Phase structure of YM theory on a small torus

Takeshi Morita

University of Crete

Ref) JHEP 1002:034,2010 G. Mandal, M. Mahato, T.M

JHEP 1008:015,2010. T.M.

arXiv:1103.1558 [hep-th] Mandal, T.M.

+work in progress with Mandal

based on collaboration with G. Mandal (TIFR), M. Mahato (IIT)

1. Introduction and Motivation

◆ Basic Question

Is it possible to solve the confinement/deconfinement transition of pure Yang-Mills theory analytically in any special situation?

A. Yes. In some finite volume cases, we can solve it in large N limit.

ex) YM on $S^1_\beta \times S^3$ (Aharony, Marsano, Minwalla Papadodimas, Raamsdonk 2005)

YM on $S^1_\beta \times S^2$ (Papadodimas, Shieh, Raamsdonk 2006)

(weak coupling)

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(weak coupling)

We found that $D+2$ dim YM on T^{D+2} is also solvable in a small T^D limit.

$\longleftrightarrow$ 2dim YM + D adjoint scalars on T^2

KK reduction

$$S = \int_0^\beta dt \int_0^L dx\, Tr\left(\frac{1}{2g^2} F_{01}^2 + \sum_{I=1}^{D} \frac{1}{2} \left(D_\mu Y^I\right)^2 - \sum_{I,J}^{D} \frac{g^2}{4} [Y^I, Y^J][Y^I, Y^J] \right)$$

1. Introduction and Motivation

- Confinement/deconfinement in SU(N) Large N YM theory

$$S=\int_0^\beta dt\int d^{D+1}x\frac{1}{4g_{D+2}^2}\mathrm{Tr}F_{\mu\nu}^2$$

β: inverse temperature

- temporal Polyakov loop

$$W_0=\frac{1}{N}\mathrm{Tr}P\left(\exp\left[i\int_0^\beta A_0 dx^0\right]\right)$$

- Z_N symmetry

$W_0 \to h_0 W_0 : h_0 \in Z_N$, center of $SU(N)$

$$\begin{cases}\langle W_0\rangle = 0 & : Z_N \text{ symmetry} & \text{confinement} \\ \langle W_0\rangle \neq 0 & : Z_N \text{ symmetry breaking} & \text{deconfinement}\end{cases}$$

The confinement/deconfinement transition is characterized by

$$W_0=0/W_0\neq 0$$

1. Introduction and Motivation

◆ SU(N) Large N YM on T^{D+2}

$$S=\int_0^\beta dt\int d^{D+1}x\frac{1}{4g_{D+2}^2}\mathrm{Tr}F_{\mu\nu}^2 \rightarrow S=\left(\prod_{\mu=0}^{D+1}\int_0^{L_\mu}dx^\mu\right)\frac{1}{4g_{D+2}^2}\mathrm{Tr}F_{\mu\nu}^2$$

β: inverse temperature $\rightarrow$ L_μ $(\mu=0,\cdots,D+1)$: periods of the torus $(L_0=\beta)$

◆ temporal Polyakov loop $\rightarrow$ D+2 Polyakov loops

$$W_0=\frac{1}{N}\mathrm{Tr}P\left(\exp\left[i\int_0^\beta A_0dx^0\right]\right)\rightarrow W_\mu=\frac{1}{N}\mathrm{Tr}P\left(\exp\left[i\int_0^{L_\mu} A_\mu dx^\mu\right]\right)$$

◆ $(Z_N)^{D+2}$ symmetry

$W_\mu\to h_\mu W_\mu$: $h_\mu\in Z_N$, center of $SU(N)$

$$\begin{cases}\langle W_\mu\rangle=0 & : \ Z_N \text{ symmetry}\\ \langle W_\mu\rangle\neq 0 & : \ Z_N \text{ symmetry breaking}\end{cases}$$

} Sharp transition at large N

$\longrightarrow$ At least, 2^{D+2} phases characterized by W_μ

$\longrightarrow$ generalization of the confinement/deconfinement transition $(W_0=0/W_0\neq 0)$

1. Introduction and Motivation

◆ Large N SU(N) YM on T^{D+2} in small T^D limit

⟶ 2dim YM + D adjoint scalars on $S^1_\beta \times S^1_L$

$$S = \int_0^\beta dt \int_0^L dx\, Tr\left(\frac{1}{2g^2} F_{01}^2 + \sum_{I=1}^{D} \frac{1}{2} \left(D_\mu Y^I\right)^2 - \sum_{I,J}^{D} \frac{g^2}{4} [Y^I, Y^J][Y^I, Y^J] \right)$$

We can solve this model by using a 1/D expansion.

↑

(The validity would be $D \geq 2$.)

◆ Interesting features of this model ⟷ YM on $S^1_\beta \times S^n$

- Richer phase structre (4+2 phases will appear)
- Large volume ($L \to \infty$) is possible.
- In D=8, Holographic dual can be constructed from D2 branes on a Scherk-Schwarz circle .

  $\to$ Test of holography
- Comparison with numerical studies.

1. Introduction and Motivation

◆ Large N SU(N) YM on T^{D+2} in small T^D limit

⟶ 2dim YM + D adjoint scalars on $S^1_\beta \times S^1_L$

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Plan of this talk

1. Introduction and Motivation
2. 1/D expansion
3. Small L limit
4. Large L limit
5. Conclusions

2. 1/D expansion

In general p-dim YM + D adjoint scalars, Y can be integrated as follows

$$S = \int d^p x \, Tr\left(\frac{1}{4g^2} F_{\mu\nu}^2 + \sum_{I=1}^{D} \frac{1}{2} \left(D_\mu Y^I \right)^2 - \sum_{I,J}^{D} \frac{g^2}{4} [Y^I, Y^J][Y^I, Y^J] \right)$$

Large D limit (p=0: Hotta-Nishimura-Tsuchiya 1999, p=1: Mahato-Mandal-T.M. 2009, general p: T.M. 2010)

$$g \to 0, \quad N, D \to \infty \quad \text{with fixed } \tilde{\lambda} = g^2 DN,$$

We can always find a non-trivial saddle point characterized by a condensation

$$\left\langle \mathrm{Tr} Y^I Y^I \right\rangle = \frac{DN^2}{\tilde{\lambda}} \Delta_0^2$$

Y^I behaves as a weakly interacting ($\sim 1/D$) massive ($m \sim \Delta_0$) scalar around this saddle point.

↓ Integrate out Y

$$S_{eff}(A) = S_0(A) + \frac{1}{D} S_1(A) + \frac{1}{D^2} S_2(A) + \cdots$$

2. 1/D expansion

In general p-dim YM + D adjoint scalars, Y can be integrated as follows

$$S = \int d^p x \, Tr \left(\frac{1}{4g^2} F_{\mu\nu}^2 + \sum_{I=1}^{D} \frac{1}{2} \left(D_\mu Y^I \right)^2 - \sum_{I,J}^{D} \frac{g^2}{4} [Y^I, Y^J][Y^I, Y^J] \right)$$

↓ Integrate out Y

$$S_{eff}(A) = S_0(A) + \frac{1}{D} S_1(A) + \frac{1}{D^2} S_2(A) + \cdots$$

This effective action is

- difficult to solve if $p \geq 3$, owing to the existence of the dynamical gluon.
- (partially) solvable if $p \leq 2$. (No dymanical gluon in 2 dim YM)

3. small L

(Mahato-Mandal-T.M. 2009)

$$S=\int_0^\beta dt\int_0^L dx\,Tr\left(\frac{1}{2g^2}F_{01}^2+\sum_{I=1}^{D}\frac{1}{2}\left(D_\mu Y^I\right)^2-\sum_{I,J}^{D}\frac{g^2}{4}[Y^I,Y^J][Y^I,Y^J]\right)$$

$L\to 0$

$$\longrightarrow\ S=\int_0^\beta dt\,Tr\left(\sum_{I=1}^{D+1}\frac{1}{2}\left(D_0 Y^I\right)^2-\sum_{I,J}^{D+1}\frac{g^2}{4}[Y^I,Y^J][Y^I,Y^J]\right)$$

$1/D$ expansion

$$\longrightarrow\ S_{eff}(A)=S_0(A)+\frac{1}{D}S_1(A)+\frac{1}{D^2}S_2(A)+\cdots$$

We computed the effective action up to O(1/D)

$$S/(DN^2)=a_1|W_0|^2+b_1|W_0|^4+\cdots,$$

$$W_0=\frac{1}{N}\mathrm{Tr}P\left(\exp\left[i\int_0^\beta A_0 dt\right]\right)$$

$$\begin{cases} a_1=\dfrac{1}{D}-\bar{x}-\dfrac{\tilde{\lambda}^{1/3}\beta}{D}\left(\dfrac{203}{160}-\dfrac{\sqrt{5}}{3}\right)\bar{x}, \\ b_1=\dfrac{\tilde{\lambda}^{1/3}\beta}{3}\bar{x}^2-\dfrac{\tilde{\lambda}^{1/3}\beta}{D}\left(\tilde{\lambda}^{1/3}\beta\left(\dfrac{2\sqrt{5}}{9}-\dfrac{229}{300}\right)+\dfrac{391\sqrt{5}}{1800}-\dfrac{3181}{2400}\right)\bar{x}^2. \end{cases}$$

We obtain a Landau-Ginzburg type effective action for Polyakov loops.

(Aharony, Marsano, Minwalla, Papadodimas, Raamsdonk 2003)

3. small L

Two phase transitions (2nd+3rd) occur.

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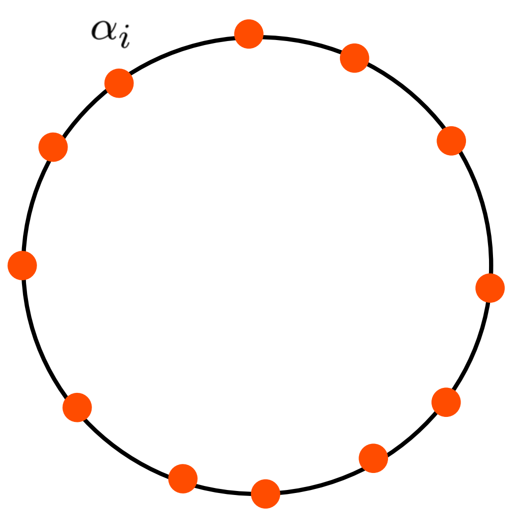
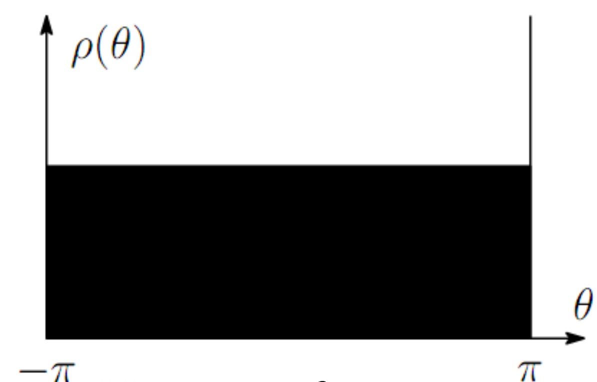
◆ Convenient quantity to investigate the phase structure

- Eigenvalue distribution of $A_{0ij} = \alpha_i \delta_{ij}$ ⟶ Eigenvalue density

$$\rho(\theta) = \frac{1}{N}\sum_{n=1}^{N}\delta(\theta - \alpha_i).$$



$$\alpha_i = \alpha_i + 2\pi/\beta$$



$W_0 = 0$ confinement

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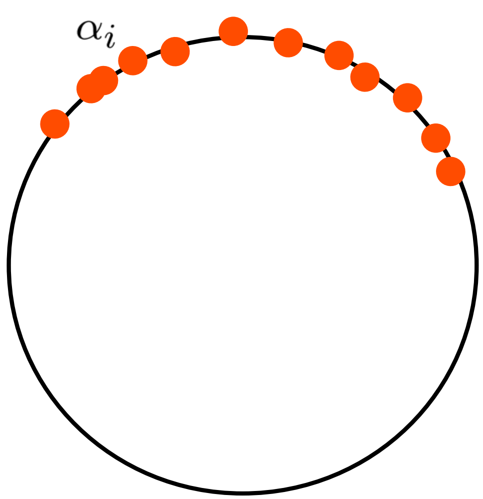

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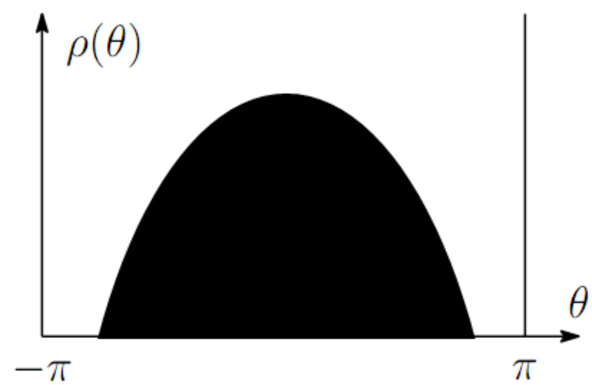

$W_0 \neq 0$ deconfinement

3. small L — Two phase transitions (2nd+3rd) occur.

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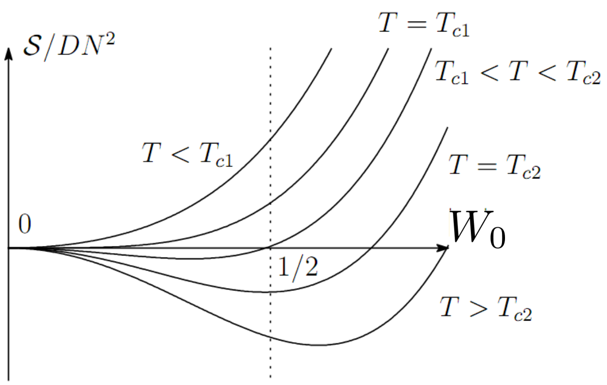
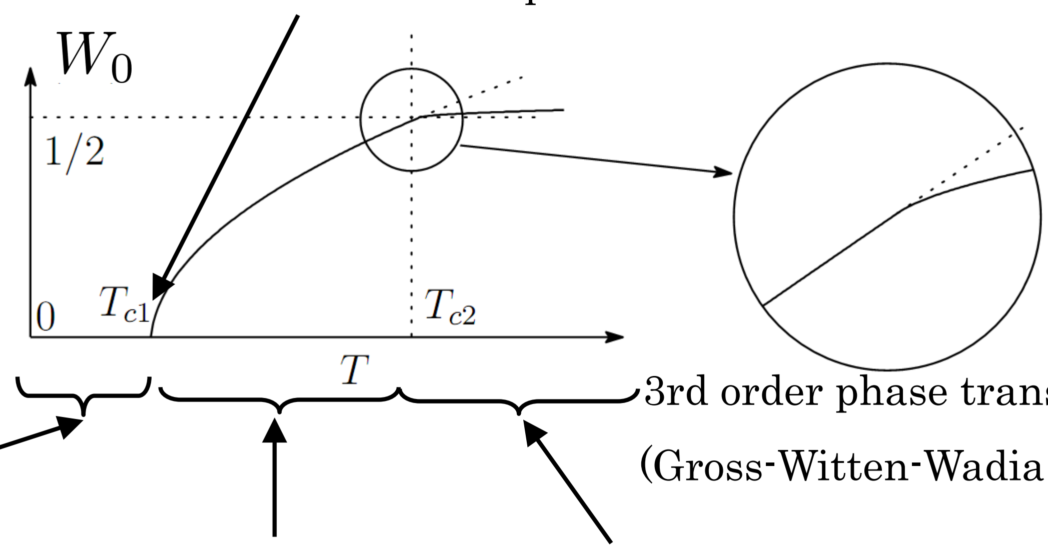
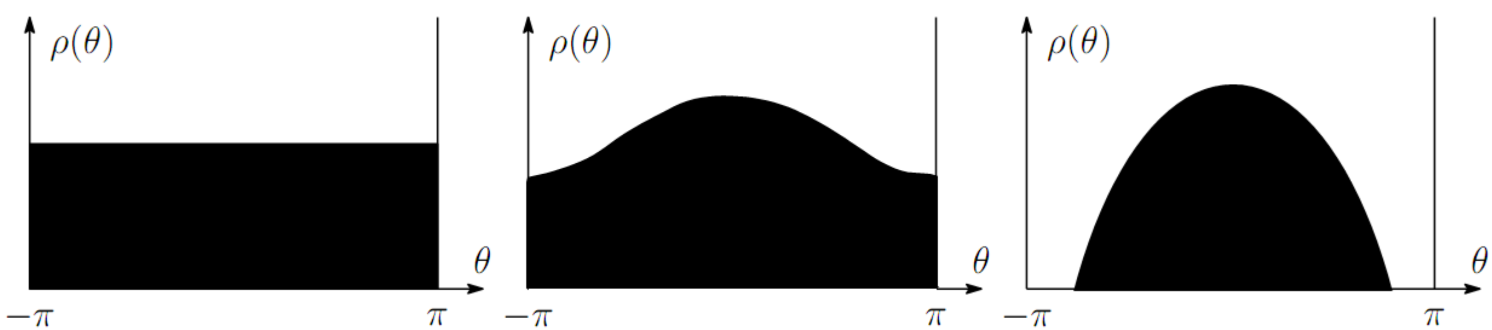


2nd order phase transition



3rd order phase transition

(Gross-Witten-Wadia type)



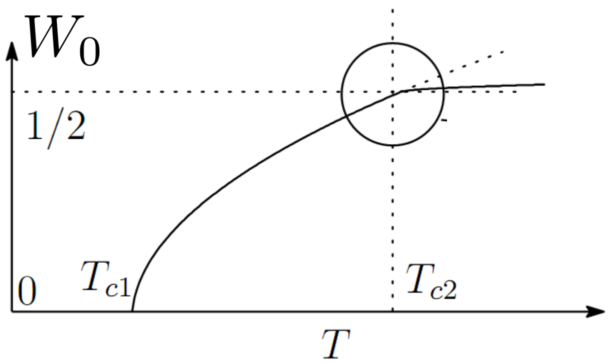
$W_0 = 0$ confinement

$W_0 \neq 0$ deconfinement

$W_0 \neq 0$ deconfinement

3. small L

Comparison with numerical studies



$$S_{eff}(A) = S_0(A) + \frac{1}{D} S_1(A) + \frac{1}{D^2} S_2(A) + \cdots$$

D=9	T_{c1}	T_{c2}	R^2	F_0
Numerical result	0.8761	0.905	2.291	6.695
$1/D$ expansion	0.895	0.917	2.28	6.72
error	2%	1%	0.5%	0.3%

← Kawahara, Nishimura, Takeuchi 2007

Errors $\sim 1/D^2 = 1/9^2 = 1\% \rightarrow$ Expected agreement.

	T_{c1} $(D=2)$	T_{c2} $(D=2)$	T_{c1} $(D=3)$	T_{c2} $(D=3)$
Our result	1.4	1.6	1.1	1.2
Numerical result	1.12	1.3	0.93	1.1

Azeyanagi, Hanada, Hirata, Shimada 2009

Qualitative agreement even in small D.

4. Large L

Mandal, T.M. 2011

$$S=\int_0^\beta dt\int_0^L dx\,Tr\left(\frac{1}{2g^2}F_{01}^2+\sum_{I=1}^{D}\frac{1}{2}\left(D_\mu Y^I\right)^2-\sum_{I,J}^{D}\frac{g^2}{4}[Y^I,Y^J][Y^I,Y^J]\right)$$

$$\xrightarrow{L\to\infty} S=\int_0^\beta dt\int_0^\infty dx\,Tr\left(\frac{1}{2g^2}F_{01}^2+\sum_{I=1}^{D}\frac{1}{2}\left(D_\mu Y^I\right)^2-\sum_{I,J}^{D}\frac{g^2}{4}[Y^I,Y^J][Y^I,Y^J]\right)$$

$1/D$ expansion

$$\longrightarrow S_{eff}(A)=\underline{S_0(A)}+\frac{1}{D}S_1(A)+\frac{1}{D^2}S_2(A)+\cdots$$

We evaluate the leading term of the effective action in a weak coupling.

We obtain a unitary matrix model as an effective theory.

$$S/DN^2=\int_0^\infty dx\left[\frac{1}{2N}Tr\left(|\partial_x U|^2\right)-\frac{\xi}{N^2}\left|TrU\right|^2\right]$$

$$U(x)=P\exp\left(i\int_0^\beta dt A_0(x,t)\right)$$

Semenoff, Tirkkonen, Zarembo 1996

Basu, Ezhuthachan and Wadia 2005

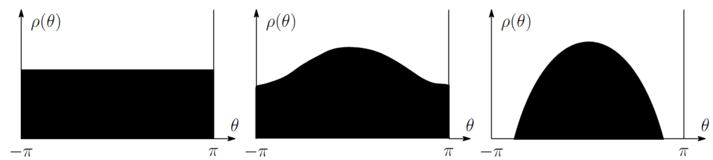
4. Large L

One 1st order transition occurs.

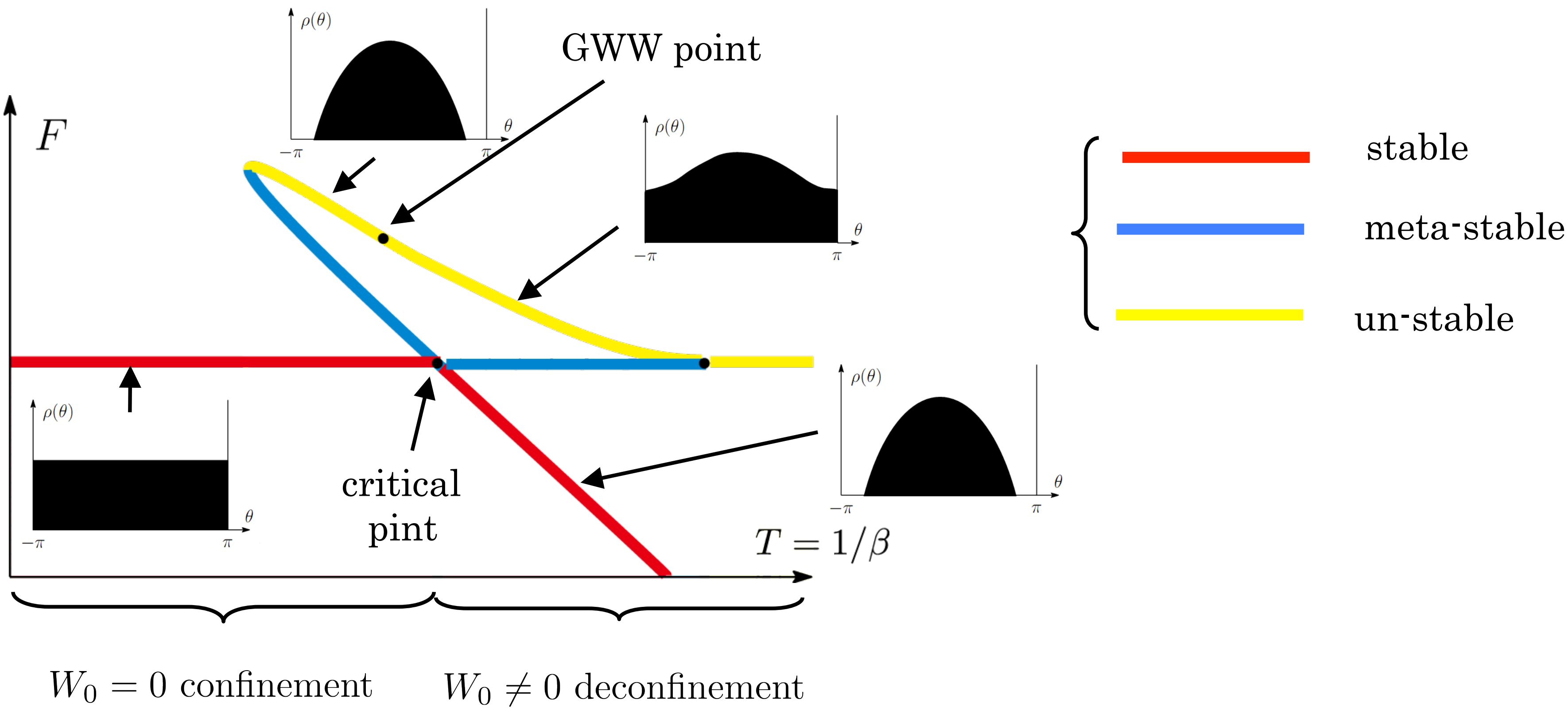
$$S/DN^2 = \int_0^\infty dx \left[\frac{1}{2N} Tr \left(|\partial_x U|^2 \right) - \frac{\xi}{N^2} \left| TrU \right|^2 \right] \qquad U(x) = P \exp \left(i \int_0^\beta dt A_0(x,t) \right)$$

This model also has three type of solutions.

$$\rho(\theta) = \frac{1}{N} \sum_{n=1}^{N} \delta(\theta - A_{0i}).$$



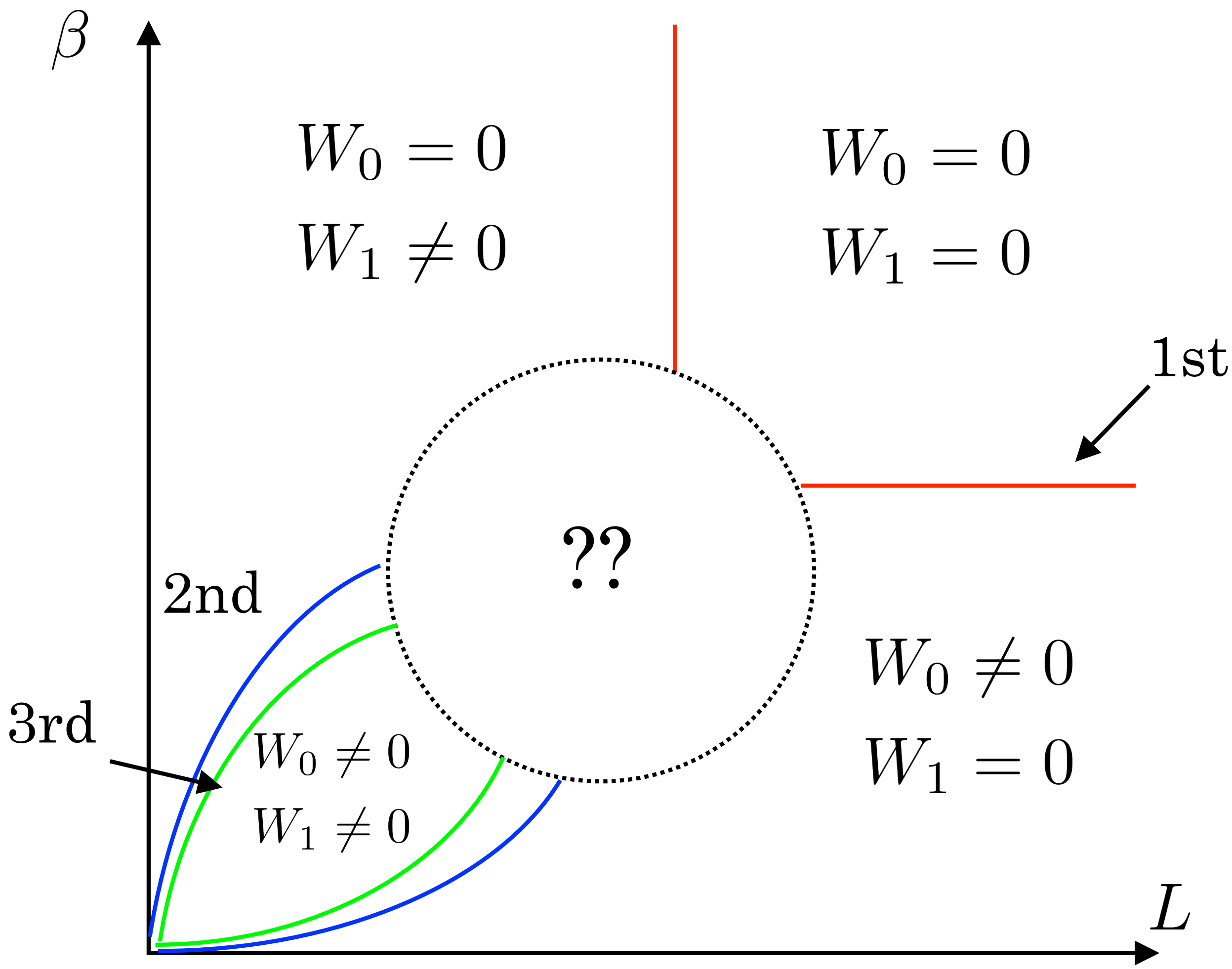
The free energies of these three solutions satisfy the following relation:



4. Large L

Whole phase structure

This model has a $\beta \leftrightarrow L$ symmetry.



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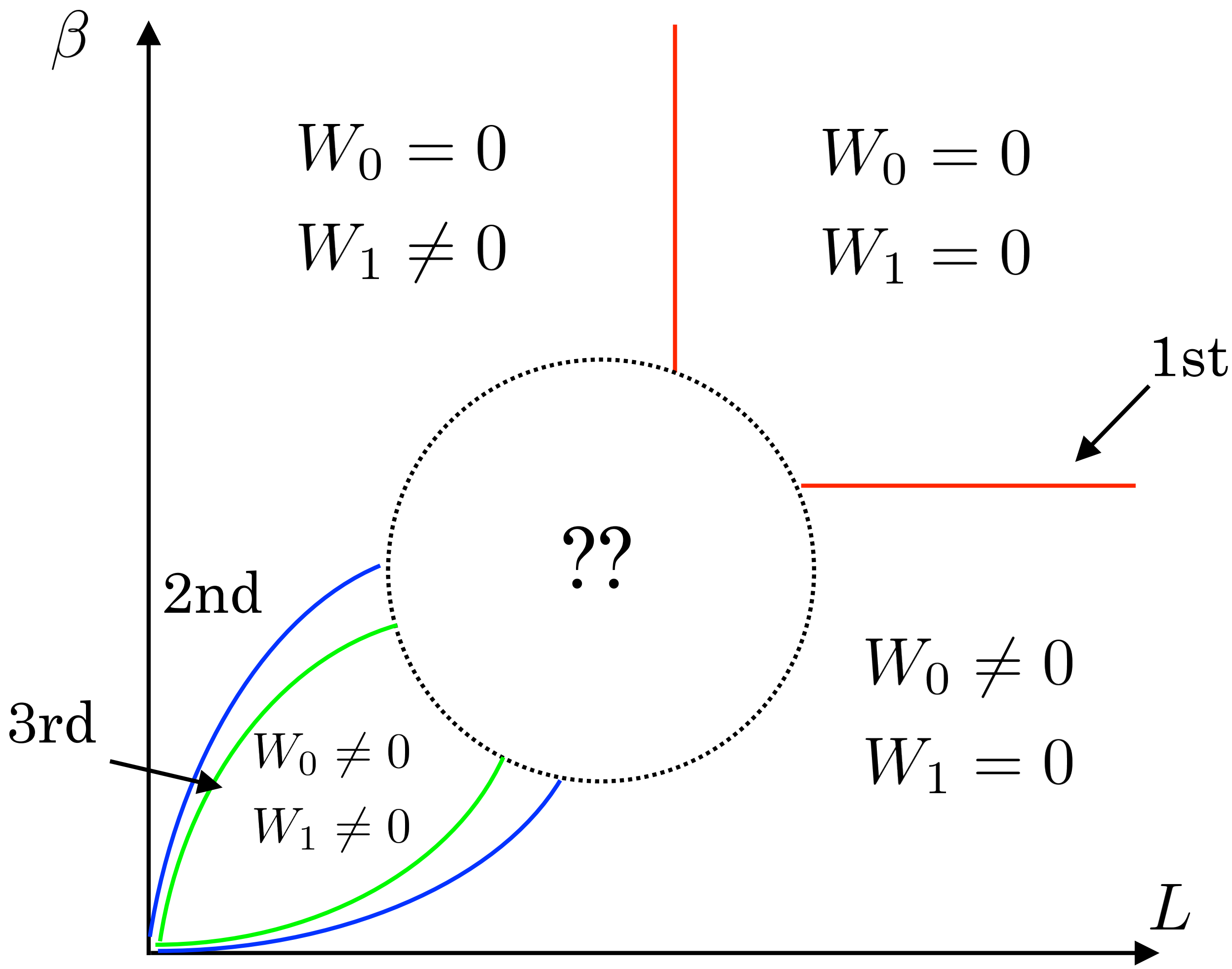

cf. (weak coupling)

YM on $S^1_\beta \times S^3$

$\rightarrow$ 1st order

YM on $S^1_\beta \times S^2$

$\rightarrow$ 2nd+3rd order

The properties of the confinement/deconfinement transition in YM theories depend on the volume and topology.

4. Large L

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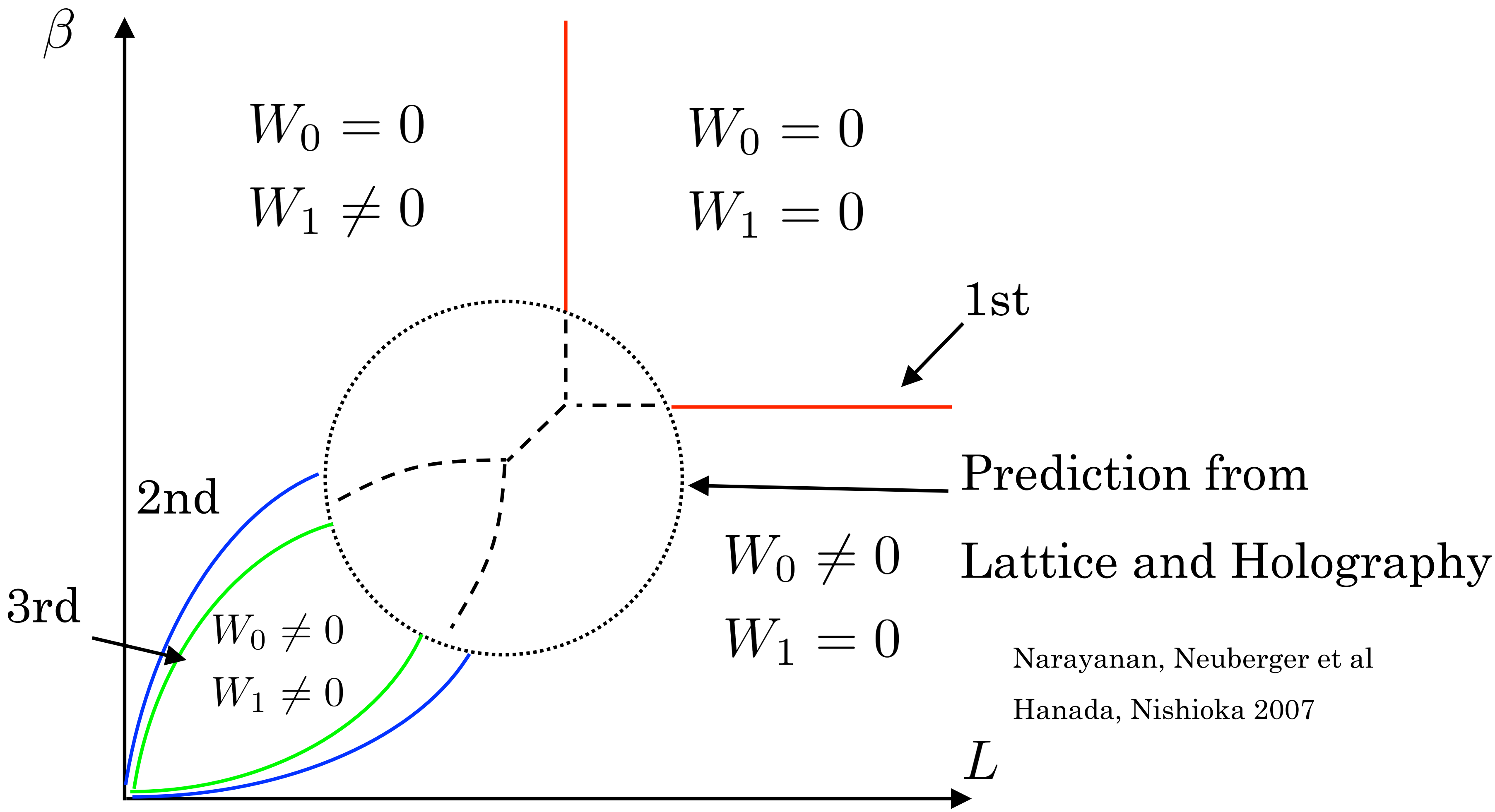
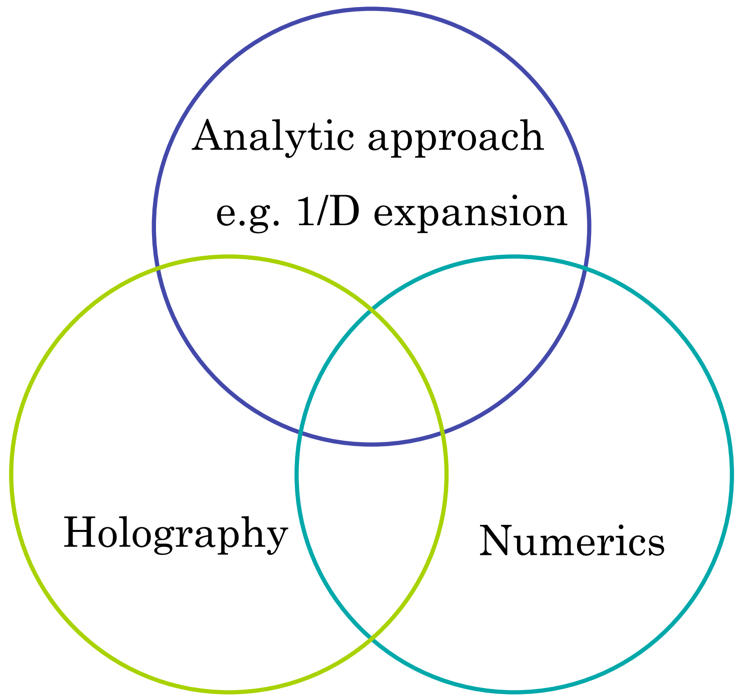


Narayanan, Neuberger et al

Hanada, Nishioka 2007

5. Conclusion

$D+2$ dim YM on a small T^D = 2 dim Yang-Mills + D adjoint scalars



Each approach has advantages and disadvantages.

None of them can solve the whole phase structure.

Interesting laboratory to understand the natures of YM theory!

Related progress

- Problems of finite temperature holographic QCD.

  AdS soliton/black brane transition does not correspond to the confinement/deconfinement transition in QCD. (Aharony, Sonnenschein, Yankielowicz 2006, Aharony, Minwalla, Wiseman)

  → We found a resolution of this issue.

- The 1/D expansion works even in finite N in some cases.

Future Problems

- Understanding of the confinement/deconfinement transitions in holographic QCD and chiral symmetry restoration in the Sakai-Sugimoto model (Mandal, T.M. work in progress)
- Toward the analytically understanding of the confinement/deconfinement transition of the 4d Yang-Mills theory.