

Anomalous Dimensions @ Strong Coupling

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based on [Brenno Carlini Vallilo, LM, arXiv:1102.1219 [hep-th]]
for a pedagogical review, [LM, arXiv:1104.2604][hep-th]]

XI Workshop on Nonperturbative QCD, Paris

Outline

- Motivations: *conformal dimensions*
- *AdS/CFT* recap
- *@ Strong coupling* with string theory
- *Konishi* multiplet on the string worldsheet

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Motivations

What are anomalous dimensions in QFT?

- Take a local gauge invariant op $\mathcal{O}(x) = \text{Tr } \phi^i(x) \phi^i(x)$
- In a CFT e.g. $\mathcal{N} = 4$ SYM: global symmetry $\supset$ dilations D

$$[D, \mathcal{O}(x)] = i\Delta \mathcal{O}(x)$$

- $\Delta \equiv$ conformal dimension $\leftrightarrow$ 2 pt functions

$$\langle \mathcal{O}(x) \mathcal{O}(y) \rangle \sim \frac{1}{|x - y|^{2\Delta}}$$

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What do we know about Δ ?

hereafter: Planar limit $N \rightarrow \infty$

$g_{YM} = 0$: classically $\Delta = \Delta_0 \equiv$ "engineering dimension"

E.g. $\text{Tr } \phi^i \phi^i$ has $\Delta_0 = 2$.



Turn on 't Hooft coupling $\lambda = g_{YM}^2 N \Rightarrow$ generate
anomalous dimensions $\equiv \Delta - \Delta_0 \neq 0$

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$\lambda \ll 1$: *weak coupling* planar loop expansion

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protected (BPS): $\Delta - \Delta_0 = 0$

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@ Strong coupling? Use holography!

[Maldacena '97]

gauge		gravity
$\mathcal{N} = 4$ SYM	=	IIB string theory on $AdS_5 \times S^5$
$\sqrt{\lambda}$	=	radius ² / α'
$\lambda \gg 1$	=	small curvature
strongly coupled CFT	=	perturbative string theory

$$CFT \text{ conformal dim. } \Delta = E \text{ energy in AdS}$$

State of the Art: [Gromov et al. '09] conjectured a set of Y-system eqs. whose numerical solution gives $\Delta(\lambda)$ at any value of the coupling.

Problem: very hard coupled integral eqs.!

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AdS/CFT dictionary

<i>BPS</i> :	$\Delta - \Delta_0 =$	0		massless vertex op = SUGRA
<i>short non-BPS</i> :	$\Delta - \Delta_0 \sim$	$\sqrt[4]{\lambda} + \dots$		<i>massive</i> vertex op = quantum strings
<i>long non-BPS</i> :	$\Delta - \Delta_0 \sim$	$\sqrt{\lambda} + \dots$		semi-classical strings

[Beisert et al. '10]

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Anomalous dimensions @ strong coupling

- $\Delta = E \Rightarrow$ Compute the spectrum of *perturbative* string states in *AdS* in the near flat space limit

$$E \sim c_0 \sqrt[4]{\lambda} + c_1 + \frac{c_2}{\sqrt[4]{\lambda}} + \mathcal{O}(1/\sqrt{\lambda})$$

- Flat space is mass $\sim \sqrt{n/\alpha'}$

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- Simple string: sitting still at the center of AdS (with same quantum #s as SYM op. in the Konishi multiplet)
- Type IIB string σ model in $AdS_5 \times S^5$
- Compute worldsheet physical state condition (Virasoro) at 1-loop

$$T \sim \frac{E(E-4)}{\sqrt{\lambda}} + \text{quantum corrections} = 0$$

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[LM+Vallilo '11]

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Strings propagating in $AdS_5 \times S^5$ are described by a σ model on the
 supercoset $PSU(2, 2|4)/SO(1, 4) \times SO(5)$

$$\text{coset representative : } g(\sigma, \tau) \rightarrow \underbrace{g_0}_{\text{global}} g(\sigma, \tau) \underbrace{h(\sigma, \tau)}_{\text{local}}$$

[Metsaev+Tseytlin '98]

- σ model action in terms of $J = g^{-1} dg$ Maurer-Cartan one-forms
 + appropriate ghosts (called pure spinors).

- Physical states:

- $\#_{ghost} = (1, 1)$ vertex operators V
- in the BRST cohomology $QV = 0, \quad V \neq QA.$
- satisfying Virasoro constraint $T_{\bar{1}} V = 0.$

- Massless vertex op's: sugra+KK modes with $E \sim \mathcal{O}(1)$

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- solves the worldsheet eoms
- vanishing BRST charge

■ Quantize the σ model action in the background field method around $\tilde{g}(\sigma, \tau)$.

■ Identify massive vertex operator

■ Compute the 1-loop quantum corrections to Virasoro [LM&Vallilo '11]

$\phi_i E t$



$\uparrow t$

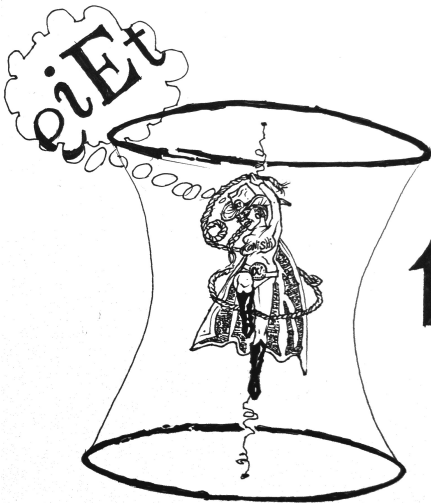
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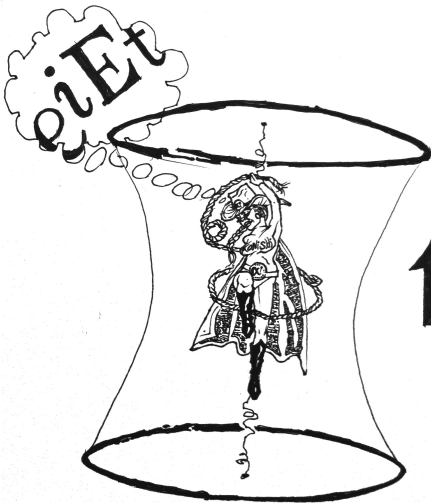
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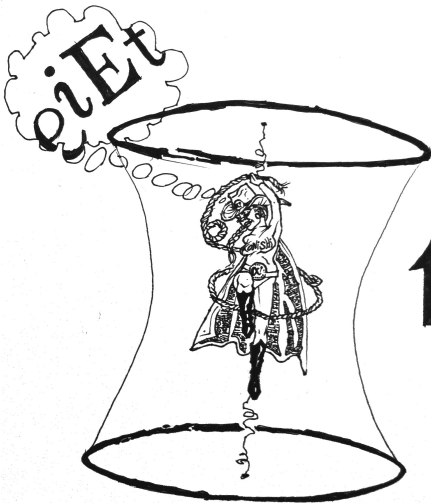
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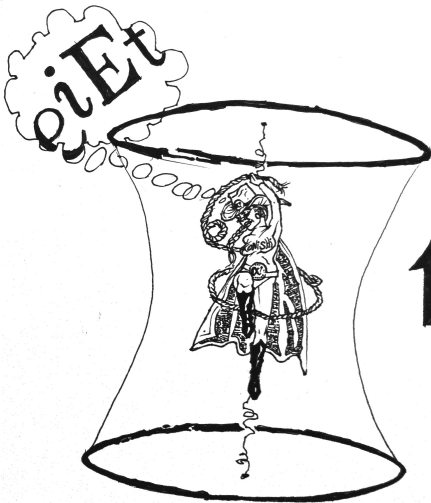
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[LM&Vallilo '11]

- **Konishi multiplet** = Vertex operators at 1st **massive** level

$$V = X_{(-1)} \bar{X}_{(-1)} (L\bar{L}) \quad \begin{array}{l} \text{Lorentz spin } (1, 1) \\ SU(4) \text{ singlet} \\ \Delta_0 = 6 \end{array}$$

- Virasoro constraint at 1-loop $T|V\rangle = 0$

$$-\underbrace{\frac{\overbrace{E(\overbrace{E}^{\text{class. centr. charge}} \overbrace{-4}^{\text{charge}})}^{2\sqrt{\lambda}}}{2\sqrt{\lambda}}}_{SO(2,4) \text{ Casimir}} + \underbrace{2\sqrt{1 + (E/\sqrt{\lambda})^2}}_{\text{massive osc.}} - \underbrace{\frac{2}{\sqrt{\lambda}}}_{\text{quartic}} = 0$$

$$\text{anomalous dim. : } \Delta - \Delta_0 = 2\sqrt[4]{\lambda} - 4 + \frac{2}{\sqrt[4]{\lambda}} + \mathcal{O}(1/\sqrt{\lambda})$$

[LM&Vallilo '11]

[Gromov et al. '11]

[Roiban&Tseytlin '11]

Application: *Pomeron*

[Brower et al. '06]

■ Leading Regge trajectory

$$\mathcal{O}_{string} \sim (\partial X \bar{\partial} X)^{j/2} e^{jEt} \longleftrightarrow \text{leading twist op's : } \text{Tr } F_{+\nu} D^{j-2} F_{+}^{\nu}$$

■ Compute its energy at strong coupling

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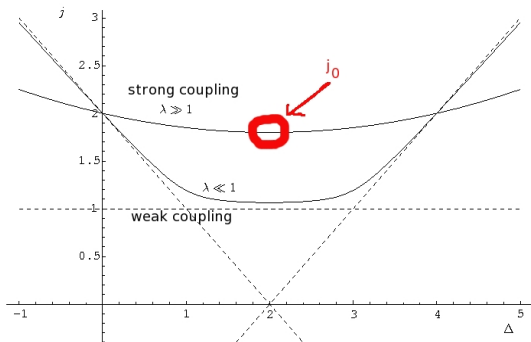
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$$j_0 \equiv j(\Delta = 2) = 2 - \underbrace{\frac{2}{\sqrt{\lambda}}}_{\text{loops @ strong coupl.}} \pm \underbrace{\quad}_{\text{loops @ strong coupl.}}$$

[Brower et al. '06]

■ Interpolating $j_0(\lambda)$ at finite λ using integrability?



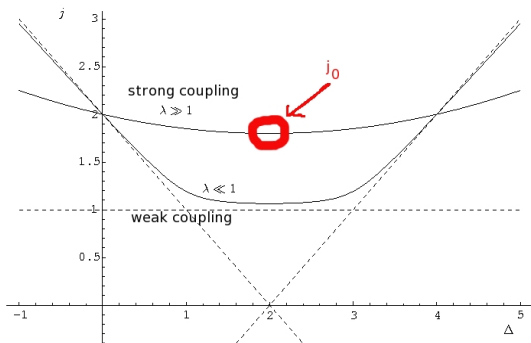
pic from [Brower et al. '06]

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pic from [Brower et al. '06]

THANK YOU!

Artwork courtesy of Miriam W. Carothers