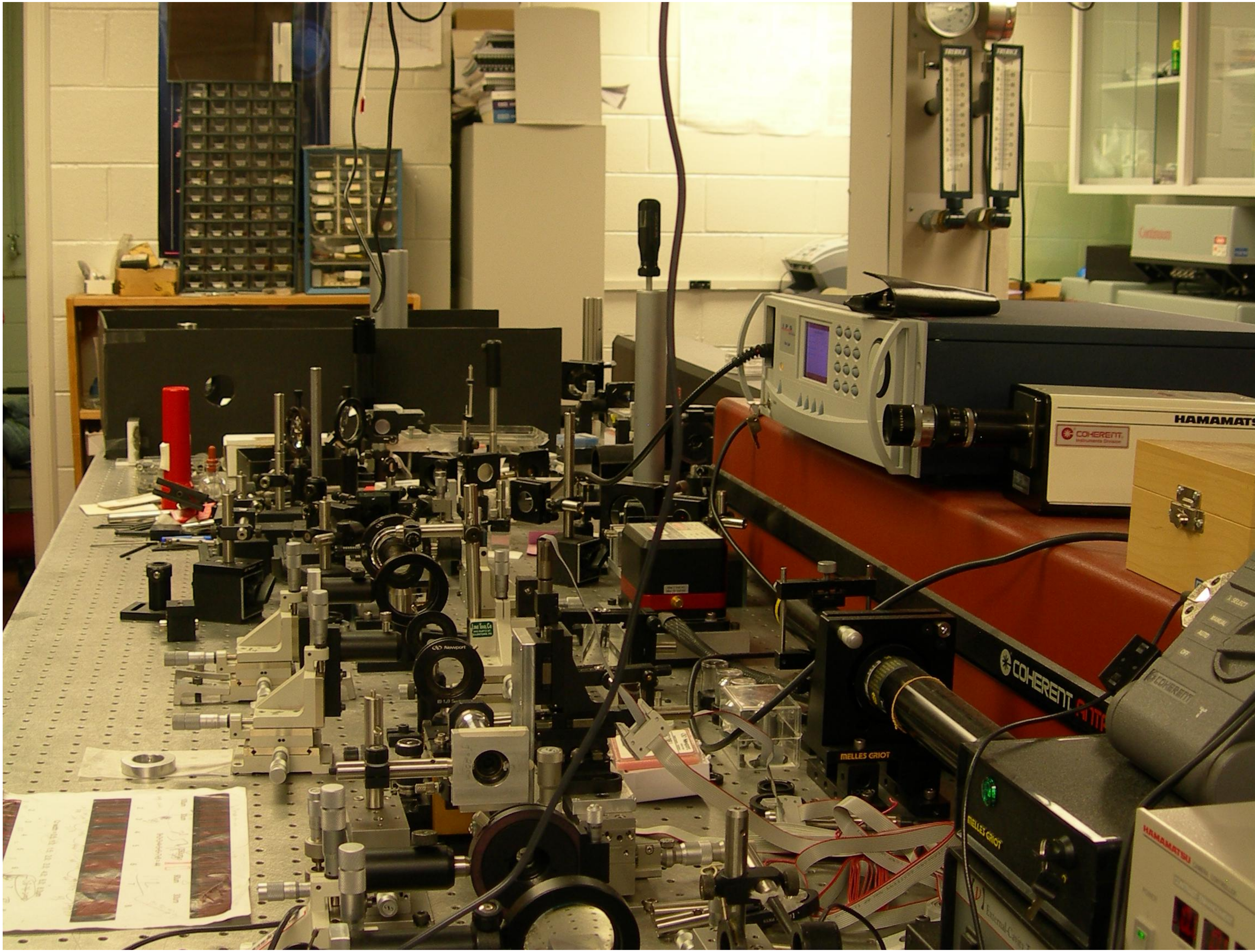
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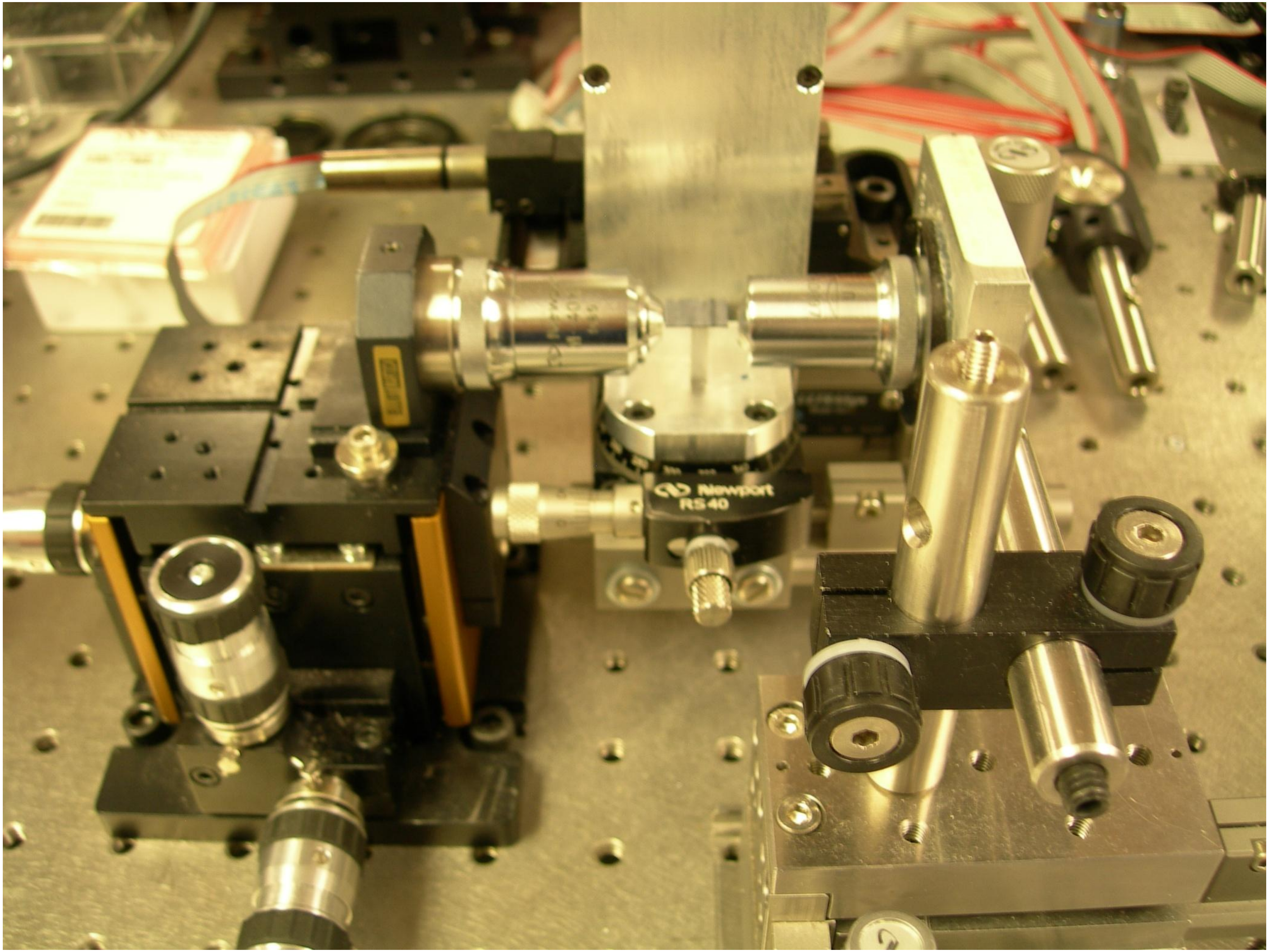
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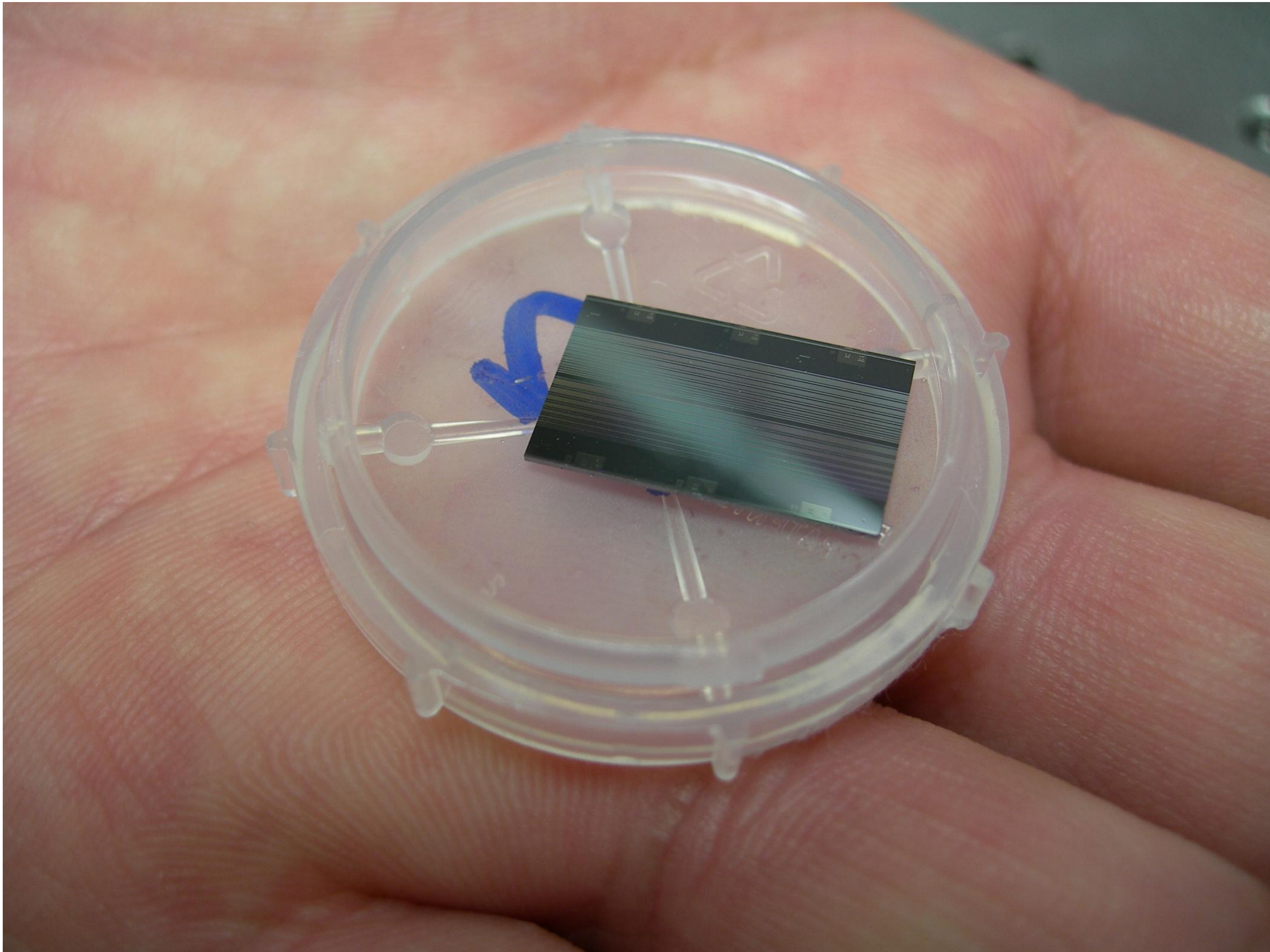
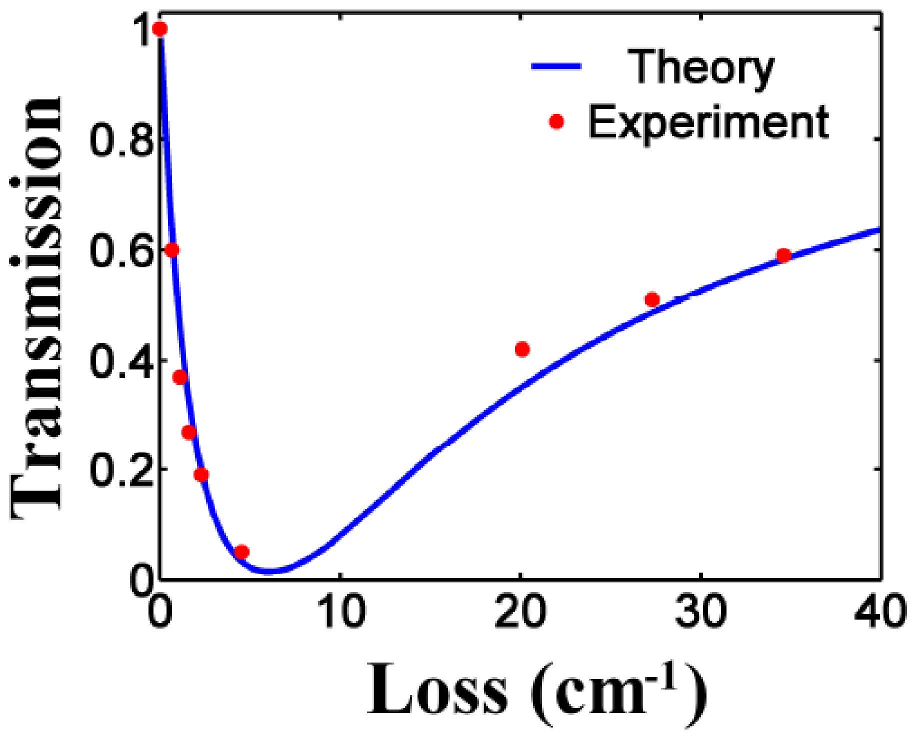


The observed *PT* phase transition

Figure 4: Experimental observation of spontaneous passive $\mathcal{PT}$-symmetry breaking. Output transmission of a passive $\mathcal{PT}$ complex system as the loss in the lossy waveguide arm is increased. The transmission attains a minimum at 6 cm^{-1}.



nature physics

LETTERS

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Observation of parity–time symmetry in optics

Christian E. Rüter[1], Konstantinos G. Makris[2], Ramy El-Ganainy[2], Demetrios N. Christodoulides[2], Mordechai Segev[3] and Detlef Kip[1]★

One of the fundamental axioms of quantum mechanics is associated with the Hermiticity of physical observables[1]. In the case of the Hamiltonian operator, this requirement not only implies real eigenenergies but also guarantees probability conservation. Interestingly, a wide class of non-Hermitian Hamiltonians can still show entirely real spectra. Among these are Hamiltonians respecting parity–time (*PT*) symmetry[2–7]. Even though the Hermiticity of quantum observables was never in doubt, such concepts have motivated discussions on several fronts in physics, including quantum field theories[8], non-Hermitian Anderson models[9] and open quantum systems[10,11], to mention a few. Although the impact of *PT* symmetry in these fields is still debated, it has been recently realized that optics can provide a fertile ground where *PT*-related notions can be implemented and experimentally investigated[12–15]. In this letter we report the first observation of the behaviour of a *PT* optical coupled system that judiciously involves a complex index potential. We observe both spontaneous *PT* symmetry breaking and power oscillations violating left–right symmetry. Our results may pave the way towards a new class of *PT*-synthetic materials with intriguing and unexpected properties that rely on non-reciprocal light propagation and tailored transverse energy flow.

($\varepsilon > \varepsilon_{th}$), the spectrum ceases to be real and starts to involve imaginary eigenvalues. This signifies the onset of a spontaneous *PT* symmetry-breaking, that is, a 'phase transition' from the exact to broken-*PT* phase[7,20].

In optics, several physical processes are known to obey equations that are formally equivalent to that of Schrödinger in quantum mechanics. Spatial diffraction and temporal dispersion are perhaps the most prominent examples. In this work we focus our attention on the spatial domain, for example optical beam propagation in *PT*-symmetric complex potentials. In fact, such *PT* 'optical potentials' can be realized through a judicious inclusion of index guiding and gain/loss regions[7,12–14]. Given that the complex refractive-index distribution $n(x) = n_R(x) + i n_I(x)$ plays the role of an optical potential, we can then design a *PT*-symmetric system by satisfying the conditions $n_R(x) = n_R(-x)$ and $n_I(x) = -n_I(-x)$.

In other words, the refractive-index profile must be an even function of position x whereas the gain/loss distribution should be odd. Under these conditions, the electric-field envelope E of the optical beam is governed by the paraxial equation of diffraction[13]:

$$i\frac{\partial E}{\partial z} + \frac{1}{2k}\frac{\partial^2 E}{\partial x^2} + k_0[n_R(x) + i n_I(x)]E = 0$$

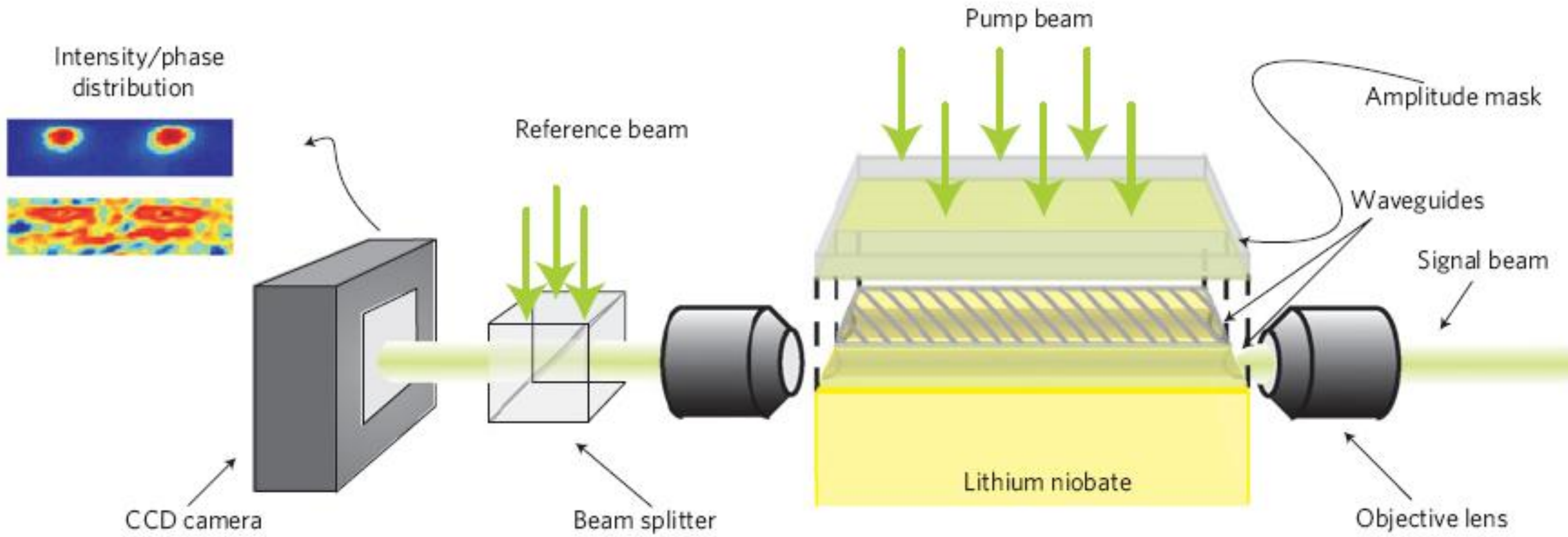

Figure 2 | Experimental set-up. An Ar^+ laser beam (wavelength 514.5 nm) is coupled into the arms of the structure fabricated on a photorefractive $LiNbO_3$ substrate. An amplitude mask blocks the pump beam from entering channel 2, thus enabling two-wave mixing gain only in channel 1. A CCD camera is used to monitor both the intensity and phases at the output.

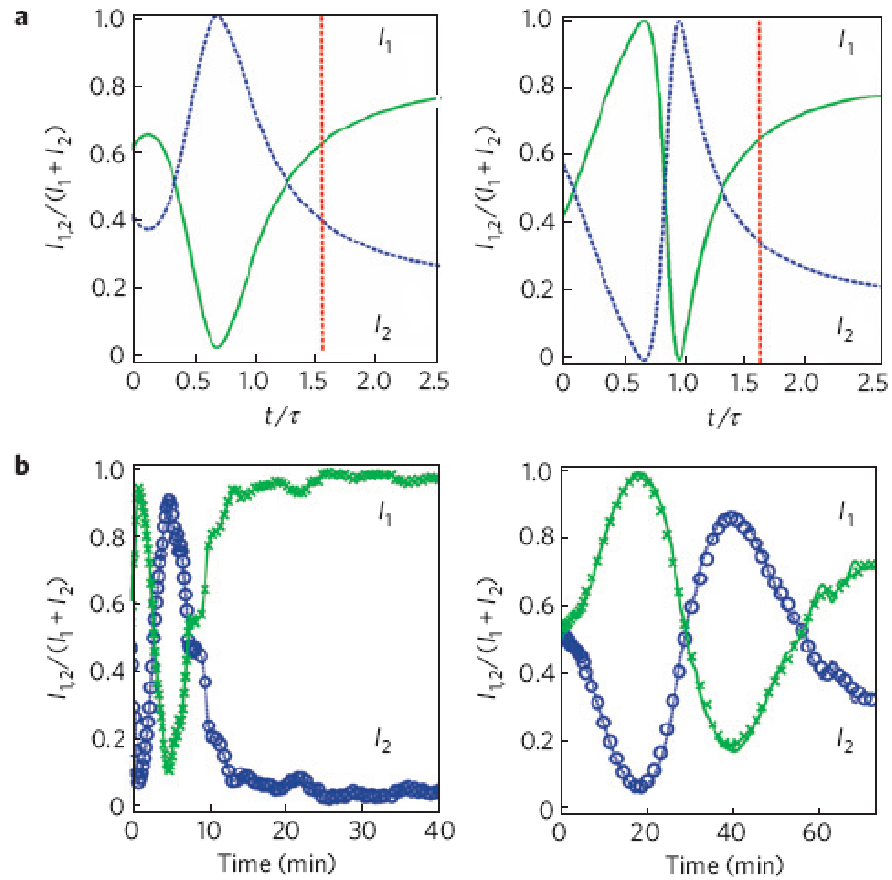

Figure 3 | Computed and experimentally measured response of a *PT*-symmetric coupled system. a, Numerical solution of the coupled equations (1) describing the *PT*-symmetric system. The left (right) panel shows the situation when light is coupled into channel 1 (2). Red dashed lines mark the symmetry-breaking threshold. Above threshold, light is predominantly guided in channel 1 experiencing gain, and the intensity of channels 1 and 2 depends solely on the magnitude of the gain. **b**, Experimentally measured (normalized) intensities at the output facet during the gain build-up as a function of time.

Another experiment...

"Enhanced magnetic resonance signal of spin-polarized Rb Atoms near surfaces of coated cells"
K. F. Zhao, M. Schaden, and Z. Wu
Physical Review A **81**, 042903 (2010)

And another experiment...

Spontaneous PT-symmetry breakdown in superconducting weak links

N. M. Chtchelkatchev, A. A. Golubov, T. I. Baturina, V. M. Vinokur
(arXiv:1008.3590v2 [cond-mat.supr-con], submitted on 21 Aug 2010 (v1), last revised 1 Sep 2010(v2))

Abstract: We formulate a description of transport in a superconducting weak link in terms of the non-Hermitian quantum mechanics. We find that the applied electric field exceeding a certain critical value change the topological structure of the effective non-Hermitian Hamiltonian of the weak link in the Hilbert space causing the parity reflection – time reversal symmetry (PT-symmetry) breakdown. We derive the expression of the critical electric field and show that the PT-symmetry breakdown gives rise to the switching instability in the current-voltage characteristic of the weak link. Taking into account superconducting fluctuations we quantitatively describe the experimentally observed differential resistance of the weak link in the vicinity of the critical temperature.

And yet another...

Spontaneous Parity--Time Symmetry Breaking and Stability of Solitons in Bose-Einstein Condensates

Zhenya Yan, Bo Xiong, Wu-Ming Liu

(arXiv:1009.4023v1 [cond-mat.quant-gas], submitted on 21 Sep 2010)

Abstract: We report explicitly a novel family of exact PT-symmetric solitons and further study their spontaneous PT symmetry breaking, stabilities and collisions in Bose-Einstein condensates trapped in a PT-symmetric harmonic trap and a Hermite-Gaussian gain/loss potential. We observe the significant effects of mean-field interaction by modifying the threshold point of spontaneous PT symmetry breaking in Bose-Einstein condensates. Our scenario provides a promising approach to study PT-related universal behaviors in non-Hermitian quantum system based on the manipulation of gain/loss potential in Bose-Einstein condensates.

Armchair graphene nanoribbons: $\mathcal{PT}$-symmetry breaking and exceptional points without dissipation

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We consider a single layer graphene nanoribbon with armchair edges in a longitudinally constant external potential and point out that its transport properties can be described by means of an effective non-Hermitian Hamiltonian. We show that this system has some features typical of dissipative systems, namely the presence of exceptional points and of $\mathcal{PT}$ symmetry breaking, although it is not dissipative.

PACS numbers: 72.80.Vp , 11.30.Er , 03.65.-w

PRL **106,** 093902 (2011) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2011

$\mathcal{PT}$-Symmetry Breaking and Laser-Absorber Modes in Optical Scattering Systems

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(Received 30 August 2010; revised manuscript received 27 January 2011; published 2 March 2011)

Using a scattering matrix formalism, we derive the general scattering properties of optical structures that are symmetric under a combination of parity and time reversal ($\mathcal{PT}$). We demonstrate the existence of a transition between $\mathcal{PT}$-symmetric scattering eigenstates, which are norm preserving, and symmetry-broken pairs of eigenstates exhibiting net amplification and loss. The system proposed by Longhi [Phys. Rev. A **82**, 031801 (2010).], which can act simultaneously as a laser and coherent perfect absorber, occurs at discrete points in the broken-symmetry phase, when a pole and zero of the S matrix coincide.

DOI: 10.1103/PhysRevLett.106.093902 PACS numbers: 42.25.Bs, 42.25.Hz, 42.55.Ah

$\mathcal{PT}$-symmetry breaking in complex nonlinear wave equations and their deformations

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ABSTRACT: We investigate complex versions of the Korteweg-deVries equations and an Ito type nonlinear system with two coupled nonlinear fields. We systematically construct rational, trigonometric/hyperbolic, elliptic and soliton solutions for these models and focus in particular on physically feasible systems, that is those with real energies. The reality of the energy is usually attributed to different realisations of an antilinear symmetry, as for instance $\mathcal{PT}$-symmetry. It is shown that the symmetry can be spontaneously broken in two alternative ways either by specific choices of the domain or by manipulating the parameters in the solutions of the model, thus leading to complex energies. Surprisingly the reality of the energies can be regained in some cases by a further breaking of the symmetry on the level of the Hamiltonian. In many examples some of the fixed points in the complex solution for the field undergo a Hopf bifurcation in the $\mathcal{PT}$-symmetry breaking process. By employing several different variants of the symmetries we propose many classes of new invariant extensions of these models and study their properties. The reduction of some of these models yields complex quantum mechanical models previously studied.

PT-Symmetric Oligomers: Analytical Solutions, Linear Stability and Nonlinear Dynamics

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In the present work we focus on the case of (few-site) configurations respecting the PT-symmetry. We examine the case of such "oligomers" with not only 2-sites, as in earlier works, but also the cases of 3- and 4-sites. While in the former case of recent experimental interest, the picture of existing stationary solutions and their stability is fairly straightforward, the latter cases reveal a considerable additional complexity of solutions, including ones that exist past the linear PT-breaking point in the case of the trimer, and more complex, even asymmetric solutions in the case of the quadrimer with nontrivial spectral and dynamical properties. Both the linear stability and the nonlinear dynamical properties of the obtained solutions are discussed.

Unidirectional Invisibility Induced by $\mathcal{PT}$-Symmetric Periodic Structures

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(Received 11 January 2011; revised manuscript received 17 March 2011; published 25 May 2011)

Parity-time ($\mathcal{PT}$) symmetric periodic structures, near the spontaneous $\mathcal{PT}$-symmetry breaking point, can act as unidirectional invisible media. In this regime, the reflection from one end is diminished while it is enhanced from the other. Furthermore, the transmission coefficient and phase are indistinguishable from those expected in the absence of a grating. The phenomenon is robust even in the presence of Kerr nonlinearities, and it can also effectively suppress optical bistabilities.

DOI: 10.1103/PhysRevLett.106.213901 PACS numbers: 42.25.Bs, 03.65.Nk, 11.30.Er

Making the Case for Conformal Gravity

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Abstract. We review some recent developments in the conformal gravity theory that has been advanced as a candidate alternative to standard Einstein gravity. As a quantum theory the conformal theory is both renormalizable and unitary, with unitarity being obtained because the theory is a PT symmetric rather than a Hermitian theory. We show that in the theory there can be no a priori classical curvature, with all curvature having to result from quantization. In the conformal theory gravity requires no independent quantization of its own, with it being quantized solely by virtue of its being coupled to a quantized matter source. Moreover, because it is this very coupling that fixes the strength of the gravitational field commutators, the gravity sector zero-point energy density and pressure fluctuations are then able to identically cancel the zero-point fluctuations associated with the matter sector. In addition, we show that when the conformal symmetry is spontaneously broken, the zero-point structure automatically readjusts so as to identically cancel the cosmological constant term that dynamical mass generation induces. We show that the macroscopic classical theory that results from the quantum conformal theory incorporates global physics effects that provide for a detailed accounting of a comprehensive set of 110 galactic rotation curves with no adjustable parameters other than the galactic mass to light ratios, and with the need for no dark matter whatsoever. With these global effects eliminating the need for dark matter, we see that invoking dark matter in galaxies could potentially be nothing more than an attempt to describe global physics effects in purely local galactic terms. Finally, we review some recent work by 't Hooft in which a connection between conformal gravity and Einstein gravity has been found.

Keywords: conformal gravity, quantum gravity, cosmological constant problem
PACS: 04.60.-m, 04.50.Kd, 04.90.+e

Impact of a global quadratic potential on galactic rotation curves

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Abstract

We have made a conformal gravity fit to an available sample of 110 spiral galaxies, and report here on the 20 of those galaxies whose rotation curve data points extend the furthest from galactic centers. We identify the impact on the 20 galaxy data set of a universal de Sitter-like potential term $V(r) = -\kappa c^2 r^2/2$ that is induced by inhomogeneities in the cosmic background. This quadratic term accompanies a universal linear potential term $V(r) = \gamma_0 c^2 r/2$ that is associated with the cosmic background itself. We find that when these two potential terms are taken in conjunction with the contribution generated by the local luminous matter within the galaxies, the conformal theory is able to account for the rotation curve systematics that is observed in the entire 110 galaxy sample, without the need for any dark matter whatsoever. With the two universal coefficients being found to be of global magnitude, viz. $\kappa = 9.54 \times 10^{-54}$ cm^{-2} and $\gamma_0 = 3.06 \times 10^{-30}$cm^{-1}, our study suggests that invoking the presence of dark matter may be nothing more than an attempt to describe global effects in purely local galactic terms. With the quadratic potential term having negative sign, galaxies are only able to support bound orbits up to distances of order $\gamma_0/\kappa = 3.21 \times 10^{23}$ cm, with global physics thus imposing a natural limit on the size of galaxies.

Interesting recent developments...

(1) K. Jones-Smith and H. Mathur (Case Western): ***PT***-symmetric Dirac equation and neutrino oscillations
(2) G. ‘t Hooft: cosmological models
(3) J. Moffat (Perimeter): cosmological constant
(4) M. de Kieviet (Heidelberg): experimental observations of ***PT***-symmetric quantum brachistochrone
(5) P. Dorey, C. Dunning, R. Tateo: ODE-IM correspondence
(6) D. Masoero (Trieste): cubic ***PT*** oscillator and Painleve I; quartic ***PT*** oscillator and Painleve II
(7) S. Longhi (Milan): Bloch waves
(8) Classical ***PT***-symmetric equations: KdV, Camassa-Holm, Sine-Gordon, Boussinesq, Lotka-Volterra, Euler’s; complex extension of chaos
(9) Complex quantum mechanics: Complex correspondence principle
(10) A. LeClair (Cornell): Generalization of spin and statistics
(11) H. Schomerous (Lancaster): ***PT*** quantum noise
(12) D. Christodoulides (Florida): Random ***PT*** dimers
(13) ... And lots more!

OK, but how do we interpret a non-Hermitian Hamiltonian??

Solve the quantum brachistochrone problem…

Quantum brachistochrone

$$|\psi_I\rangle \to |\psi_F\rangle = e^{-iHt/\hbar}|\psi_I\rangle$$

Constraint: $\omega = E_{\max} - E_{\min}$

$$|\psi_I\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |\psi_F\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

Hermitian case

$$H = \begin{pmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{pmatrix} \qquad (r,\ s,\ u,\ \theta \text{ real}) \qquad \omega^2 = (s-u)^2 + 4r^2$$

$$|\psi_F\rangle = e^{-iH\tau/\hbar}|\psi_I\rangle$$

becomes:

$$\begin{pmatrix} a \\ b \end{pmatrix} = e^{-\frac{1}{2}i(s+u)t/\hbar} \begin{pmatrix} \cos\frac{\omega t}{2\hbar} - i\frac{s-u}{\omega}\sin\frac{\omega t}{2\hbar} \\ -i\frac{2r}{\omega}e^{i\theta}\sin\frac{\omega t}{2\hbar} \end{pmatrix}$$

$$t = \frac{2\hbar}{\omega}\arcsin\frac{\omega|b|}{2r}$$

Minimize *t* over all positive *r* while maintaining constraint

$$\omega^2 = (s-u)^2 + 4r^2.$$

Minimum evolution time: $\omega\tau = 2\hbar \arcsin|b|$

Looks like uncertainty principle but is merely rate times time = distance

If $a = 0$ and $b = 1$, the smallest time required to transform $\binom{1}{0}$ to the orthogonal state $\binom{0}{1}$ is $\tau = \pi\hbar/\omega$

Non-Hermitian *PT*-symmetric Hamiltonian

$$H = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \qquad (r,\ s,\ \theta \text{ real}) \qquad \omega^2 = 4s^2 - 4r^2\sin^2\theta.$$

$\mathcal{T}$ is complex conjugation and $\mathcal{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$E_\pm = r\cos\theta \pm \sqrt{s^2 - r^2\sin^2\theta} \qquad \text{real if } s^2 > r^2\sin^2\theta$$

$$\mathcal{C} = \frac{1}{\cos\alpha}\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$$ where $$\sin\alpha = (r/s)\sin\theta.$$

$$e^{-iHt/\hbar}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{e^{-itr\cos\theta/\hbar}}{\cos\alpha}\begin{pmatrix} \cos(\frac{\omega t}{2\hbar} - \alpha) \\ -i\sin\left(\frac{\omega t}{2\hbar}\right) \end{pmatrix}$$

The pair of vectors used in the Hermitian case, $|\psi_I\rangle = \binom{1}{0}$ and $|\psi_F\rangle = \binom{0}{1}$, are not orthogonal with respect to the $\mathcal{CPT}$ inner product. The time needed for $|\psi_I\rangle$ to evolve into $|\psi_F\rangle$ is $t = (2\alpha - \pi)\hbar/\omega$. Optimizing this result over α indicates that the optimal time τ is ZERO!

The bottom line…

*So, what does **PT** symmetry really mean?*

Interpretation…

Finding the optimal ***PT****-symmetric Hamiltonian amounts to constructing a wormhole in Hilbert space!*

Observation of Fast Evolution in Parity-Time-Symmetric System

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To find and realize the optimal evolution between two states is significant both in theory and application. In quantum mechanics, the minimal evolution is bounded by the gap between the largest and smallest eigenvalue of the Hamiltonian. In the parity-time-symmetric(PT-symmetric) Hamiltonian theory, it was predicted that the optimized evolution time can be reduced drastically comparing to the bound in the Hermitian case, and can become even zero. In this Letter, we report the experimental observation of the fast evolution of a PT-symmetric Hamiltonian in an nuclear magnetic resonance (NMR) quantum system. The experimental results demonstrate that the PT-symmetric Hamiltonian can indeed evolve much faster than that in a quantum system, and time it takes can be arbitrary close to zero.

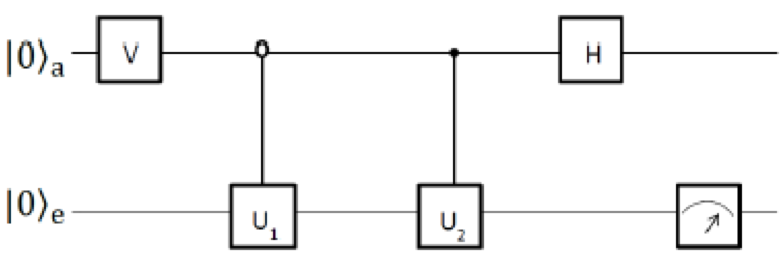


FIG. 1: Quantum circuit for a PT-symmetric system. Operations from to left to right are a single-qubit operation V, 0-controlled U_1, 1- controlled U_2, and a Hadamard operation H. When the auxiliary qubit is in $|0\rangle_a$, the lower qubit is in state $e^{-\frac{i}{\hbar}\tilde{H}t}|0\rangle_e$. See text for details.

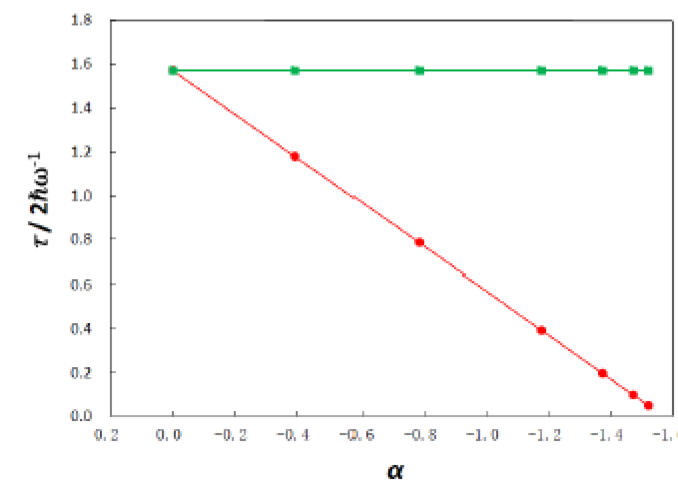


Figure 5: The evolution time τ versus α. he red circles are for the PT-symmetric case, and the green squares are for the Hermitian cases. The connected lines are used to guide the eyes only.

"The shortest path between two truths in the real domain passes through the complex domain."

-- Jacques Hadamard

The Mathematical Intelligencer **13** (1991)

Thanks for listening!


Pt
Lake
TRAILHEAD
SAN ISABEL
National Forest
FOREST SERVICE
US
DEPARTMENT OF AGRICULTURE
U.S. DEPARTMENT · OF · AGRICULTURE

OK, but what exactly is this ***PT*** phase transition?

Examining the CLASSICAL limit of ***PT*** quantum mechanics provides an intuitive explanation of ***PT*** symmetry…

Motion on the real axis

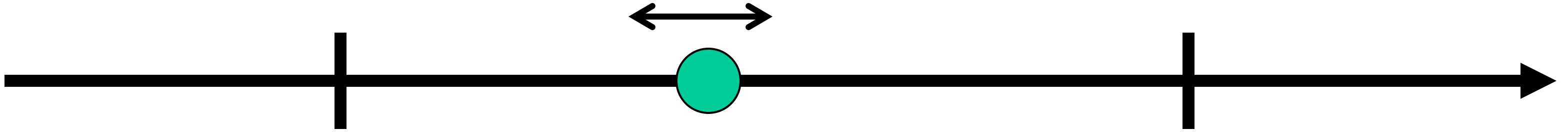
Motion of particles is governed by Newton’s Law:

$$F=ma$$

In freshman physics this motion is restricted to the REAL AXIS.

Harmonic oscillator: Particle on a spring

Back and forth motion on the real axis:



Turning point *Turning point*

$$H = p^2 + x^2 \qquad (\epsilon = 0)$$

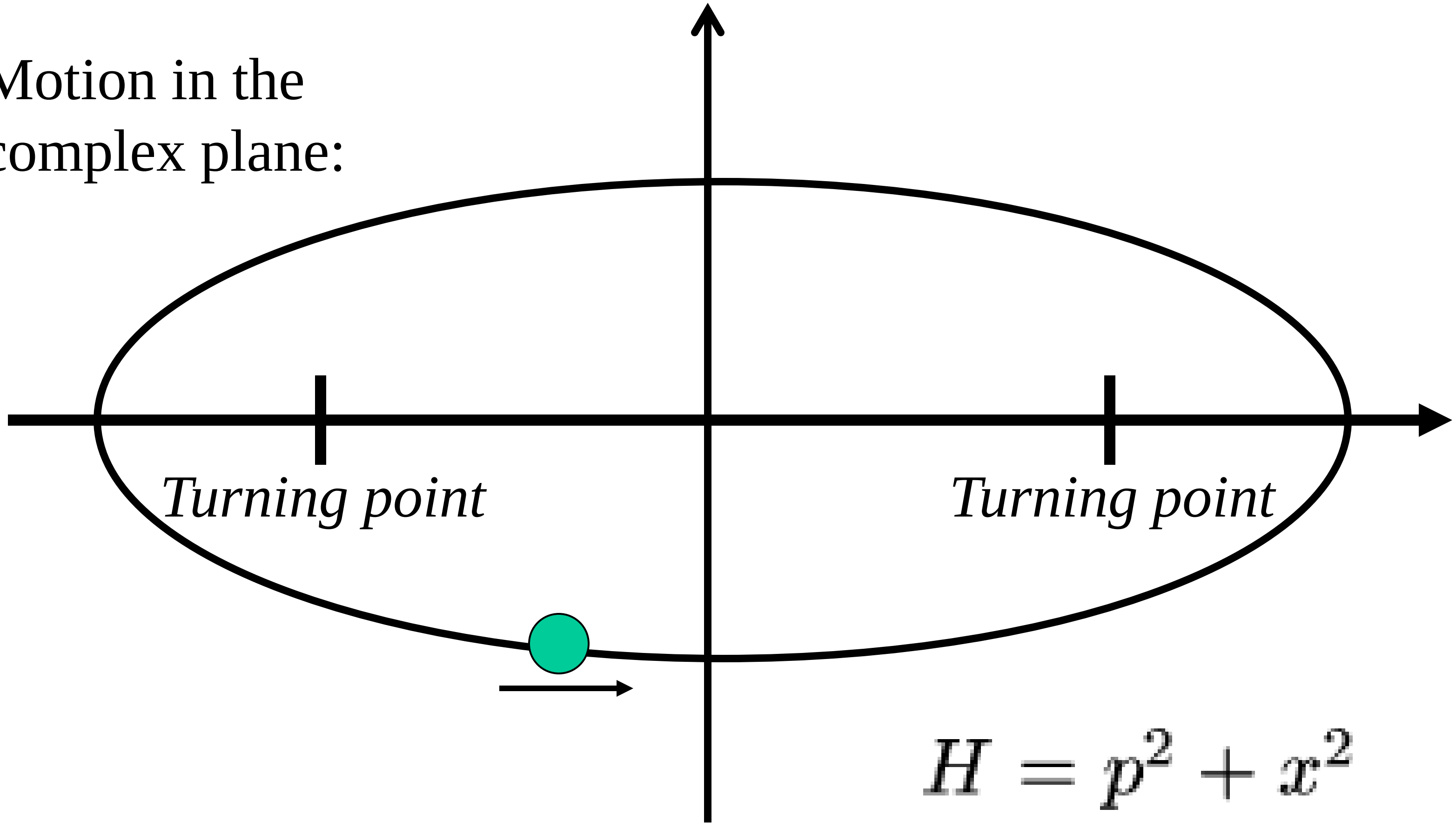
Hamilton's equations

$$\dot{x} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial x}$$

Harmonic oscillator:

Motion in the complex plane:

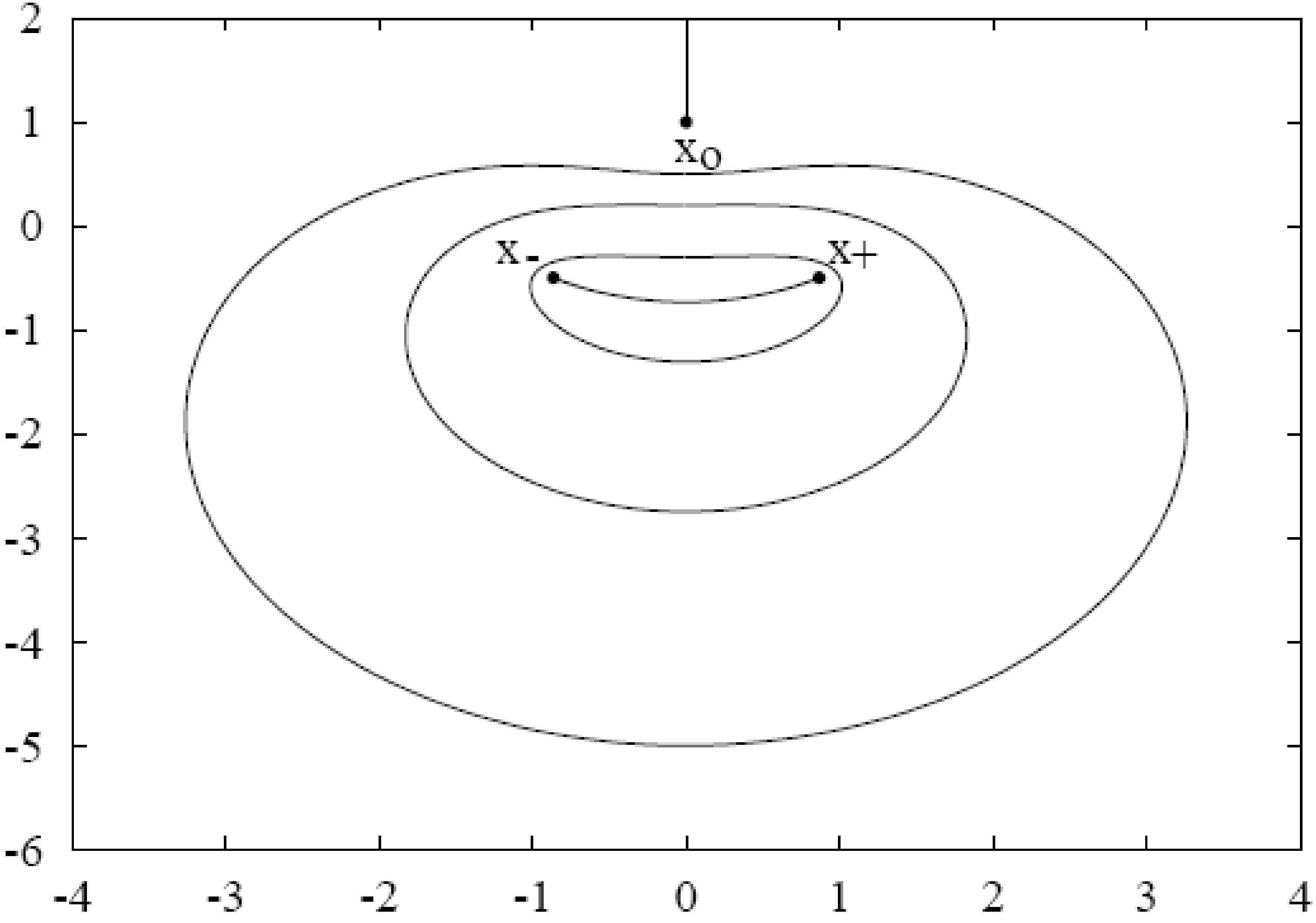


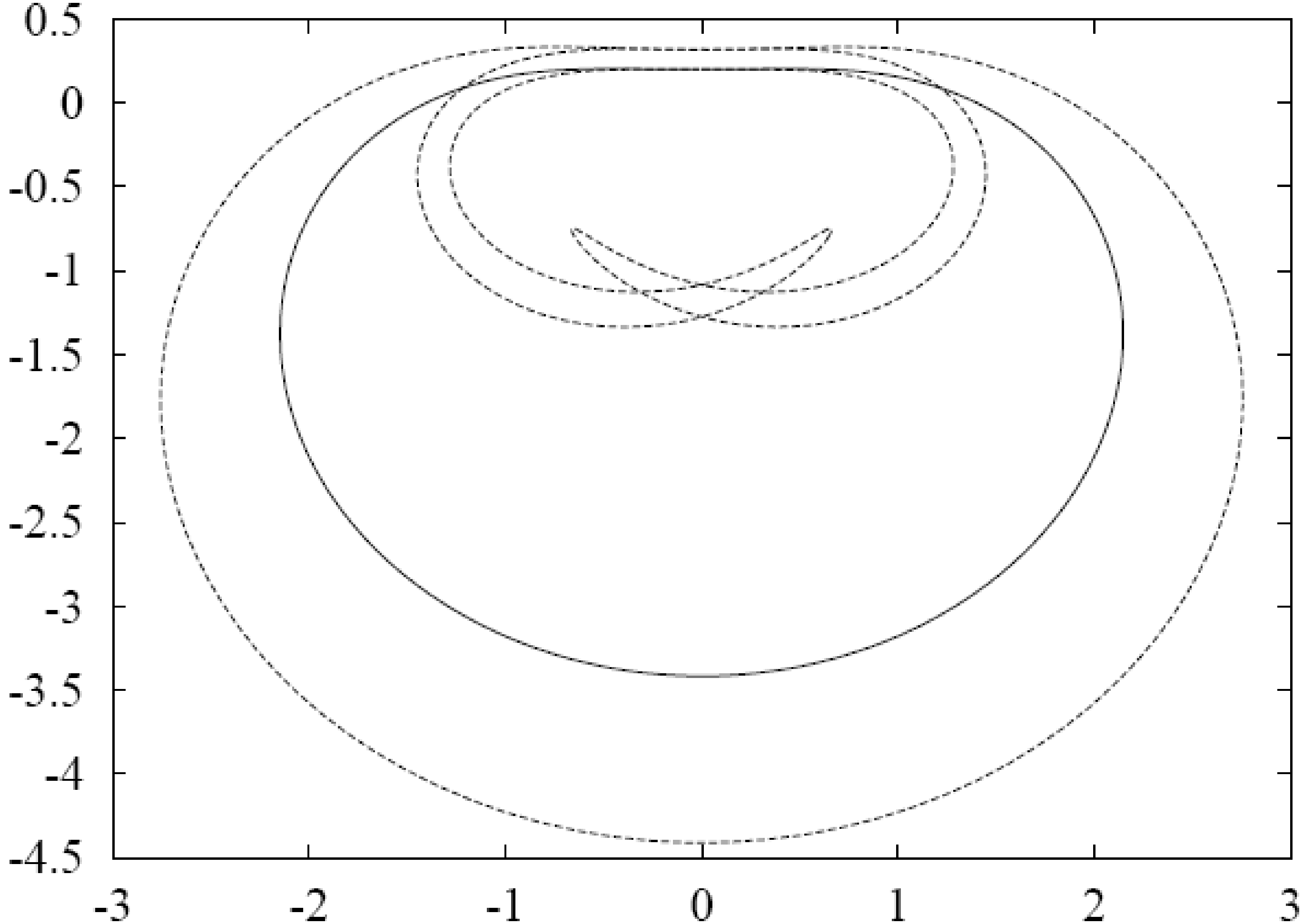
$$H = p^2 + x^2$$

(ϵ = 0)

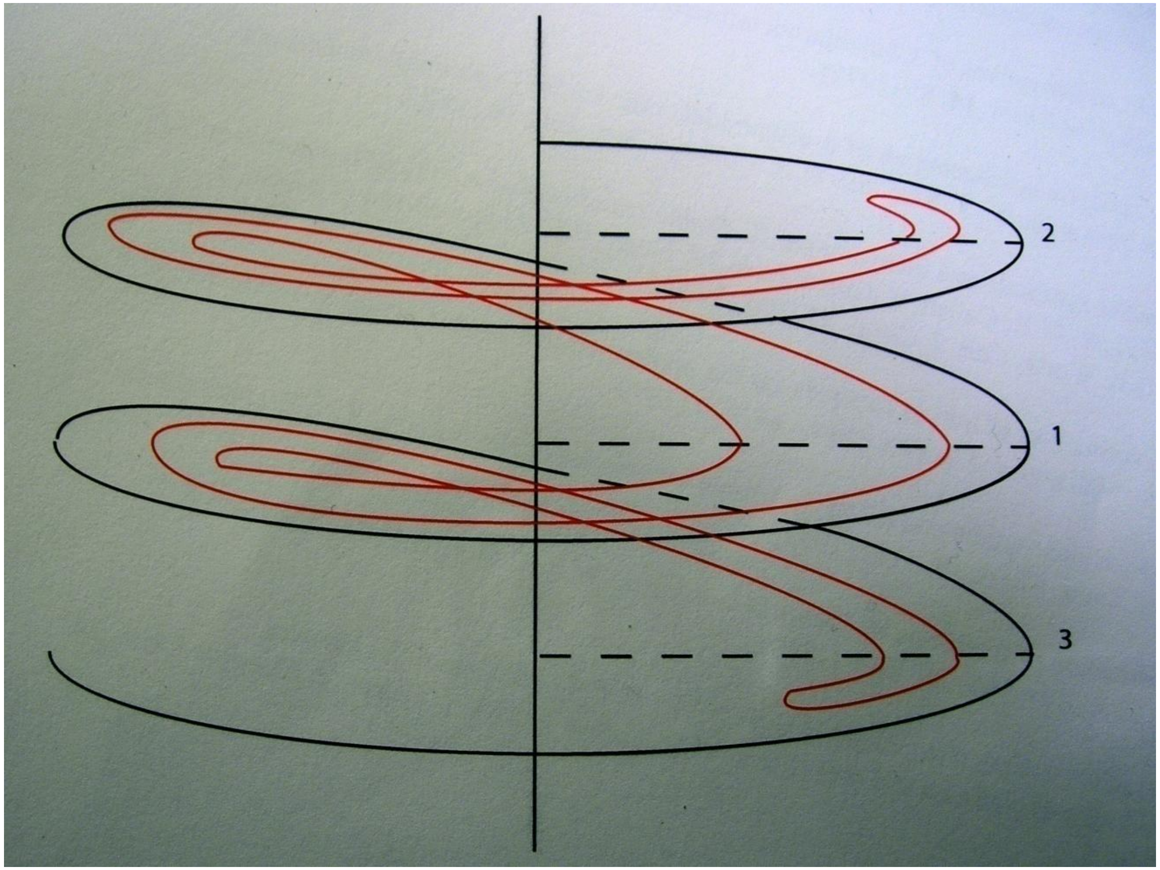
$$H = p^2 + ix^3$$

($\epsilon = 1$)

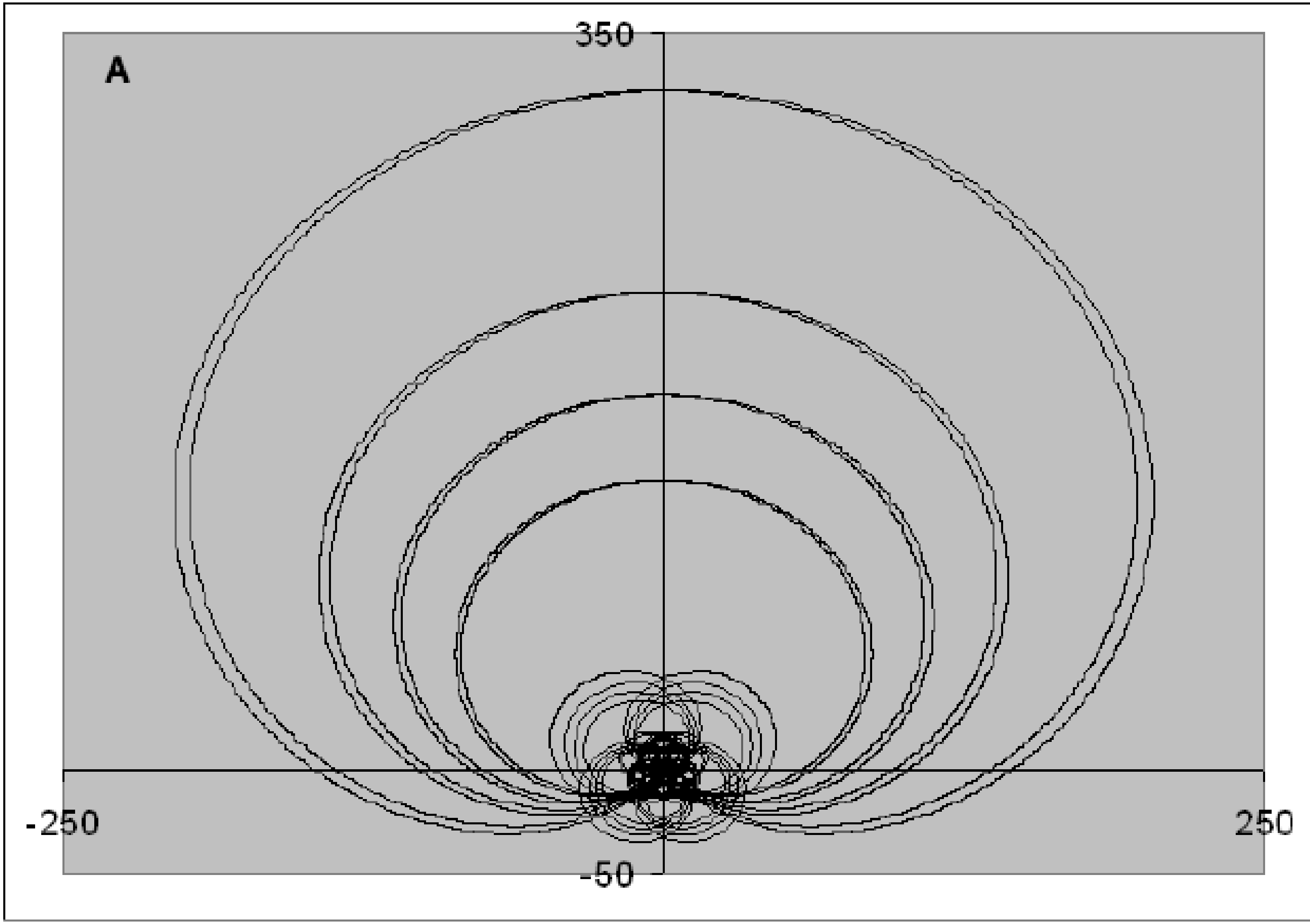




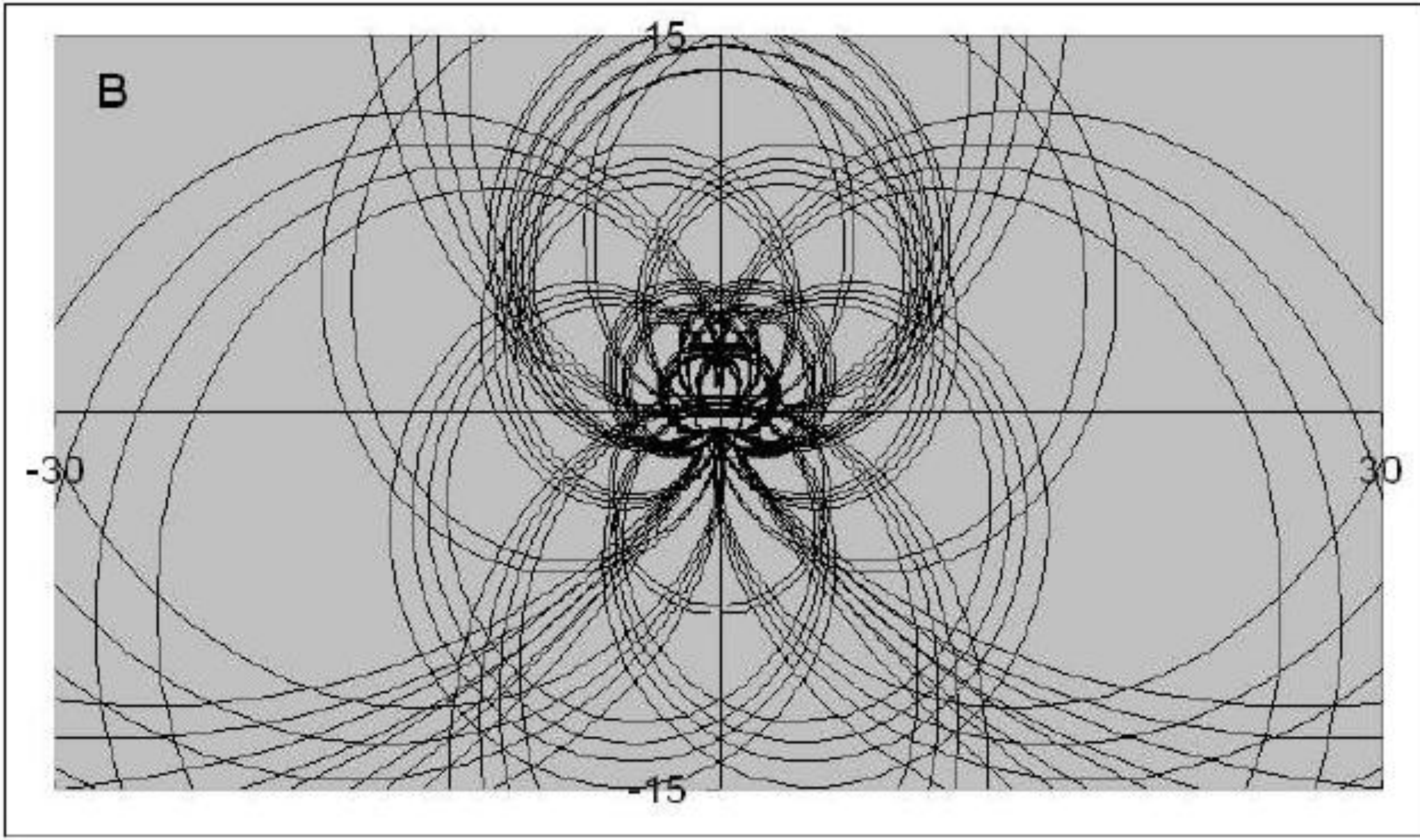
$\varepsilon = \pi - 2$



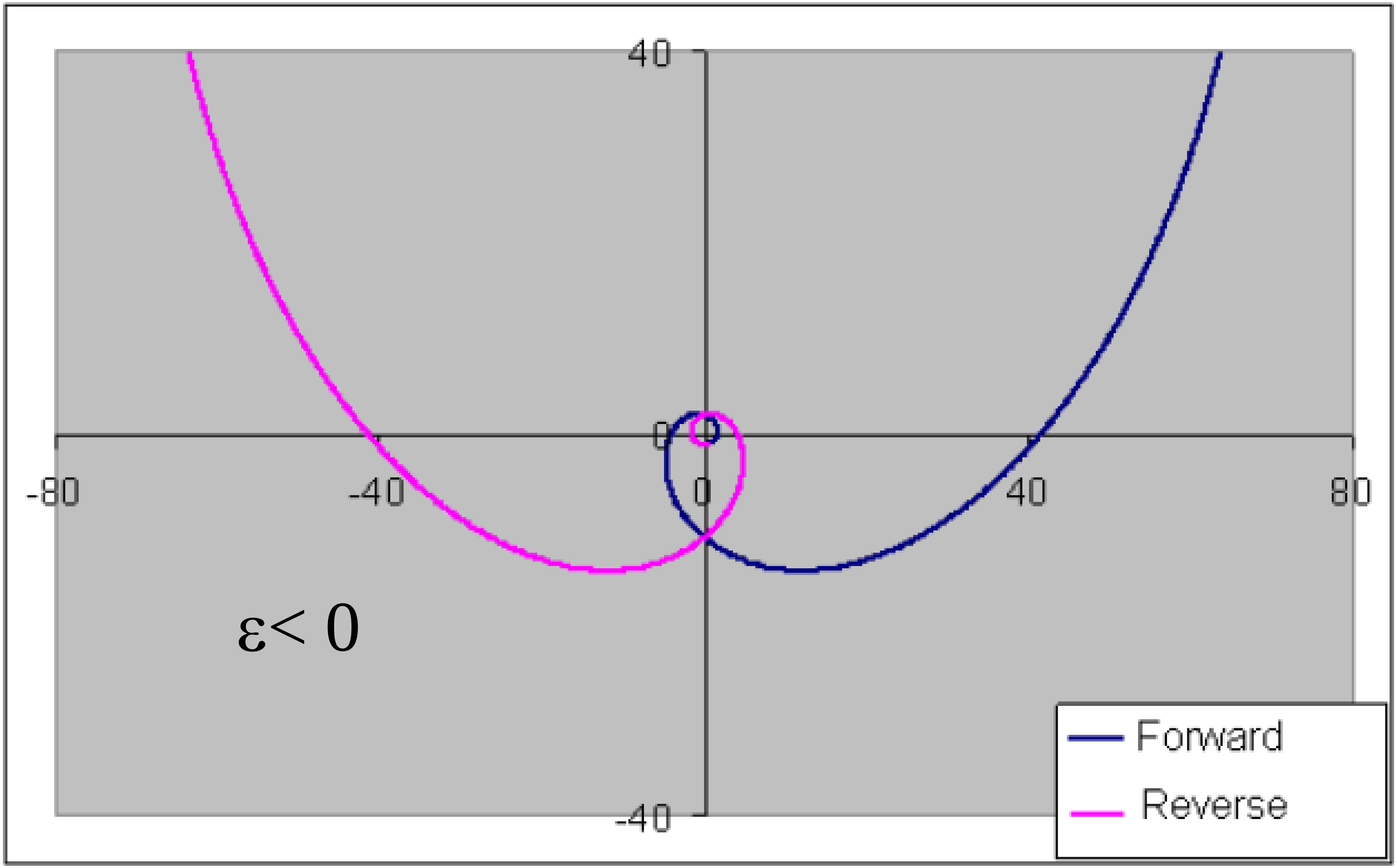
Classical orbit that visits three sheets of the Riemann surface



$\epsilon = \pi - 2$ 11 sheets


B
15
-30
30
-15

Broken *PT* symmetry – orbit not closed



Bohr-Sommerfeld Quantization of a complex atom

$$\oint dx\, p = \left(n + \frac{1}{2}\right)\pi$$