Electromagnetic scattering of vector mesons in the Sakai-Sugimoto model

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Summary

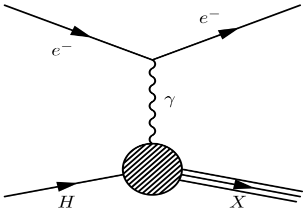
- Introduction : photon-hadron scattering
- The Sakai-Sugimoto model
- The ρ meson form factors in the Sakai-Sugimoto model
- The ρ meson structure functions
- Conclusions and perspectives

1. Introduction : photon-hadron scattering

Consider the electromagnetic scattering

$$e^-(k) + H(p) \rightarrow e^-(k') + X \tag{1}$$

The electron and hadron exchange a **virtual photon** with momentum $\mathbf{q}^\mu = \mathbf{k}^\mu - \mathbf{k}'^\mu$.



Kinematical variables

- Mass of the initial hadron and lepton : M , m ,
- **Virtuality** : $q^2 = -q_0^2 + \vec{q}^2 > 0$,
- Energy transfer : $\nu = (p \cdot q)/M$,
- **Bjorken variable** : $x = -q^2/(2p \cdot q) = -q^2/(2M\nu)$.
- Photon-hadron **center of mass energy squared** :

$$W^2 = -(p+q)^2 = M^2 + q^2 \left(\frac{1}{x} - 1 \right) .$$

The scattering **physical region** is given by $\mathbf{0 < x \leq 1}$.

An **elastic** (**inelastic**) scattering corresponds to $\mathbf{x = 1}$ ($\mathbf{x < 1}$) .

Unpolarized cross section and the hadronic tensor

$$d\sigma = \left(\frac{1}{4ME}\right)\frac{d^3\vec{k}}{(2\pi)^3 2E'}\left(\frac{1}{2}\sum_{\sigma,\sigma'}\right)\left(\frac{1}{2s_H+1}\sum_{\sigma_H}\sum_X\right) \times (2\pi)^4\delta^4(p+q-p_X)|\mathcal{M}|^2\,, \tag{2}$$

where $\mathcal{M}$ is the **scattering amplitude** :

$$\mathcal{M} = e^2\bar{u}(k',\sigma')\gamma^\mu u(k,\sigma)\,\frac{\eta_{\mu\nu}}{q^2}\,\langle X|J_H^\nu(0)|p,\sigma_H\rangle\,. \tag{3}$$

$\sum_X$: sum over final states X (inclusive cross section) ,

p_X : final momentum for each state X .

σ,σ' : lepton polarizations ,
$s_H\,,\,\sigma_H$: spin and polarization of the initial hadron .

It is convenient to write the cross section as

$$d\sigma = \left(\frac{1}{4ME}\right) \frac{d^3\vec{k}}{(2\pi)^3 2E'} \frac{e^4}{q^4}(4\pi)\, L_{\mu\nu}\, W^{\mu\nu}\,, \tag{4}$$

where

$$\begin{aligned} L_{\mu\nu} &= \frac{1}{2}\sum_{\sigma,\sigma'} \bar{u}(k,\sigma)\gamma_\mu u(k',\sigma')\, \bar{u}(k',\sigma')\gamma_\nu u(k,\sigma) \\ &= 2\left[k_\mu k'_\nu + k_\nu k'_\mu + (m^2 - k\cdot k')\eta_{\mu\nu}\right]\,, \end{aligned} \tag{5}$$

is the **leptonic tensor** and

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{4\pi}\left(\frac{1}{2s_H+1}\right)\sum_{\sigma_H}\sum_X (2\pi)^4\delta^4(p+q-p_X) \\ &\times \langle p,\sigma_H|J_H^\mu(0)|X\rangle\langle X|J_H^\nu(0)|p,\sigma_H\rangle\,, \end{aligned} \tag{6}$$

is the **hadronic tensor**.

Structure functions

We can use **current conservation**, **P**, **T** and **Lorentz invariance** to decompose the hadronic tensor.

In the unpolarized case it takes the form

$$W^{\mu\nu} = F_1(x,q^2)\Big(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\Big) + \frac{2x}{q^2}F_2(x,q^2)\Big(p^\mu + \frac{q^\mu}{2x}\Big)\Big(p^\nu + \frac{q^\nu}{2x}\Big)\,, \quad (7)$$

where $F_1(x,q^2)$ and $F_2(x,q^2)$ are known as the **structure functions** of the hadron.

To calculate the structure functions we need to **sum over all possible final states**.

Form factors

In the hadronic tensor, each final state X contributes through the current element $\langle p, \sigma_H | J^\mu_H(0) | X \rangle$. The final state X can be one particle or many particles.

Here we are interested in the case $\mathbf{X} = \mathbf{H}$, where the final state is **one hadron with the same mass and spin** as the initial hadron. In that case the current matrix element can be decomposed as

$$\langle p, n, \sigma_H | J^\mu_H(0) | p + q, n_X, \sigma_X \rangle = \sum_i \Gamma^\mu_i(p, q)\, F^i_{n,n_X}(q^2)\,, \tag{8}$$

where n (n_X) is an index associated to the mass of the initial (final) hadron. The functions $\mathbf{F^i_{n,n_X}(q^2)}$ are the **hadronic form factors**.

Example : The elastic ρ meson form factor

In this case we have that $n_X = n = 1$ and the current can be decomposed as

$$\langle p, \epsilon | J^\mu(0) | p+q, \epsilon' \rangle = \epsilon \cdot \epsilon' (2p+q)^\mu \mathbf{F_1}(\mathbf{q^2})$$
$$+ \left[\epsilon^\mu \epsilon' \cdot q - \epsilon'^\mu \epsilon \cdot q\right] \left[\mathbf{F_1}(\mathbf{q^2}) + \mathbf{F_2}(\mathbf{q^2})\right] - \frac{q \cdot \epsilon' q \cdot \epsilon}{M^2} (2p+q)^\mu \mathbf{F_3}(\mathbf{q^2}) . \quad (9)$$

It is also convenient to define the **electric, magnetic, and quadrupole form factors**

$$\mathbf{G_E} = F_1 - \frac{q^2}{6M^2}\left[F_2 - (1 + \frac{q^2}{4M^2})F_3\right] \quad , \quad \mathbf{G_M} = F_1 + F_2 ,$$
$$\mathbf{G_Q} = -F_2 + \left(1 + \frac{q^2}{4M^2}\right) F_3 . \quad (10)$$

2. The Sakai-Sugimoto model

Motivation

- In the regime of **low momentum transfer** ($\sqrt{q^2}$ lower than some *GeVs*) **soft processes** become dominant and **perturbative QCD fails**.
- In this region we need to construct **non-perturbative models** since the precision of lattice QCD is still very far from the experiments.
- The **AdS/QCD models** provide new insights into this old problem. These models are motivated by the **AdS/CFT** correspondence.

The D4-brane background (*Witten 1998*)

N_c coincident **D4-branes in type IIA Supergravity** generate the metric

$$ds^2 = \frac{u^{3/2}}{R^{3/2}} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{u^{3/2}}{R^{3/2}} f(u) d\tau^2 + \frac{R^{3/2}}{u^{3/2}} \frac{du^2}{f(u)} + R^{3/2} u^{1/2} d\Omega_4^2 \,,$$
$$f(u) = 1 - \frac{u_\Lambda^3}{u^3} \,, \tag{11}$$

where $R = (\pi g_s N_c)^{1/3} \sqrt{\alpha'}$.

In addition, we have a dilaton and a four-form

$$e^\phi = g_s \frac{u^{3/4}}{R^{3/4}} \quad , \quad F_4 = \frac{(2\pi l_s)^3 N_c}{V_{S^4}} \epsilon_4 \,. \tag{12}$$

The τ coordinate is compactified in a circle with period

$$\delta\tau = \frac{4\pi}{3}\frac{R^{3/2}}{U_\Lambda^{1/2}} \equiv \frac{2\pi}{M_\Lambda}\,, \tag{13}$$

where M_Λ is a 4-d mass scale.

Imposing anti-periodic conditions for the fermionic states we arrive to a **4-d non-supersymmetric strongly coupled $U(N_c)$ theory** at large N_c.

The 4-d 't Hooft constant is given by

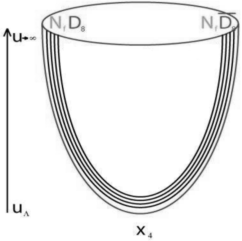
$$\lambda = g_{YM}^2 N_c = (2\pi M_\Lambda)\, g_s N_c\, l_s\,. \tag{14}$$

The D4-D8 brane intersection (*Sakai & Sugimoto 2004*)

Consider N_f coincident D8-$\overline{\text{D8}}$ probe branes living in the background generated by the N_c D4-branes. The **probe approximation** is guaranteed by the condition $N_f \ll N_c$.

The D8 and $\overline{\text{D8}}$ branes separated in the UV region **merge in the infrared**. This is a **geometrical realization of chiral symmetry breaking**

$$U(N_f) \times U(N_f) \to U(N_f)$$



The **DBI action** describing 9-d gauge field fluctuations in the D8-$\overline{\text{D8}}$ branes can be integrated in S^4 leading to

$$S_{YM} = -\kappa \int d^4x dz \mathrm{Tr} \left[\frac{1}{2} K_z^{-1/3} \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + M_\Lambda^2 K_z \eta^{\mu\nu} F_{\mu z} F_{\nu z} \right] \quad (15)$$

where $K_z = 1 + z^2$ and $\kappa = \lambda N_c / (216\pi^3)$.

The 5-d gauge field can be expanded, in the $A_z = 0$ gauge, as

$$A_\mu(x,z) = \hat{\mathcal{V}}_\mu(x) + \hat{\mathcal{A}}_\mu(x) \psi_0(z) + \sum_{n=1}^{\infty} v_\mu^n(x) \psi_{2n-1}(z) + \sum_{n=1}^{\infty} a_\mu^n(x) \psi_{2n}(z) \,,$$

where the $\psi_n(z)$ modes satisfy

$$\kappa \int dz \, K_z^{-1/3} \psi_n(z) \psi_m(z) = \delta_{nm} \quad , \quad -K_z^{1/3} \partial_z \left[K_z \partial_z \psi_n(z) \right] = \lambda_n \, \psi_n(z) \,. \quad (16)$$

Integrating the z coordinate we get a 4-d effective lagrangian. The **vector (axial)** fields $v_\mu^n(x)$ ($a_\mu^n(x)$) correspond to the modes $\psi_{2n-1}(z)$ ($\psi_{2n}(z)$) and the **pion** $\pi(x)$ is related to $\psi_0(z)$.

The 4-d fields $\hat{\mathcal{V}}_\mu(x)$ and $\hat{\mathcal{A}}_\mu(x)$ can be expressed in terms of the

$$\begin{aligned}\hat{\mathcal{V}}_\mu(x) &= \frac{1}{2}U^{-1}\left[\mathcal{A}_\mu^L + \partial_\mu\right]U + \frac{1}{2}U\left[\mathcal{A}_\mu^R + \partial_\mu\right]U^{-1}\\ \hat{\mathcal{A}}_\mu(x) &= \frac{1}{2}U^{-1}\left[\mathcal{A}_\mu^L + \partial_\mu\right]U - \frac{1}{2}U\left[\mathcal{A}_\mu^R + \partial_\mu\right]U^{-1}\end{aligned}$$

where

$$U(x) = e^{\frac{i\pi(x)}{f_\pi}} \quad , \quad \mathcal{A}_\mu^{L(R)} = \mathcal{V}_\mu \pm \mathcal{A}_\mu \qquad (17)$$

with $\mathcal{V}$ interpreted as the **photon**.

In order to have a diagonal kinetic term, the vector mesons are redefined as $\tilde{v}_\mu^n = v_\mu^n + (g_{v^n}/M_{v^n}^2)\mathcal{V}_\mu$ (similarly for the axial mesons).

The quadratic terms in the vector sector take the form

$$\mathcal{L}_2 = \frac{1}{2}\sum_n \left[\mathrm{Tr}\left(\partial_\mu \tilde{v}_\nu^n - \partial_\nu \tilde{v}_\mu^n\right)^2 + 2M_{v^n}^2 \mathrm{Tr}\left(\tilde{v}_\mu^n - \frac{g_{v^n}}{M_{v^n}^2}\mathcal{V}_\mu\right)^2 \right] ,$$

where

$$M_{v^n}^2 = \lambda_{2n-1} M_\Lambda^2 \quad , \quad g_{v^n} = \kappa\, M_{v^n}^2 \int dz\, K_z^{-1/3} \psi_{2n-1}(z) \, .$$

The term $g_{v^n}\tilde{v}_\mu^n \mathcal{V}^\mu$ represents the **decay of the photon into vector mesons** realizing **vector meson dominance**.

3. The ρ meson form factors in the Sakai-Sugimoto model

(*BB, Boschi-Filho, Braga, Torres 2009*)

From the cubic terms in the 4-d effective meson theory discussed above we can extract the vector meson interaction term

$$\mathcal{L}_{vvv} = \sum_{n,\ell,m} g_{v^n v^\ell v^m} \mathrm{Tr}\Big\{ (\partial^\mu \tilde{v}^{\nu\, n} - \partial^\nu \tilde{v}^{\mu\, n}) [\tilde{v}^\ell_\mu, \tilde{v}^m_\nu] \Big\} \quad (18)$$

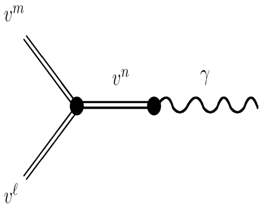
where $g_{v^n v^\ell v^m}$ are 4-d effective couplings given by the integral

$$g_{v^n v^\ell v^m} = \kappa \int dz\, K_z^{-1/3} \psi_{2n-1}(z) \psi_{2\ell-1}(z) \psi_{2m-1}(z) \,. \quad (19)$$

Using the Feynman rules associated to the interaction term we find the current matrix element for vector mesons

$$\langle v^m(p), \epsilon | J^\mu(0) | v^\ell(p+q), \epsilon' \rangle = \left[\sum_{n=1}^{\infty} g_{v^n} g_{v^m v^n v^\ell} \Delta^{\mu\sigma}(q, m_n^2) \right]$$
$$\epsilon^\nu \epsilon'^\rho \Big[\eta_{\sigma\nu}(q-p)_\rho + \eta_{\nu\rho}(2p+q)_\sigma - \eta_{\rho\sigma}(p+2q)_\nu \Big] \qquad (20)$$

where $\Delta^{\mu\sigma}(q, m_n^2)$ is the propagator of a massive vector particle with momentum q and mass m_n^2.



v^m : initial vector meson

v^ℓ : final vector meson

Using the holographic sum rule

$$\sum_{n=1}^{\infty} \frac{g_{v^n} g_{v^n v^m v^\ell}}{M^2_{v^n}} = \delta_{m\ell} \tag{21}$$

and the transversality of the initial and final polarizations we get the simple expression

$$\begin{aligned} \langle v^m(p), \epsilon | J^\mu(0) | v^\ell(p+q), \epsilon' \rangle &= \epsilon^\nu \epsilon'^\rho \Big[\eta_{\nu\rho}(2p+q)_\sigma \\ + \quad & 2(\eta_{\sigma\nu} q_\rho - \eta_{\rho\sigma} q_\nu) \Big] \left(\eta^{\mu\sigma} - \frac{q^\mu q^\sigma}{q^2} \right) \mathcal{F}_{v^m v^\ell}(q^2) \, . \end{aligned} \tag{22}$$

where

$$\mathcal{F}_{v^m v^\ell}(q^2) = \sum_{n=1}^{\infty} \frac{g_{v^n} g_{v^n v^m v^\ell}}{q^2 + M^2_{v^n}} \ . \tag{23}$$

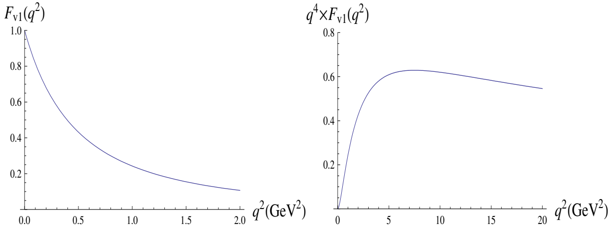
is a generalized vector meson form factor.

The elastic ρ meson form factors can be extracted from the case $m = \ell = 1$:

$$\langle v^1(p), \epsilon | J^\mu(0) | v^1(p+q), \epsilon' \rangle = \Big\{ (\epsilon \cdot \epsilon')(2p+q)^\mu + 2 \left[\epsilon^\mu (\epsilon' \cdot q) - \epsilon'^\mu (\epsilon \cdot q) \right] \Big\} \mathcal{F}_{v^1 v^1}(q^2) \,, \tag{24}$$

Comparing this result with the expansion (9) we find the ρ meson form factors:

$$F_1(q^2) = F_2(q^2) = \mathcal{F}_{v^1 v^1}(q^2) \;,\; F_3(q^2) = 0 \,, \tag{25}$$



Hence the electric, magnetic and quadrupole form factors predicted by the D4-D8 brane model for the ρ meson are

$$G_E = (1 + \frac{q^2}{6M^2})F_1 \quad , \quad G_M = 2F_1 \quad , \quad G_Q = -F_1 \,. \qquad (26)$$

From these form factors, we can extract the ρ meson electric radius

$$\langle r_\rho^2 \rangle = -6 \frac{\mathrm{d}}{\mathrm{d}q^2} G_E(q^2)|_{q^2=0} = 0.5739 \,, \qquad (27)$$

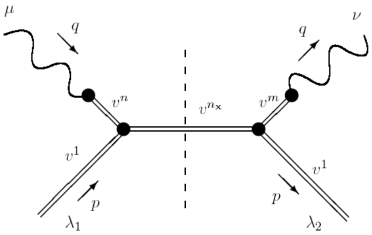
and the magnetic and quadrupole moments

$$\mu = \frac{1}{2M} G_M(q^2)|_{q^2=0} = \frac{1}{M} \quad , \quad D \equiv \frac{1}{M^2} G_Q(q^2)|_{q^2=0} = -\frac{1}{M^2} \,. \qquad (28)$$

4. The ρ meson structure functions

(*BB, Boschi-Filho, Braga, Torres 2010*)

The **optical theorem** relates the hadronic tensor to the **imaginary part** of the tensor associated to the **Compton forward scattering**. We consider the case where the **final state is one vector meson**.



Using the Feynman rules corresponding to this diagram, we find

$$
\begin{aligned}
\mathrm{Im}\, T^{\mu\nu} &= \frac{1}{3}\sum_{\epsilon}\epsilon^{\lambda_1}\epsilon^{\lambda_2}\sum_{n_x}\left\{\sum_{n} g_{v^n} g_{v^1 v^{n_x} v^n}\frac{\left[\eta^{\mu\sigma_1}+\frac{q^{\mu}q^{\sigma_1}}{M^2_{v^n}}\right]}{q^2+M^2_{v^n}}\right\} \\
&\times \left\{\sum_{m} g_{v^m} g_{v^1 v^{n_x} v^m}\frac{\left[\eta^{\nu\sigma_2}+\frac{q^{\nu}q^{\sigma_2}}{M^2_{v^m}}\right]}{q^2+M^2_{v^m}}\right\} \\
&\times f^{0ab}\left[\eta_{\sigma_1\lambda_1}(2q)_{\rho_1}+\eta_{\lambda_1\rho_1}(2p)_{\sigma_1}+\eta_{\sigma_1\rho_1}(-2q)_{\lambda_1}\right] \\
&\times f^{0ab}\left[\eta_{\sigma_2\lambda_2}(2q)_{\rho_2}+\eta_{\lambda_2\rho_2}(2p)_{\sigma_2}+\eta_{\sigma_2\rho_2}(-2q)_{\lambda_2}\right] \\
&\times \int\frac{d^4p_x}{2\pi^4}(2\pi)\delta(p_x^2+M^2_{v^{n_x}})\left[\eta^{\rho_1\rho_2}+\frac{p_x^{\rho_1}p_x^{\rho_2}}{M^2_{v^{n_x}}}\right](2\pi)^4\delta^4(p+q-p_x)
\end{aligned}
\tag{29}
$$

The polarization vectors satisfy

$$\sum_{\epsilon} \epsilon^{\lambda_1} \epsilon^{\lambda_2} = \eta^{\lambda_1 \lambda_2} + \frac{p^{\lambda_1} p^{\lambda_2}}{M_{v^1}^2} \,. \tag{30}$$

After some algebra we obtain

$$\begin{aligned} \mathrm{Im}\, T^{\mu\nu} &= \frac{N_f}{3} \left[\eta^{\mu\sigma_1} - \frac{q^\mu q^{\sigma_1}}{q^2} \right] \left[\eta^{\nu\sigma_2} - \frac{q^\nu q^{\sigma_2}}{q^2} \right] \\ &\times \left[\eta^{\lambda_1 \lambda_2} + \frac{p^{\lambda_1} p^{\lambda_2}}{M_{v^1}^2} \right] \left[\eta^{\rho_1 \rho_2} + \frac{(p+q)^{\rho_1} (p+q)^{\rho_2}}{W^2} \right] \\ &\times [\eta_{\sigma_1 \lambda_1} (2q)_{\rho_1} + \eta_{\lambda_1 \rho_1} (2p)_{\sigma_1} + \eta_{\sigma_1 \rho_1} (-2q)_{\lambda_1}] \\ &\times [\eta_{\sigma_2 \lambda_2} (2q)_{\rho_2} + \eta_{\lambda_2 \rho_2} (2p)_{\sigma_2} + \eta_{\sigma_2 \rho_2} (-2q)_{\lambda_2}] \\ &\times \sum_{n_x} \left[\mathcal{F}_{v^1 v^{n_x}} (q^2) \right]^2 (2\pi) \delta [M_{v^{n_x}}^2 - W^2] \end{aligned} \tag{31}$$

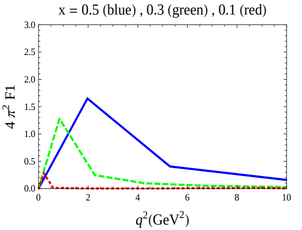
We can approximate the sum over the delta functions by an integral

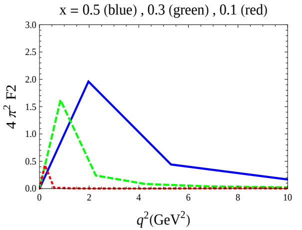
$$\sum_{n_x} \delta[M^2_{v^{n_x}} - W^2] \equiv \sum_{n_x} \delta[M^2_{v^{n_x}} - M^2_{v^{\bar{n}}}] = \int dn_x \left[\left| \frac{\partial M^2_{v^{n_x}}}{\partial n_x} \right| \right]^{-1} \delta(n_x - \bar{n})$$
$$\equiv f(\bar{n}) \,. \tag{32}$$

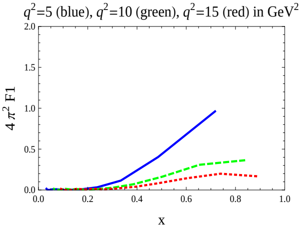
Then, the structure functions are

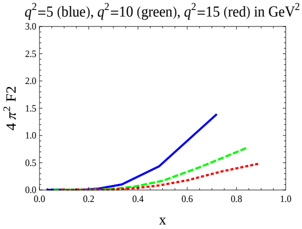
$$F_1 = \frac{4N_f}{3} f(\bar{n}) \Big[F_{v^1 v^{\bar{n}}}(q^2) \Big]^2 q^2 \left[2 + \frac{q^2}{4x^2 M^2_{v^1}} + \frac{q^2}{W^2 x^2}(x - \frac{1}{2})^2 \right]$$
$$F_2 = \frac{4N_f}{3} f(\bar{n}) \Big[F_{v^1 v^{\bar{n}}}(q^2) \Big]^2 \frac{q^2}{2x} \left[3 + \frac{q^2}{M^2_{v^1}} + \frac{(q^2)^2}{M^2_{v^1} W^2 x^2}(x - \frac{1}{2})^2 \right] \tag{33}$$

Numerical results for $N_f = 1$:

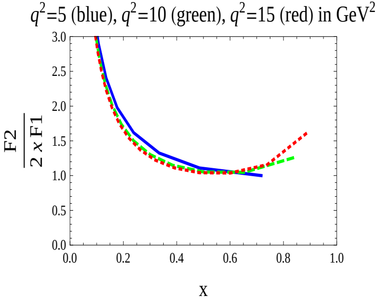








Near $x = 0.5$ we find an approximate Callan-Gross relation



Conclusions and Perspectives

- The Sakai-Sugimoto model offers a new approach for the non-perturbative regime of hadronic interactions. In particular, it realizes in a simple way the property of vector meson dominance.
- It would be interesting to investigate the regime of high energies but low momentum transfer where the **soft pomeron** is relevant.
- The perturbative QCD approach to hadron scattering involves **factorization** between **hard parton scattering** and **soft parton distribution functions**. It would be interesting to understand this factorization in the AdS/QCD approach.