String Scattering Amplitudes in High Energy Limits

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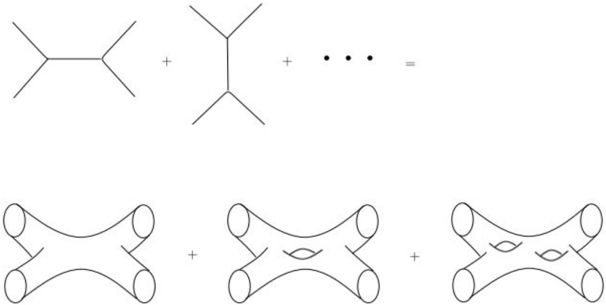
1. Outline

1. About String Scattering Amplitudes
2. High Energy Limits
   (a) $E \to \infty$, fixed angle: hard scattering
   (b) $E \to \infty$, small angle: Regge limit
3. Summary

2. String Scattering Amplitudes

- Amplitudes in field theory and string theory



- QFT amplitude

$$A^{(J)}_{tree} \sim E^{-2(1-J)}$$

$$\Rightarrow A^{(J)}_{1-loop} \sim \int d^4p \frac{\left(A^{(J)}_{tree}\right)^2}{\left(p^2\right)^2} \sim \int d^4E \frac{E^{-4(1-J)}}{E^4}$$

- Sum over intermedian states

$$A = \sum_J A^{(J)}_{tree} \sim \sum_J a_J E^{-2(1-J)} \sim e^{-E}$$

- infinite high-spin particles

- perfect coefficients a_J

- QFT amplitude

$$A_{tree}^{(J)} \sim E^{-2(1-J)}$$

$$\Rightarrow A_{1-loop}^{(J)} \sim \int d^4p \frac{\left(A_{tree}^{(J)}\right)^2}{\left(p^2\right)^2} \sim \int d^4E \frac{E^{-4(1-J)}}{E^4}$$

- Sum over intermedian states

$$A = \sum_J A_{tree}^{(J)} \sim \sum_J a_J E^{-2(1-J)} \sim e^{-E}$$

- infinite high-spin particles $\Leftrightarrow$ Regge poles

- perfect coefficients a_J $\Leftrightarrow$ symmetry: Gross' conjecture

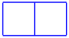
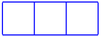
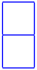
- Veneziano amplitude (4-tachyon)

$$\begin{aligned}
\mathcal{T} &= \left\langle e^{ik_1 X(z_1)\cdot} e^{ik_2\cdot X(z_2)} e^{ik_3\cdot X(z_3)} e^{ik_4\cdot X(z_4)} \right\rangle \\
&= \int dz_1 dz_2 dz_3 dz_4 \prod_{1\le i<j\le 4} |z_i - z_j|^{2k_i\cdot k_j} \\
&= \int_0^1 dx \cdot x^{-\frac{s}{2}-2} (1-x)^{-\frac{t}{2}-2} \\
&= B\left(-\frac{s}{2}-1, -\frac{t}{2}-1\right)
\end{aligned}$$

- $E \to \infty$, fixed angle

- Exponential fall-off: $\mathcal{T} \sim e^{-\alpha E}$

- String spectrum

Mass level	Positive-norm states	Zero-norm states
$M^2 = -2$	•	n/a
$M^2 = 0$		•
$M^2 = 2$		 , •
$M^2 = 4$	 , 	 , $2 \times$  , •

3. High Energy Limits

- $E \to \infty$, fixed angle ϕ

  $\Rightarrow s \sim E^2 \to \infty$, $\dfrac{t}{s} \sim \sin^2 \frac{\phi}{2} \to$ fixed.

  Gross conjecture: symmetry $\to$ linear relations.

- $E \to \infty$, small angle ϕ

  $\Rightarrow s \sim E^2 \to \infty$, $t \sim E^2 \sin^2 \frac{\phi}{2} \to$ fixed

  Regge limit

3.1. $E \to \infty$, Fixed Angle

- $E \to \infty$, fixed angle ϕ

  $\Rightarrow s \sim E^2 \to \infty$, $\dfrac{t}{s} \sim \sin^2 \frac{\phi}{2} \to$ fixed.

- 4-point scattering amplitudes

  at mass level $M^2 = 2(N-1)$,

$$\mathcal{T}^{(N,2m,q)} = \langle V_1 V^{(N,2m,q)}(k) V_3 V_4 \rangle,$$

  where the vertex of leading states are

$$V^{(N,2m,q)}(k) \sim \left(\alpha_{-1}^T\right)^{N-2m-2q} \left(\alpha_{-1}^L\right)^{2m} \left(\alpha_{-2}^L\right)^q |0; k\rangle,$$

$$\mathcal{T}^{(N,2m,q)} = \langle V_1 V^{(N,2m,q)}(k) V_3 V_4 \rangle,$$

- Linear relations

$$\boxed{\frac{\mathcal{T}^{(N,2m,q)}}{\mathcal{T}^{(N,0,0)}} = \left(-\frac{1}{2M}\right)^q \left(\frac{1}{2M^2}\right)^m (2m-1)!!}$$

1. Decoupling of zero norm states

2. Virasoro constraints

3. Saddle point

3.2. $E \to \infty$, Small Angle: Regge Limit

- $E \to \infty$, small angle ϕ

  $\Rightarrow s \sim E^2 \to \infty$, $t \sim E^2 \sin^2 \frac{\phi}{2} \to$ fixed.

- "Leading states" for the mass level $M^2 = 2\,(N-1)\,,$

$$|N, k_n, q_m\rangle = \prod_{n>0} (\alpha^T_{-n})^{k_n} \prod_{m>0} (\alpha^L_{-m})^{q_m} |0\rangle$$

$$\sum_{n,m} n k_n + m q_m = N$$

- Scattering amplitude in Regge limit,

$$\mathcal{T}^{(N,k_n,q_m)} = \left(-\frac{i}{M_2}\right)^{q_1} U\left(-q_1, \frac{t}{2}+2-q_1, \frac{t+M^2+2}{2}\right)$$
$$\cdot B\left(-1-\frac{s}{2}, -1-\frac{t}{2}\right) \cdot \prod_{n=1}\left[i\sqrt{-t}(n-1)!\right]^{k_n}$$
$$\cdot \prod_{m=2}\left[i\left(t+M^2+2\right)(m-1)!\left(-\frac{1}{2M_2}\right)\right]^{q_m}$$

- Kummer function of the first kind $U(a,c,x)$

$$xU''(x) + (c-x)U'(x) - aU(x) = 0$$

- Reproducing the ratios when $|t| \to \infty$

$$U\left(-2m, \frac{t}{2} + 2 - 2m, \frac{t}{2}\right) = \sum_{j=0}^{2m} (-2m)_j \left(-1 - \frac{t}{2}\right)_j \frac{(-2)^j}{j!} t^{2m-j}$$

$$= 0 \cdot t^{2m} + 0 \cdot t^{2m-1} + \cdots\cdots + 0 \cdot t^{m+1} + \frac{(2m)!}{(-4)^m m!} t^m + O\left(t^{m-1}\right)$$

- Universal power-law behavior: $\mathcal{T} \sim E^{\alpha(t)}$

- Compactified space

ϕ	(s,t,N,K)	$E \to \infty$	$\tilde{\phi}$
ϕ fixed	$(s,t) \gg (N,K)$	e^{-E}	$\tilde{\phi}$ fixed
ϕ fixed	$(s,t,K) \gg N$	E^{-c}	$\tilde{\phi} \sim 0$
$\phi \sim 0$	$s \gg (t,N,K)$	E^{-c}	$\tilde{\phi} \sim 0$
$\phi \sim 0$	$(s,K) \gg (t,N)$	e^{-E}	$\tilde{\phi}$ fixed

4. Summary

- High energy, fixed angle ⇒ linear relations,

$$\lim_{E\to\infty} \frac{\mathcal{T}^{(n,2p,q)}}{\mathcal{T}^{(n,0,0)}} = \left(-\frac{1}{2M}\right)^q \left(\frac{1}{2M^2}\right)^p (2p-1)!!$$

- High energy, small angle (Regge) ⇒ Kummer function

$$\mathcal{T} \sim U\left(-q_1, \frac{t}{2}+2-q_1, \frac{t+M^2+2}{2}\right)$$

- Reproduce the linear relations from Regge limit $t \to \infty$,

$$U\left(-2m, \frac{t}{2}+2-2m, \frac{t}{2}\right) = 0 + \frac{(2m)!}{m!}\left(-\frac{t}{4}\right)^m + \mathcal{O}\left(t^{m-1}\right)$$