String-Gauge Dual Description of Deep Inelastic Scattering at Small x

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work with Richard C. Brower, Ina Sarčević and Chung-I Tan

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- More specifically, a recent conjecture by Maldacena relating string theory on $AdS_5 \times S_5$ to $\mathcal{N} = 4SYM$ allows us to study QCD at strong coupling.

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- To $\mathcal{O}(\lambda)$

$$\alpha(0) \simeq 1 + \frac{\log 2}{\pi^2}\lambda$$

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- Correspondence works in the limit

$$N_C \to \infty, \quad \lambda = g^2 N_C = R^4/\alpha'^2 \gg 1, \text{fixed}$$

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where

$$Im\ \chi(s,b,z,z') = \frac{g_0^2}{16\pi}\sqrt{\frac{\rho}{\pi}}\, e^{(1-\rho)\tau}\frac{\xi}{\sinh\xi}\frac{\exp(\frac{-\xi^2}{\rho\tau})}{\tau^{3/2}}$$

- χ is a function of only two variables

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\begin{aligned}
\xi &= \log(1 + v + \sqrt{v(2+v)}) \\
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- In the limit $\tau \gg 1$, $\lambda \gg 1$ and $\lambda/\tau \to 0$

$$\Re\chi \approx \cot(\frac{\pi\rho}{2})\Im\chi$$

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  Weak coupling:

$$\mathcal{K}(k_\perp, k'_\perp, s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\log s}} e^{-(\log k_\perp - \log k'_\perp)^2/4\mathcal{D}\log s}$$

$$j_0 = 1 + \frac{\log 2}{\pi^2}\lambda, \quad \mathcal{D} = \frac{14\zeta(3)}{\pi}\lambda/4\pi^2$$

  Strong coupling:

$$\mathcal{K}(z, z', s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\log s}} e^{-(\log z - \log z')^2/4\mathcal{D}\log s}$$

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- We can expand the exponential to get

$$A(s, -\mathbf{q}_\perp{}^2) = -2is \int d^2b e^{-i\mathbf{b}_\perp \cdot \mathbf{q}_\perp} (i\chi + \frac{(i\chi)^2}{2} + \cdots) \, .$$

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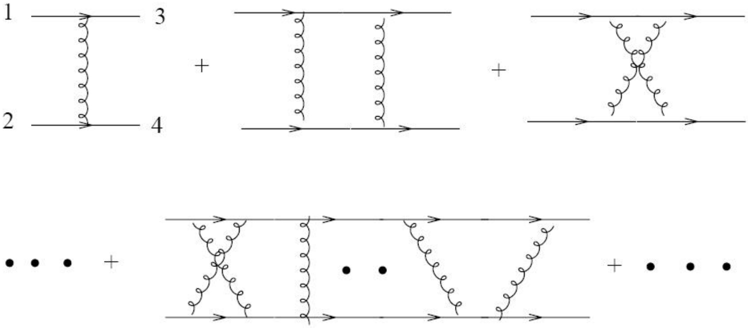

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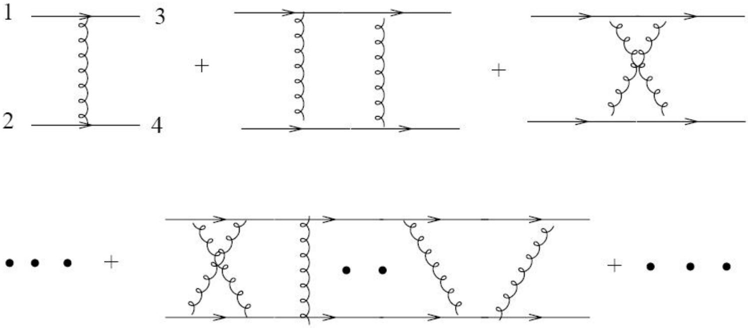

- Eikonal approximation in AdS space (Brower, Strassler, Tan; Cornalba,Costa,Penedones)

$$A(s,-{\mathbf{q}_\perp}^2) = 2is\int d^2b e^{-i\mathbf{b}_\perp\cdot\mathbf{q}_\perp}\int dzdz'\, P_{13}(z)P_{24}(z')(1-e^{i\chi(s,b,z,z')})$$

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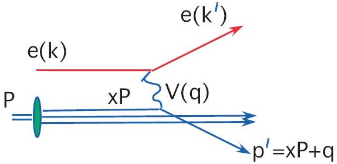

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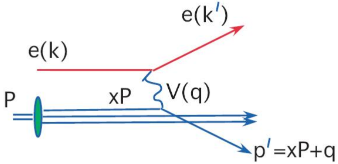

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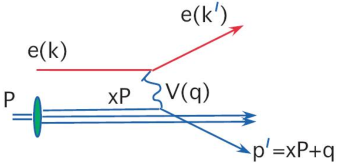

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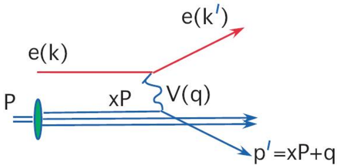

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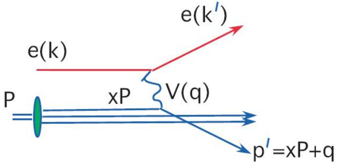

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- the scaling variable

$$x \approx \frac{Q^2}{s}$$

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- To calculate the total cross section we can use the optical theorem

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- We will look at $0.15GeV^2 < Q^2 < 400GeV^2$.
- At lower or higher Q^2 there is no experimental data with $x < 0.01$.

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We need to supply the wavefunctions for the photon and the proton. For the photon we will consider an R boson propagating through the bulk that couples to leptons on the boundary (Polchinski, Strassler 2003)

$$P_{13}(z, Q^2) = \frac{1}{z}(Qz)^2 (K_0^2(Qz) + K_1^2(Qz)),$$

We would also need a wavefunction associated to the proton $\phi_p(z)$. For the current analysis, we will assume that the wave function is sharply peaked near the IR boundary z_0, with $1/Q' \leq z_0$, with Q' of the order of the proton mass. For simplicity, we will simply replace P_{24} by a sharp delta-function

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- The structure function F_2 can be expressed as

$$F_2(x,Q^2) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \int dz dz' P_{13}(z,Q^2) P_{24}(z')(zz'Q^2) \times e^{(1-\rho)\tau} \frac{\exp(\frac{-(\log z - \log z')^2}{\rho\tau})}{\tau^{1/2}}$$

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leading to the expression for F_2 with confinement

$$F_2(x,Q^2) = \frac{g_0^2\rho^{3/2}}{32\pi^{5/2}} \int dz dz' P_{13}(z,Q^2) P_{24}(z')(zz'Q^2)\ e^{(1-\rho)\tau} \times \left(\frac{e^{-\frac{\log^2 z/z'}{\rho\tau}}}{\tau^{1/2}} + \mathcal{F}(z,z',\tau) \frac{e^{-\frac{\log^2 zz'/z_0^2}{\rho\tau}}}{\tau^{1/2}} \right)$$

Hard-wall

Similarly for the hard-wall model we would have

$$Im\ \chi_{hw}(s,t=0,z,z') = Im\ \chi_c(\tau,0,z,z') + \mathcal{F}(z,z',\tau)\ Im\ \chi_c(\tau,0,z,z_0^2/z'),$$

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Where

$$\mathcal{F}(u,u',\tau) = 1 - 4\sqrt{\pi\tau}\,e^{\eta^2}\operatorname{erfc}(\eta)\,, \qquad \eta = \frac{u+u'+4\tau}{\sqrt{4\tau}}\,.$$

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- The difference between the conformal and confinement depends on the size of the function $\mathcal{F}$.
- $\mathcal{F}$ at fixed z, z', goes to 1 as $\tau \to 0$ and to -1 as $\tau \to \infty$. Hence, at small x, $\mathcal{F} \to -1$ and confinement leads to a partial cancelation for the growth rate. Since $\mathcal{F}$ is continuous, there will be a region over which $\mathcal{F} \sim 0$, and, in this region, there is little difference between the hard-wall and the conformal results.

Let us look at the graph of $\mathcal{F}$ in the region where there is data

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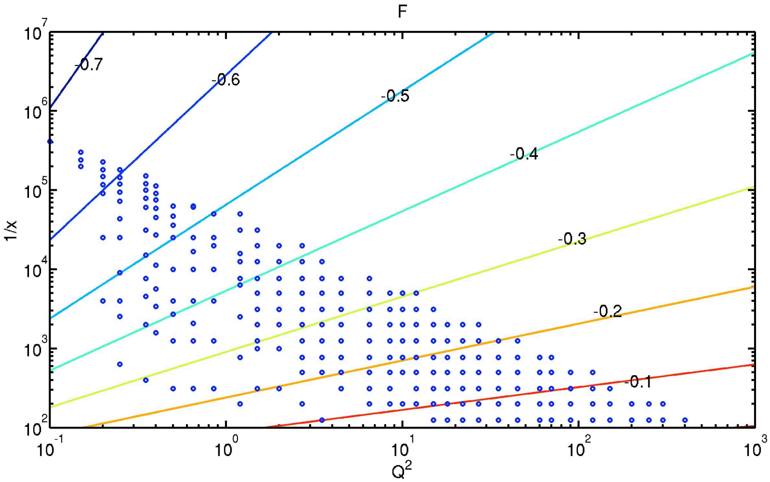

Figure: Contour plot for coefficient function $\mathcal{F}$ as a function of $log(1/z)$ and $\log(1/x)$, with $z' \simeq z_0$ fixed, $z_0 \sim \Lambda_{QCD}^{-1}$.

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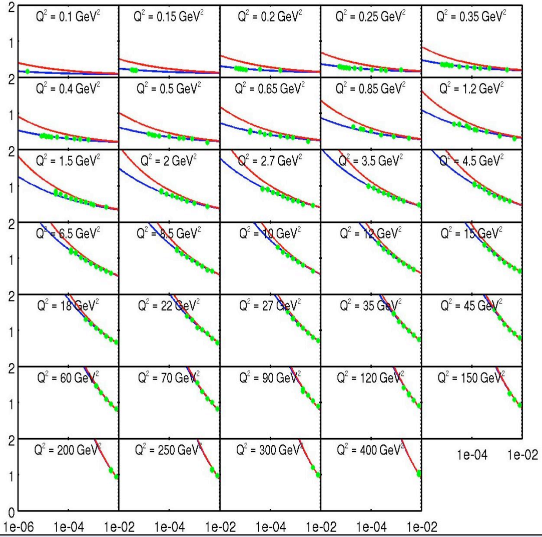
Plots


Q² = 0.1 GeV²
Q² = 0.15 GeV²
Q² = 0.2 GeV²
Q² = 0.25 GeV²
Q² = 0.35 GeV²
Q² = 0.4 GeV²
Q² = 0.5 GeV²
Q² = 0.65 GeV²
Q² = 0.85 GeV²
Q² = 1.2 GeV²
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Q² = 2.7 GeV²
Q² = 3.5 GeV²
Q² = 4.5 GeV²
Q² = 6.5 GeV²
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Q² = 27 GeV²
Q² = 35 GeV²
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Q² = 90 GeV²
Q² = 120 GeV²
Q² = 150 GeV²
Q² = 200 GeV²
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0
1
2
1e-06
1e-04
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$$\begin{aligned} Im\, \chi_{hw}(\tau, t, z, z') &= Im\, \chi_{hw}(\tau, 0, z, z') \\ &+ \frac{\alpha_0 t}{2} \int_0^{\tau} d\tau' \int_0^{z_0} d\tilde{z}\, \tilde{z}^2 \, \times \\ &\times Im\, \chi_{hw}(\tau', 0, z, \tilde{z}) Im\, \chi_{hw}(\tau - \tau', t, \tilde{z}, z') \end{aligned}$$

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Work is underway in evaluating this. We used an approximate treatment which incorporates some of the important features.

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where m_1 and m_0 are solutions of

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respectively.
For b-small, we shall take $Im\ \chi_{hw}(\tau, b, z, z')$ to be of the form

$$Im\ \chi_{hw}^{(0)}(\tau, b, z, z') \sim Im\ \chi_c(\tau, b, z, z') + \mathcal{F}(\tau, z, z') Im\ \chi_c(\tau, b, z, z_0^2/z')$$

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$$D(\tau, b) = \begin{cases} 1\ , & b < z_0 \\ \frac{\exp[-m_1 b - (m_0 - m_1)^2\ b^2/4\rho\tau]}{\exp[-m_1 z_0 - (m_0 - m_1)^2\ z_0^2/4\rho\tau]}\ , & b > z_0 \end{cases}$$

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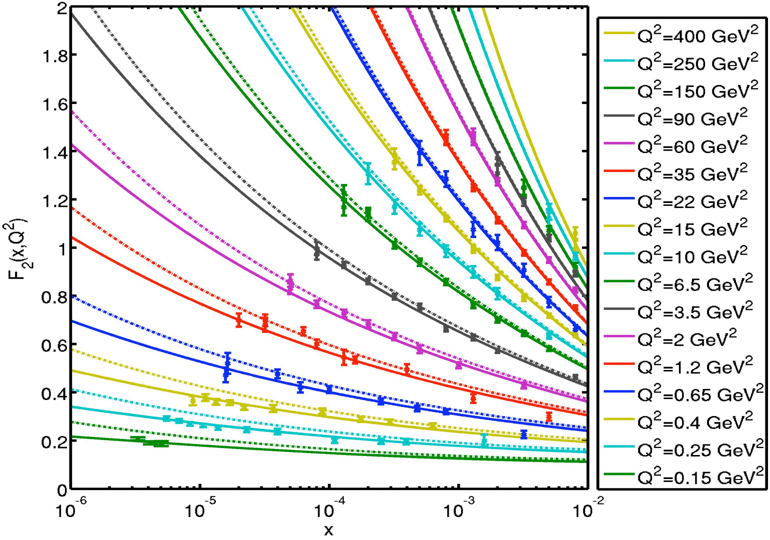
Fitting this expression we get the parameters:

$$\rho = 0.7833 \pm 0.0035,\ g_0^2 = 104.81 \pm 1.41,$$
$$z_0 = 6.04 \pm 0.15\, GeV^{-1},\ Q' = 0.4439 \pm 0.0177\, GeV$$
$$\chi^2_{d.o.f} = 1.04$$

Plots



$F_2(x,Q^2)$
x
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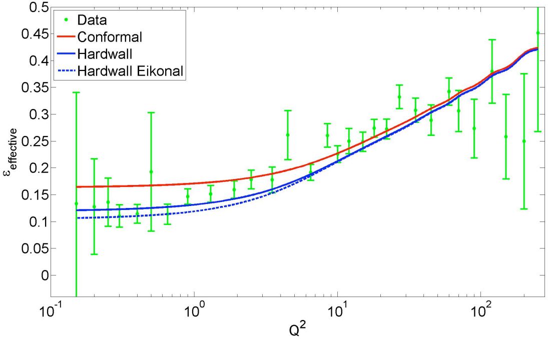

Figure: Q^2-dependence for effective Pomeron intercept, $\alpha_P = 1 + \epsilon_{eff}$.

Outline

What is DVCS?

- Scattering between a virtual photon and the proton.

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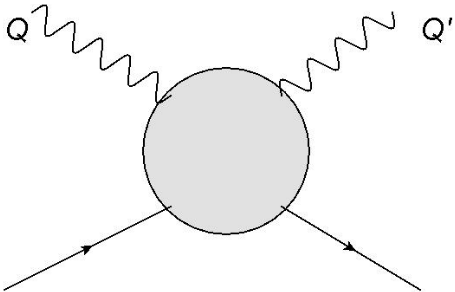

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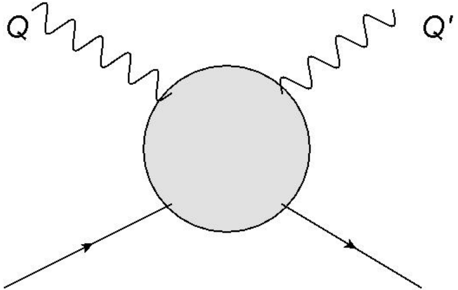

- Differences with DIS:

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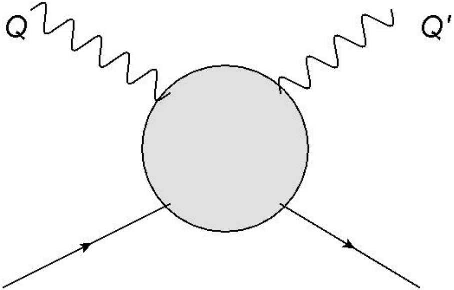

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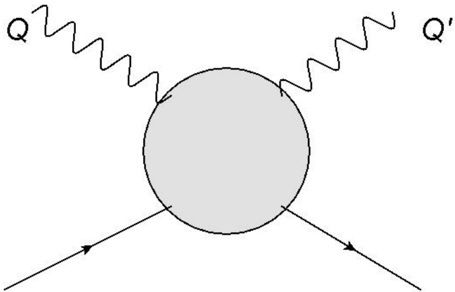

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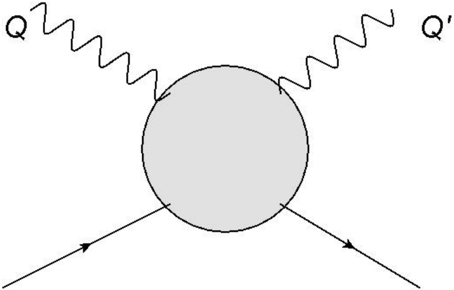

- Differences with DIS:
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DVCS using AdS/CFT (Costa, M.D., in preparation)

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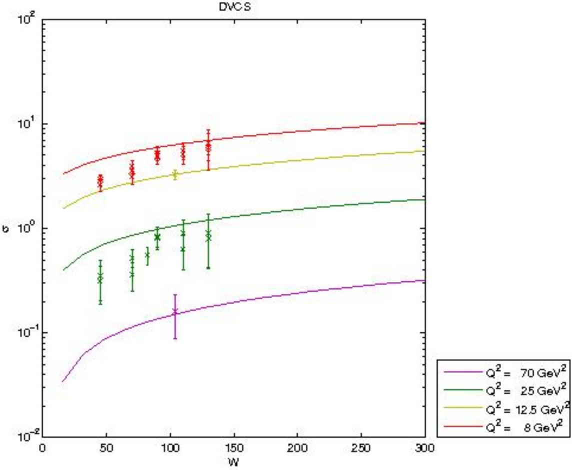

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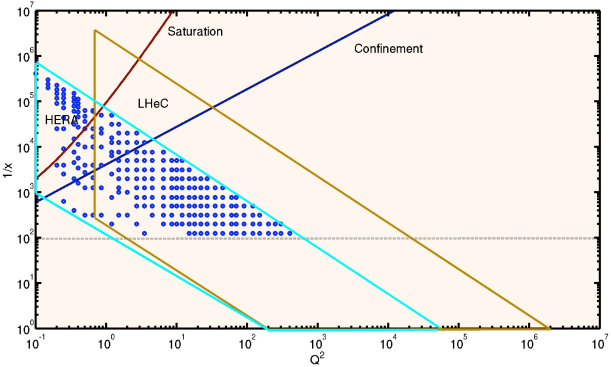

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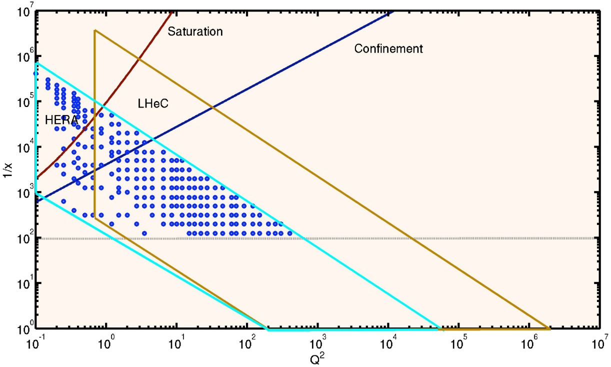

Is the above order of lines an artifact of our model?

Conclusions - continued

Some more work that is under way

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- Work is in progress to apply the same techniques to DVCS. In this process we have production of a real photon.
- Equally important is the application of gauge/string duality to the production of the Higgs boson (also known as the SX boson). One of the possible discovery channels for the Higgs could be double diffractive proton-proton scattering. Hence a thorough theoretical understanding would be very interesting. BPST Pomeron exchange could be applied to this process, but the important new feature now would be the coupling of the Higgs, described by a non-normalizable current, to two Pomerons.

Thank you!