A step to off-shell: From Amplitude to Form Factor in $\mathcal{N}$=4 SYM

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Based on the work

A. Brandhuber, B. Spence, G. Travaglini, GY

*"Form Factors in N=4 Super Yang-Mills and Periodic Wilson Loops"***,**
JHEP 1101 (2011) 134. arXiv:1011.1899

A. Brandhuber, O. Gurdogan, R. Mooney, G. Travaglini, GY

"Harmony of Super Form Factor", to appear.

Outline

- Background and Motivation
- A simple look at bosonic form factor
- Supersymmetric form factor
- Dual picture
- Summary and outlook

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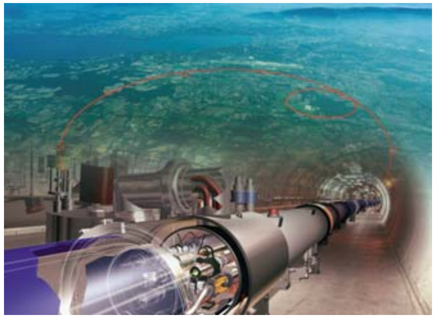
Background and Motivations

There has been continuous significant progress in the study of scattering amplitudes in past twenty years !

Practical motivation : Experimental relevance



Observables. QCD amplitudes for LHC

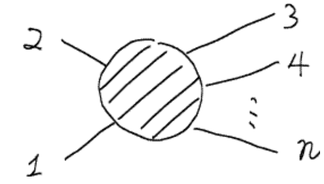
More fundamental motivation : A probe to study QFT

Underlying hidden structures, new formulation of QFT

Non-perturbative physics

Quantum Gravity

Surprising Simplicity

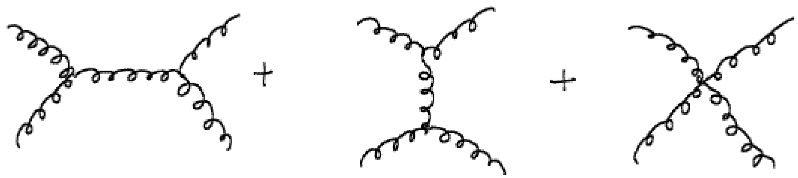


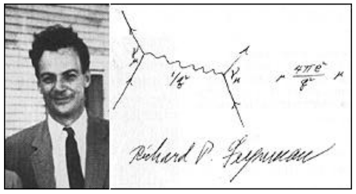
MHV (maximally-helicity-violating) scattering amplitudes:

$$A_{\mathrm{MHV}}(1^+,..,i^-,..,j^-,..,n^+) = \frac{\langle i\ j\rangle^4}{\langle 1\ 2\rangle\cdots\langle n\ 1\rangle}$$ (Parke, Taylor 1986)

Spinor helicity formalism $p_\mu \to p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}\ ,\quad \langle i\ j\rangle = \lambda_i^\alpha \lambda_{j\alpha}$

This simplicity is not expected at all from traditional textbook method,





which means that there are some underlying structure we didn't understand! One should (and can) go beyond Feynman.

(Indeed, BCFW recursion, MHV rules, unitarity method, ...)

Background and Motivations

Particular interest is for planar amplitudes in $\mathcal{N}$=4 SYM.

Since planar $\mathcal{N}$=4 SYM is integrable, we should be able to compute quantities for all values of the coupling.

(Great progress in the anomalous dimension of operators)

A non-perturbative result of amplitudes may be available !

The state of art for explicit results of amplitudes in $\mathcal{N}$=4 :

- All tree and one-loop amplitudes
- Full four and five-point amplitudes
- Six-point two-loop MHV amplitudes (general points in special kinematic regime)
- General points at strong coupling limit

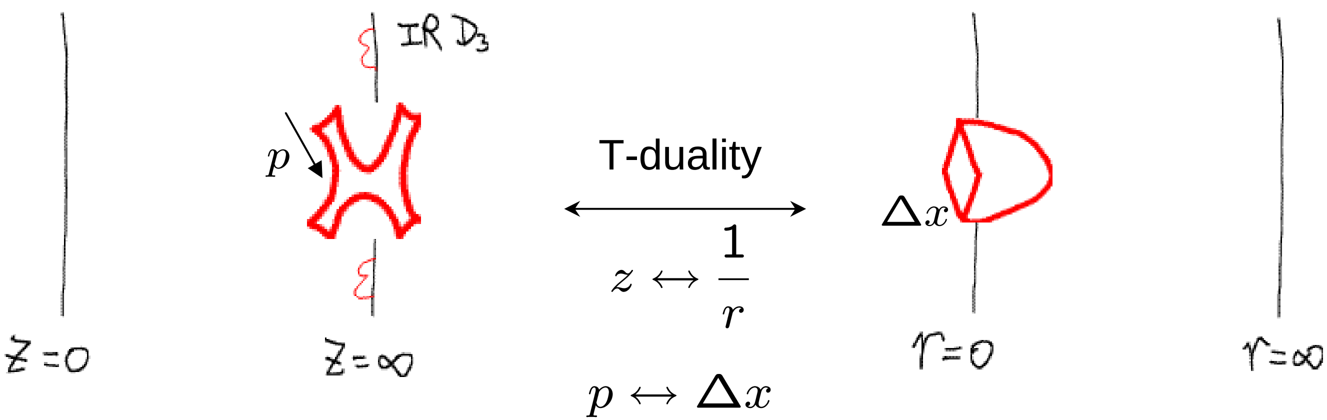
Duality plays a central role!

Dualities

One most remarkable discovery is the amplitudes / Wilson loops duality. (more recent duality to correlators)
This was first inspired by the AdS/CFT duality.

N=4 SYM ⟷ **AdS/CFT duality** ⟷ Type IIB string in AdS5 x S5

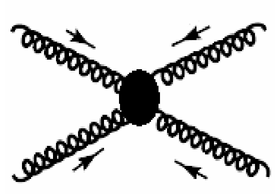
Dual picture of amplitudes in N=4: (Alday, Maldacena)

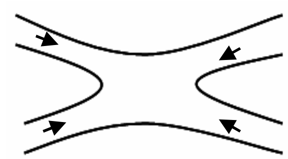


Poincare metric : $\frac{d^2x_{1,3} + d^2z}{z^2}$ $\frac{d^2y_{1,3} + d^2r}{r^2}$

Duality

Gluon scattering amplitude in N=4



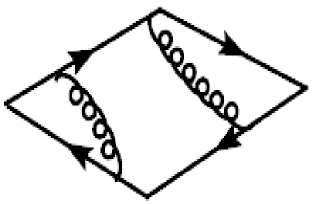


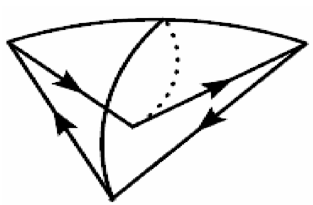
Open string on D3 in AdS5

Dual conformal

T-duality

Wilson loop in N=4





Minimal surface in AdS5

Weak coupling

Strong coupling

N=4 SYM

AdS/CFT duality

Type IIB string in AdS5 x S5

The central reason for the interest is

Surprising simplicity and dualities, indicate that there must be some underline structures and physics that we need to know better. The study of them may have far-reaching influence to fundamental physics. (some of them are not limited to N=4 SYM)

(QCD may also be simple, we haven't understand it well enough ..)

The question we want to ask here is

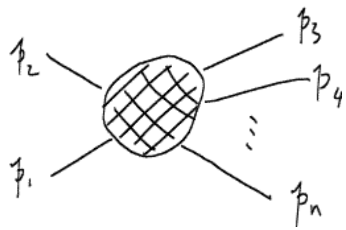
There are many other important observables, such as correlation functions.

Can we go beyond scattering amplitudes and make similar progress for other quantities?

A simple step beyond is the form factor !

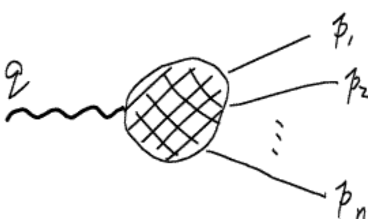
What is form factor ?

Scattering amplitude :



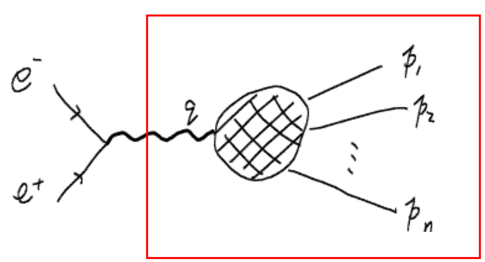
$$A_n = \langle 0 | p_1 p_2 \cdots p_n \rangle$$

Form factor is a mixture of on-shell states and off-shell operator !

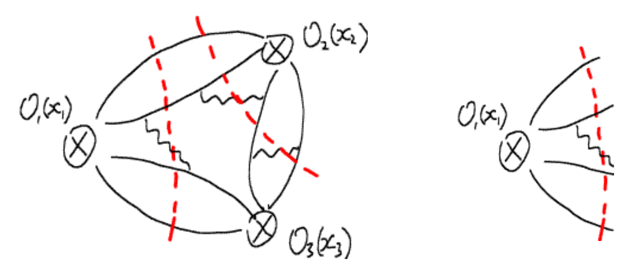


$$\begin{aligned} F_n &= \langle 0 | \mathcal{O}(q) | p_1 p_2 \cdots p_n \rangle \\ &= \int d^4 x e^{iq\cdot x} \langle 0 | \mathcal{O}(x) | p_1 p_2 \cdots p_n \rangle \end{aligned}$$

Examples:



with $\mathcal{O}(x) = \overline{\psi}\gamma^{\mu}\psi(x)$



unitarity cut

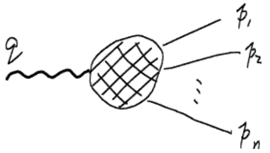
Interesting enough ?

Yes, we found form factor inherits can inherit nice properties of amplitudes. And we do have the simplicity and duality picture.

We will consider on a special half-BPS operator, which belongs to the stress-tensor supermultiplet of N=4 SYM.

The primary operator : $O(x) = \mathrm{Tr}(\phi_{12}^2)(x)$

$$F(1, ..., n; q) = \langle 0|\mathrm{Tr}(\phi_{12}^2)(q)|p_1 p_2 \cdots p_n\rangle$$



Later we consider the full supermultiplet and generalize to super form factor.

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Form factor $F(1, ..., n; q) = \langle 0|\mathrm{Tr}(\phi_{12}^2)(q)|p_1 p_2 \cdots p_n\rangle$

Simple !

$$F_{\mathrm{MHV}}(1^+, .., i_\phi, .., j_\phi, .., n^+; q) = \frac{\langle i\ j\rangle^2}{\langle 1\ 2\rangle \cdots \langle n\ 1\rangle}$$

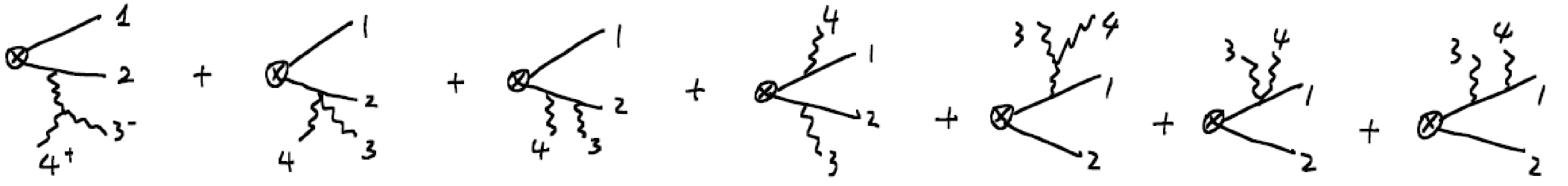
BCFW recursion, MHV rules, unitarity method can be used with simple input !

Compare to: $A_{\mathrm{MHV}}(1^+, .., i^-, .., j^-, .., n^+) = \frac{\langle i\ j\rangle^4}{\langle 1\ 2\rangle \cdots \langle n\ 1\rangle}$

A next-to-MHV example: $F_{\mathrm{NMHV}}(1_\phi, 2_\phi, 3^-, 4^+)$

Feynman diagram method: $F_{\text{NMHV}}(1_\phi, 2_\phi, 3^-, 4^+)$

All Feynman diagrams (color-ordered) :



(Operator is color singlet, therefore should be inserted in all possible positions.)

Choose $\quad \epsilon_3^- = \dfrac{\lambda_3 \tilde{\lambda}_4}{[3\ 4]}\,, \qquad \epsilon_4^+ = \dfrac{\lambda_3 \tilde{\lambda}_4}{\langle 3\ 4\rangle}$

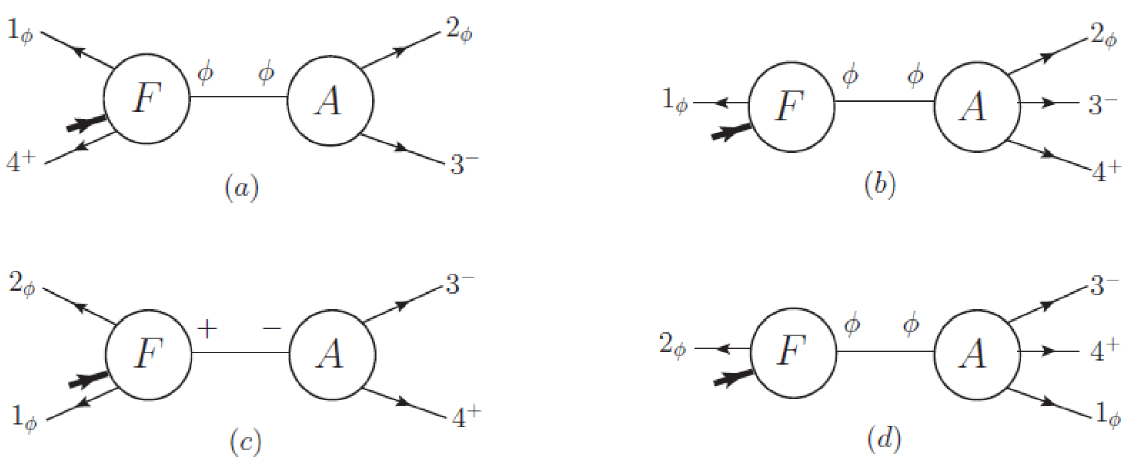
$$F = \frac{\langle 1\ 3\rangle [4\ 2]}{\langle 1\ 4\rangle [3\ 2] s_{34}} - \frac{\langle 1\ 3\rangle^2 [4\ 1]}{\langle 1\ 4\rangle s_{34} s_{341}} - \frac{[4\ 2]^2 \langle 2\ 3\rangle}{[3\ 2] s_{34} s_{234}}$$

MHV rules :

(Cachazo, Svrcek, Witten)

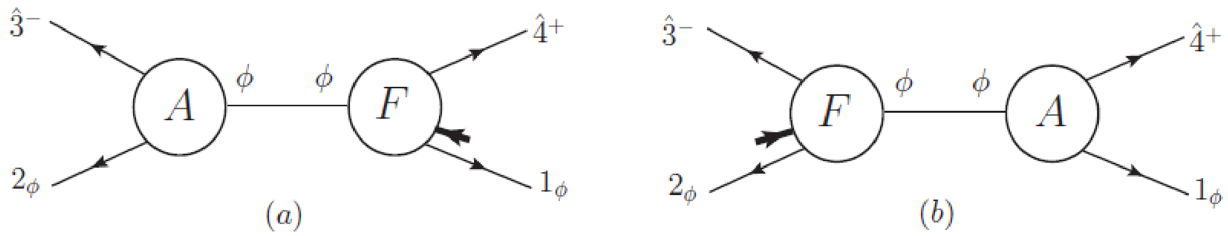
$$
\begin{aligned}
\text{Diagram } (a) &= \frac{[2\eta]}{[\eta 3]} \frac{1}{[32]\langle 41\rangle} \frac{\langle 1|q-p_4|\eta]}{|\langle 4|q-p_1|\eta]}\,, \\
\text{Diagram } (b) &= \frac{\langle 23\rangle}{\langle 34\rangle s_{234}} \frac{\langle 3|p_2+p_4|\eta]^2}{\langle 2|p_3+p_4|\eta]\langle 4|p_2+p_3|\eta]}\,, \\
\text{Diagram } (c) &= \frac{\langle 12\rangle}{[43]} \frac{[\eta 4]^3}{[3\eta]} \frac{1}{\langle 2|p_3+p_4|\eta]\langle 1|p_3+p_4|\eta]}\,, \\
\text{Diagram } (d) &= \frac{1}{s_{341}} \frac{\langle 13\rangle^2}{\langle 34\rangle\langle 41\rangle} \frac{\langle 3|p_4+p_1|\eta]}{\langle 1|p_3+p_4|\eta]}\,.
\end{aligned}
$$

Independent of the choice of reference spinor η .



BCFW recursion relation :

(Britto, Cachazo, Feng, Witten 2004)



Choose $[3\ 4\rangle$ shift :

$$F = \frac{1}{\langle 1|q|2]}\frac{[24]^2}{[23][34]}\frac{1}{s_{234}}\langle 1|q|4] + \frac{1}{\langle 1|q|2]}\frac{\langle 13\rangle^2}{\langle 34\rangle\langle 41\rangle}\frac{1}{s_{341}}\langle 3|q|2]$$

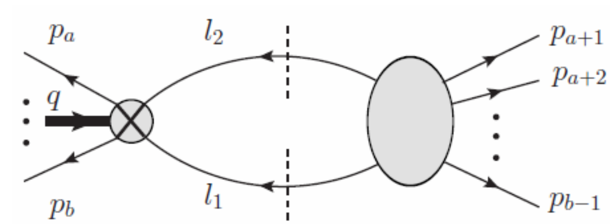
where $\langle 1|q|2]$ is indeed a spurious pole.

Methods are different, but results are all equivalent !

Unitarity method

(Bern, Dixon, Dunbar, Kosower 1994)

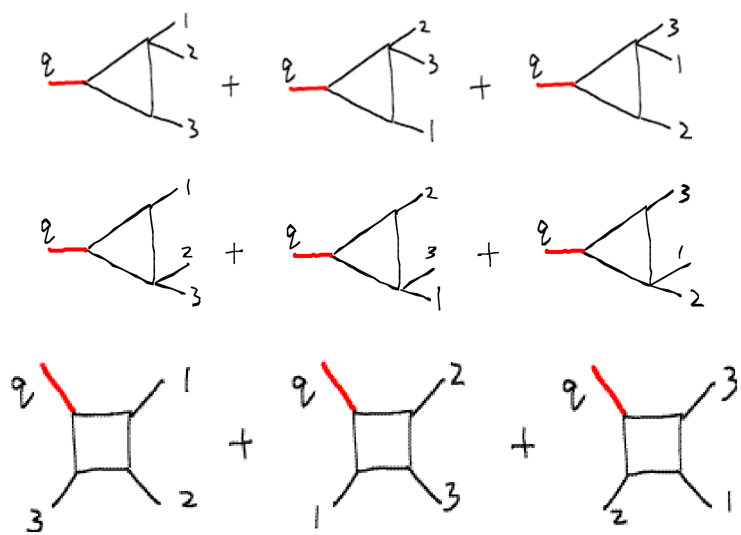
MHV one-loop result :



$$F^{(1)}_{\mathsf{MHV}} = F^{(0)}_{\mathsf{MHV}}\,[-\sum_{i=1}^{n}\frac{(-s_{ii+1})^{-\epsilon}}{\epsilon^2} + \sum_{a,b}\mathsf{Fin}^{\mathsf{2me}}(p_a,p_b,P,Q)]$$

(Operator is color singlet, therefore should be inserted in all possible positions.)

3-point example :



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Super Amplitudes $\mathcal{A}(\Phi(1)\cdots\Phi(n))$

Super states:

$$\Phi(p,\eta) = g^{+}(p) + \eta^{A}\psi_{A}(p) + \frac{\eta^{A}\eta^{B}}{2!}\phi_{AB}(p) + \epsilon_{ABCD}\frac{\eta^{A}\eta^{B}\eta^{C}}{3!}\tilde{\psi}^{D}(p) + \eta^{1}\eta^{2}\eta^{3}\eta^{4}g^{-}(p)$$

Superamplitudes combine different external states into a single nice expression, where different external states correspond to different expansion of fermionic component η.

Super MHV amplitude: $$\mathcal{A}_{\text{MHV}}(1,2,\cdots,n) = \frac{\delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1\ 2\rangle\langle 2\ 3\rangle\cdots\langle n\ 1\rangle}$$ (Nair)

$$(\eta_i)^4(\eta_j)^4 \ : \ A_{\text{MHV}}(1^{+},..,i^{-},..,j^{-},..,n^{+}) = \frac{\langle i\ j\rangle^4}{\langle 1\ 2\rangle\cdots\langle n\ 1\rangle}$$

$$(\eta_i)^4(\eta_j)^2(\eta_k)^2 \ : \ A_{\text{MHV}}(1^{+},..,i^{-},..,j_{\phi},...,k_{\phi},..,n^{+}) = \frac{\langle i\ j\rangle^2\langle i\ k\rangle^2}{\langle 1\ 2\rangle\cdots\langle n\ 1\rangle}$$

Underlying reason is the super Ward identity :

$$\langle 0|[s,\Phi(1)\Phi(2)...\Phi(n)]|0\rangle = 0, \quad s = P, Q, \bar{Q}$$

$$[P,\Phi(p,\eta)] = p\Phi(p,\eta), \ \ [Q,\Phi(p,\eta)] = \lambda\eta\Phi(p,\eta), \ \ [\bar{Q},\Phi(p,\eta)] = \tilde{\lambda}\frac{\partial}{\partial\eta}\Phi(p,\eta)$$

Super form factor

With supersymmetric expression, the calculation with various methods is generally much upgraded. The structure is also nicer, and can receive further constraints which are not present in the bosonic expression.

Therefore, we should supersymmetrize form factor.

In form factors, the situation is more complicated:

there are not only on-shell states, but also one operator inserted.

$$\langle 0|\mathrm{Tr}(\phi_{12}^2)(q)|p_1 p_2 \cdots p_n\rangle$$

First step: states → superstates

Second step: operator → supermultiplet

Super form factor : on-shell superstates

The first step, i.e. fixing the operator but with the on-shell states to be superstates, is much similar to the case of amplitudes:

$$\langle 0|\mathrm{Tr}(\phi_{12}^2)(0)|\Phi(1)\Phi(2)\ldots\Phi(n)\rangle$$

$$\mathcal{F}^{\mathrm{MHV}}_{\phi_{12}^2}(1,...,n) = \frac{\delta_{12}^{(4)}(\sum_i \lambda_i \eta_i)}{\langle 1\ 2\rangle\langle 2\ 3\rangle\cdots\langle n\ 1\rangle}$$

Different terms of η expansion give different external states.

The second step for operator is more non-trivial. We need to first look at the supermultiplet.

Stress-tensor supermultiplet

Half-BPS supermultiplet :

$$Q_{1,2}\mathrm{Tr}(\phi_{12}^2) = \bar{Q}^{3,4}\mathrm{Tr}(\phi_{12}^2) = 0 \qquad (Q_A\phi^{BC} = \delta_A^B\psi^C - \delta_A^C\psi^B, \dots)$$

$Q_{3,4}$ operate on $\mathrm{Tr}(\phi_{12}^2)$ generate the chiral sector of supermultiplet

$\bar{Q}^{1,2}$ operate on $\mathrm{Tr}(\phi_{12}^2)$ generate the anti-chiral sector of supermultiplet

---

Chiral subset : $\quad \mathcal{T}(x,\theta^{3,4},0) \; = \; e^{Q_3\theta^3+Q_4\theta^4}\mathrm{Tr}(\phi_{12}\phi_{12})(x)$

$$\begin{aligned}
\to \quad & \mathrm{Tr}(\phi_{12}\phi_{12})(x) \\
+ \; & [\ \mathrm{Tr}(\psi_{123}\psi_{123}) + \mathrm{Tr}(\phi_{12}\phi_{23}\phi_{31})(x)\ ] \times (\theta^3)^2 + \dots \\
+ \; & [\ \mathrm{Tr}(F_{\mathrm{SD}}^2) + \mathrm{Tr}(\psi_{ACD}\psi_{BCD}\phi_{AB}) + \mathrm{Tr}(\phi_{AB}\phi_{CD}\phi^{AB}\phi^{CD})(x)\ ] \times (\theta^3)^2(\theta^4)^2
\end{aligned}$$

Full stress-tensor supermultiplet (in harmonic superspace presentation) :

$$\begin{aligned}
\mathcal{T}(x,\theta^+,\bar{\theta}_-) \; &= \; e^{Q_+\theta^+ + \bar{Q}^-\bar{\theta}_-}\mathrm{Tr}(\phi^+\phi^+)(x) \\
&= \; \mathcal{T}(x,\theta^+,0) \; + \; \mathcal{T}(x,0,\bar{\theta}_-) \; + \; (\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta})T_{\mu\nu} \; + \; \cdots
\end{aligned}$$

Chiral super form factor:

The chiral super form factor can be derived as :

$$\int d^4x e^{iq\cdot x} d^4\theta^{3,4} e^{\gamma_3\theta^3+\gamma_4\theta^4} \langle 0|\mathcal{T}(x,\theta^{3,4},0)|\Phi(1)\cdots\Phi(n)\rangle$$

$$= \frac{\delta_{34}^{(4)}(\gamma - \sum_i \lambda_i\eta_i)\delta_{12}^{(4)}(\sum_i \lambda_i\eta_i)}{\langle 1\ 2\rangle\langle 2\ 3\rangle\cdots\langle n\ 1\rangle}$$

Different operators are given by the expansion of fermionic momentum γ .

Examples:

$$(\gamma)^4 \ : \ \mathcal{F}_{\phi_{12}^2} = \frac{\delta_{12}^{(4)}(\sum_i \lambda_i\eta_i)}{\langle 1\ 2\rangle\langle 2\ 3\rangle\cdots\langle n\ 1\rangle}$$

$$(\gamma)^0 \ : \ \mathcal{F}_{\mathrm{SD}} = \frac{\delta_{12}^{(8)}(\sum_i \lambda_i\eta_i)}{\langle 1\ 2\rangle\langle 2\ 3\rangle\cdots\langle n\ 1\rangle}$$

$$\mathrm{SD} \to \mathrm{Tr}(F_{\mathrm{SD}}^2) + \mathrm{Tr}(\psi_{ACD}\psi_{BCD}\phi_{AB}) + \mathrm{Tr}(\phi_{AB}\phi_{CD}\phi^{AB}\phi^{CD})$$

Non-chiral super form factor

How about the full supermultiplet, which is really a non-chiral presentation?

$$\langle 0|\mathcal{T}(x,\theta^+,\bar\theta_-)|\Phi(1)\cdots\Phi(n)\rangle$$

Solution: using non-chiral superspace by doing Fourier transformation to change half of chiral superspace to anti-chiral superspace !

(amplitudes considered by Huang)

$$\begin{aligned}\Phi(p,\eta_+,\tilde\eta^-) &= \int d^2\eta_- e^{\eta_-\tilde\eta^-}\Phi(p,\eta)\\ &= g^+(p)(\tilde\eta^-)^2+\ldots+\phi(p)+\ldots+g^-(p)(\eta_+)^2\end{aligned}$$

$$\mathcal{F}^{\rm nc}_{\phi^2}(\lambda,\tilde\lambda,\eta_+,\tilde\eta^-)=\prod_{i=1}^n\int d^2\eta_{-,i}e^{i\eta_{-,i}\tilde\eta_i^-}\times\mathcal{F}_{\phi^2}(\lambda,\tilde\lambda,\eta_+,\eta_-)$$

We have:

$$\int d^4x e^{iq\cdot x}d^4\theta^+e^{\gamma_+\theta^+}d^4\bar\theta_-e^{\tilde\gamma^-\bar\theta_-}\langle 0|\mathcal{T}(x,\theta^+,\bar\theta_-)|\Phi(1)\cdots\Phi(n)\rangle$$

$$\mathcal{F}=\delta^{(4)}\big(\gamma_+-\sum_{i=1}^n\eta_{+,i}\lambda_i\big)\,\delta^{(4)}\big(\tilde\gamma^--\sum_{i=1}^n\tilde\eta_i^-\tilde\lambda_i\big)\,\mathcal{F}^{\rm nc}_{\phi^2}$$

$$\begin{aligned}&Q_+: && (\eta_+\lambda-\gamma_+)\mathcal{F}=0\,,\\ &\bar Q^-: && (\tilde\eta^-\tilde\lambda-\tilde\gamma^-)\mathcal{F}=0\,,\\ &Q_-: && \left(q\frac{\partial}{\partial\tilde\gamma^-}-\lambda\frac{\partial}{\partial\tilde\eta^-}\right)\mathcal{F}=0\,,\\ &\bar Q^+: && \left(q\frac{\partial}{\partial\gamma_+}-\tilde\lambda\frac{\partial}{\partial\eta_+}\right)\mathcal{F}=0\,,\end{aligned}$$

Non-chiral super form factor

$$\mathcal{F} = \delta^{(4)}\big(\gamma_+ - \sum_{i=1}^{n} \eta_{+,i}\lambda_i\big)\, \delta^{(4)}\big(\tilde{\gamma}^- - \sum_{i=1}^{n} \tilde{\eta}_i^- \tilde{\lambda}_i\big)\, \mathcal{F}^{\rm nc}_{\phi^2}$$

Different operators are given by the expansion of fermionic momentum γ .

In components:

$$\begin{aligned} \mathcal{T}(x,\theta^+,\bar{\theta}_-) &= e^{Q_+\theta^+ + \bar{Q}^-\bar{\theta}_-}\mathrm{Tr}(\phi^+\phi^+)(x) \\ &= \mathcal{T}(x,\theta^+,0) + \mathcal{T}(x,0,\bar{\theta}_-) + (\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta})T_{\mu\nu} + \cdots \end{aligned}$$

$$(\gamma_+)^4(\tilde{\gamma}^-)^4 \; : \; \mathcal{F}_{\phi^2}$$

$$(\gamma)^0(\tilde{\gamma}^-)^4 \; : \; \mathcal{F}_{\mathrm{SD}} = \delta^{(4)}(\sum_i \lambda_i \eta_{i,+}) \times \mathcal{F}^{\mathrm{nc}}_{\phi^2}$$

$$(\gamma)^2(\tilde{\gamma}^-)^2 \; : \; \mathcal{F}_{T_{\mu\nu}} = \delta^{(2)}(\sum_i \lambda_i \eta_{i,+})\delta^{(2)}(\sum_i \tilde{\lambda}_i \tilde{\eta}_i^-) \times \mathcal{F}^{\mathrm{nc}}_{\phi^2}$$

$$(\gamma)^4(\tilde{\gamma}^-)^0 \; : \; \mathcal{F}_{\mathrm{ASD}} = \delta^{(4)}(\sum_i \tilde{\lambda}_i \tilde{\eta}_i^-) \times \mathcal{F}^{\mathrm{nc}}_{\phi^2} \longrightarrow$$

What is MHV case for this anti-chiral operator?

A non-trivial check:

$$\begin{aligned}
\mathcal{F}^{\text{MHV}}_{\text{ASD}} &= \delta^{(4)}(\sum_{i=1}^{n} \tilde{\eta}_i^{-} \tilde{\lambda}_i) \mathcal{F}^{\text{MHV,nc}}_{\phi^2} \\
&= \delta^{(4)}(\sum_{i=1}^{n} \tilde{\eta}_i^{-} \tilde{\lambda}_i) \prod_{i=1}^{n} \int d^2\eta_{-,i} \; e^{i\eta_{-,i}\tilde{\eta}_i^{-}} \; \frac{\delta^{(4)}(\sum_{i=1}^{n} \eta_{-,i}\lambda_i)}{\langle 12 \rangle \cdots \langle n1 \rangle} \\
&= \frac{\sum_{i<j} \langle ij \rangle [ij] \sum_{k<l} \langle kl \rangle [kl]}{\langle 12 \rangle \cdots \langle n1 \rangle} \prod_{i=1}^{n} (\tilde{\eta}_i^{-})^2 \\
&= \frac{q^4}{\langle 12 \rangle \cdots \langle n1 \rangle} \prod_{i=1}^{n} (\tilde{\eta}_i^{-})^2
\end{aligned}$$

This is exactly the anti-self-dual operator with all plus helicity gluons.

$$F_{\text{ASD}}(1^+, \cdots, n^+) = \frac{q^4}{\langle 12 \rangle \cdots \langle n1 \rangle}$$

(Dixon, Glover and Khoze)

This result was a surprise and was proved recursively, here we see that it is simply from SUSY Ward identity !

Supersymmetric methods

Applying BCFW recursion relation, MHV rules, and unitarity method at supersymmetric level.

In particular, MHV rules is important to study the dual picture.

We will consider dual MHV rules later.

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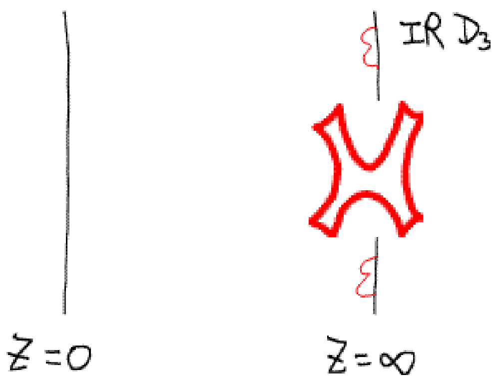
- Supersymmetric form factor
- Dual picture

  --- Periodic Wilson line
- Summary and open problems

Dual picture from AdS/CFT duality (Alday, Maldacena)

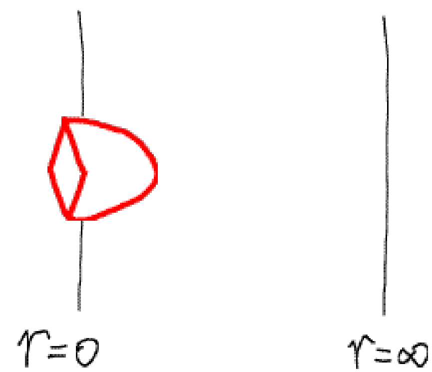
Scattering amplitudes:

Poincare metric : $\frac{d^2x_{1,3} + d^2z}{z^2}$ $\frac{d^2y_{1,3} + d^2r}{r^2}$

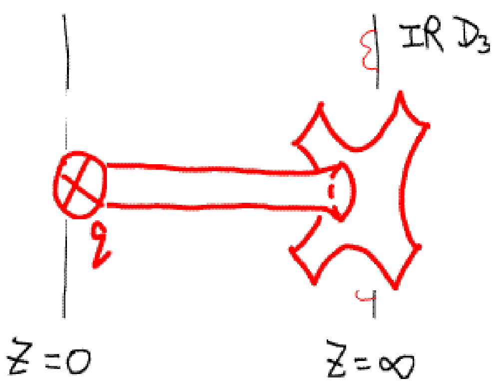


T-duality

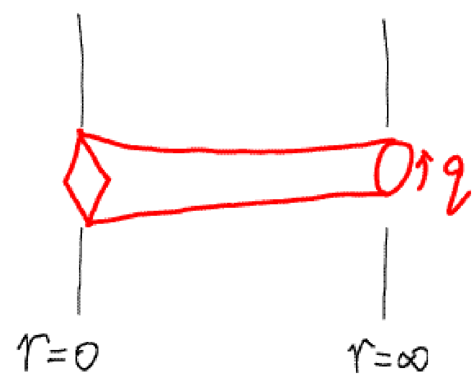
$$z \leftrightarrow \frac{1}{r}$$



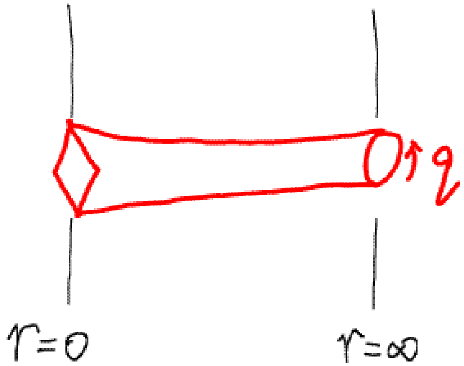
Form factor:



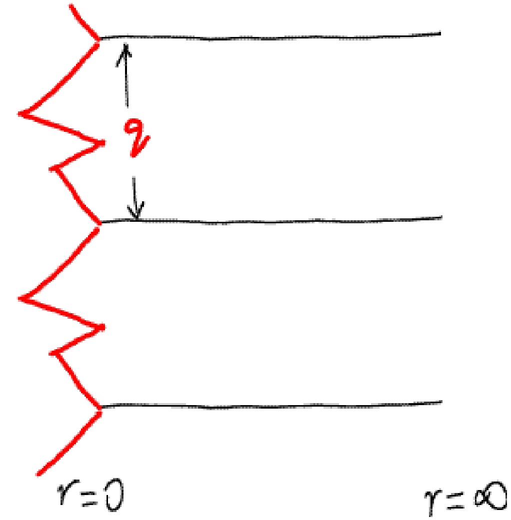
T-duality



Form factor / Periodic Wilson line duality



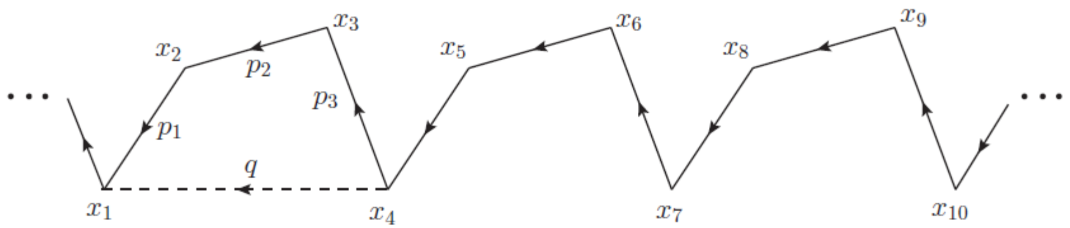
equivalent
periodic
picture



Strong coupling considered. (Maldacena, Zhiboedov)

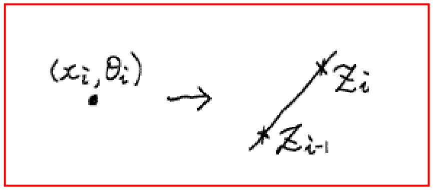
Question is if there a duality at weak coupling ?

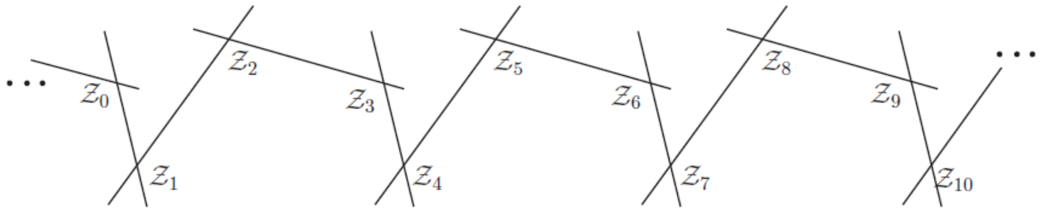
Periodic configuration in momentum space



$$x_i - x_{i+1} = p_i = \lambda_i \tilde{\lambda}_i \ , \qquad \theta_i - \theta_{i+1} = \lambda_i \eta_i$$

One can also define momentum twistor space :





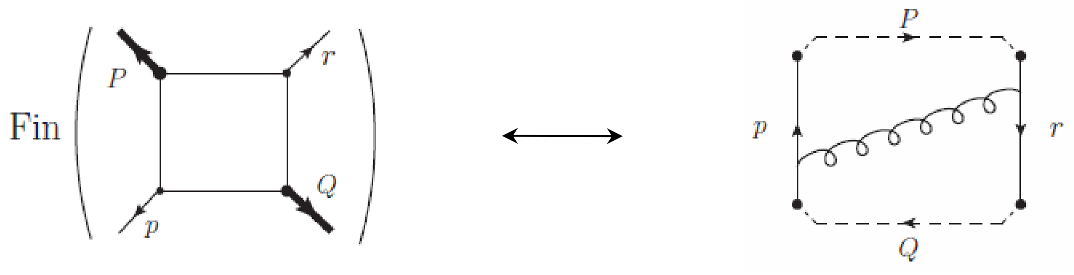
General NMHV form factor can be written in terms of momentum twistors as:

$$\mathcal{F}^{(0)}_{\mathrm{NMHV}} = \mathcal{F}^{(0)}_{\mathrm{MHV}} \sum_{i=1}^{n} \sum_{j=i+2}^{i+n-1} [*, i-1, i, j-1, j] \qquad \mathcal{Z}_* = (0, \xi, 0)$$

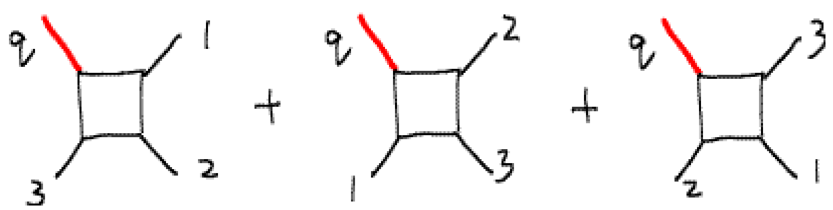
Duality at MHV one loop

$$F^{(1)}_{\text{MHV}} = F^{(0)}_{\text{MHV}} [- \sum_{i=1}^{n} \frac{(-s_{ii+1})^{-\epsilon}}{\epsilon^2} + \sum_{a,b} \text{Fin}^{2\text{me}}(p_a, p_b, P, Q)]$$

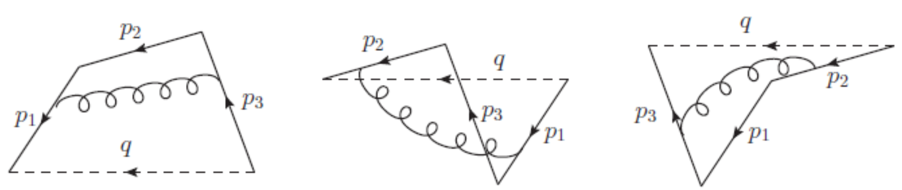
(Brandhuber, Heslop, Travaglini)

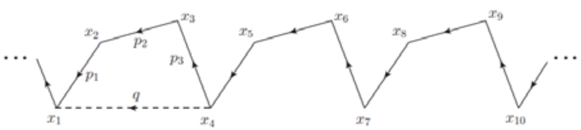


3-point example:



Dual picture:





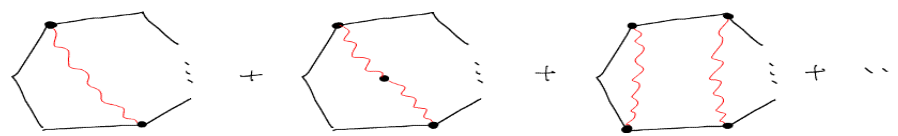
Dual MHV rules

One can obtain dual MHV rule in momentum space. Dual MHV diagrams are deeply related to amplitudes/Wilson loops duality.

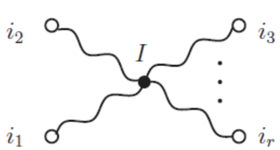
They can be given by Wilson loop! (Mason, Skinner)

Superamplitudes =

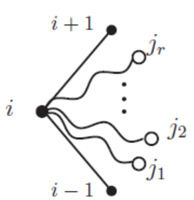
Super Wilson loop =



$$\frac{1}{x_{ij}^2}\int d^4\eta_{ij}\ \delta^{0|8}(\ell_{ij}\eta_{ij}+\theta_{ij})$$



$$g_{\mathrm{YM}}^2\int d^4x_I d^8\theta_I\, \frac{1}{\langle \ell_{Ii_1}\ell_{Ii_2}\rangle\langle \ell_{Ii_2}\ell_{Ii_3}\rangle\cdots\langle \ell_{Ii_{r-1}}\ell_{Ii_r}\rangle\langle \ell_{Ii_r}\ell_{Ii_1}\rangle}$$



$$\frac{\langle i{-}1\ i\rangle}{\langle i{-}1\ \ell_{ij_1}\rangle\langle \ell_{ij_1}\ell_{ij_2}\rangle\langle \ell_{ij_2}\ell_{ij_3}\rangle\cdots\langle \ell_{ij_{r-1}}\ell_{ij_r}\rangle\langle \ell_{ij_r}\ i\rangle}$$

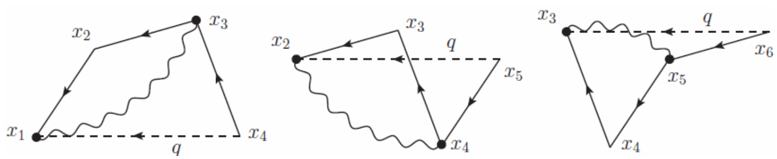
(Bullimore, Mason, Skinner)

(Brandhuber, Spence, Travaglini, GY)

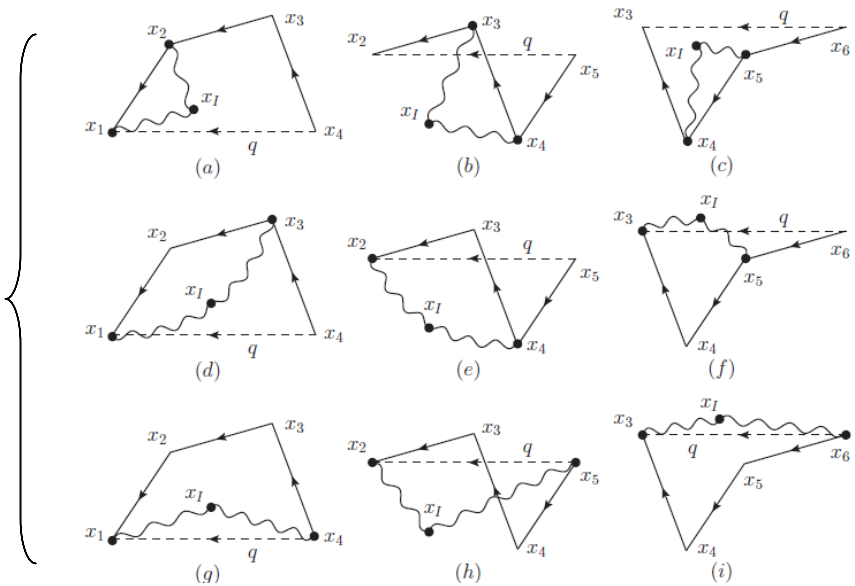
Dual MHV rules

We find there are dual MHV rules for form factor !

$$F^{\text{tree}}_{\text{NMHV}} =$$



$$F^{(1)}_{\text{MHV}} =$$



Prescription for higher-loop can also be given.

Dual MHV rules

For scattering amplitudes, the dual MHV rules can be derived from null-polygonal Wilson loops in twistor space. (Mason, Skinner)

This therefore provides a formal proof for amplitudes/Wilson loops duality.

For form factor, the dual MHV rule is obtained only by mapping to the conventional MHV diagrams.

It is not yet clear whether it can be interpreted in terms of periodic Wilson line. A derivation of this would provide a formal proof for the duality between form factor and periodic Wilson line.

Outline

- Background and Motivation
- A simple look at bosonic form factor
- Supersymmetric form factor
- Dual picture
- Summary and outlook

Summary

- From amplitudes to form factor
- Some simple result of form factor
- Supersymmetric generalization for form factor
- Dual picture

Outlook

- Understand duality at higher loops
- With other operators
- Two and more operators inserted

Thank you.