Dualities for amplitudes in $\mathcal{N}$=4 SYM

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Paris

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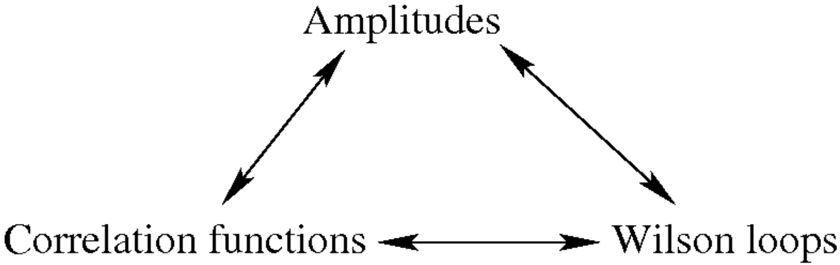
based on work with: Khoze (1007.1805)
Eden, Korchemsky, Sokatchev (1103.4353,1103.3714)
Previous work with: Anastasiou, Brandhuber, Khoze, Spence, Travaglini

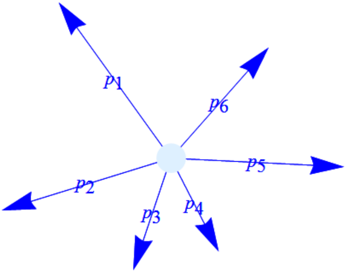
Introduction

- Three objects of interest in $N = 4$ SYM: Amplitudes (S-matrix), Correlation functions of guage invariant operators, Wilson loops
- AdS/CFT: can now consider all at strong coupling
- Increasing evidence of a triality between all three objects in $\mathcal{N}$=4 SYM



Amplitude/Wilson loop duality

[Alday Maldacena 2007, Drummond Korchemsky Sokatchev 2007, Brandhuber Travaglini PH]



planar *MHV* amplitude $\mathcal{R}_n$ '=' $\langle W[\mathcal{C}_n]\rangle$

$(D = 4 - 2\epsilon)$ $(D = 4 + 2\epsilon)$

- Wilson loop over the polygonal contour $\mathcal{C}_n$

vertices, region momenta $p_i = x_{i\,i+1} = x_i - x_{i+1}$

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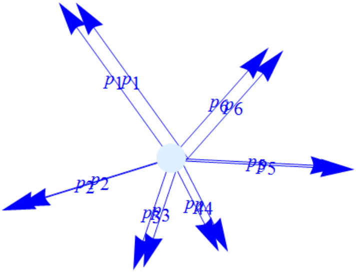

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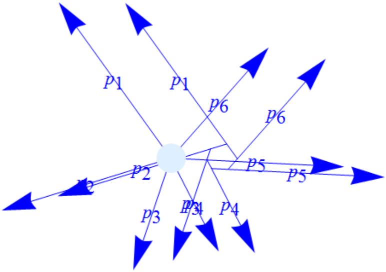

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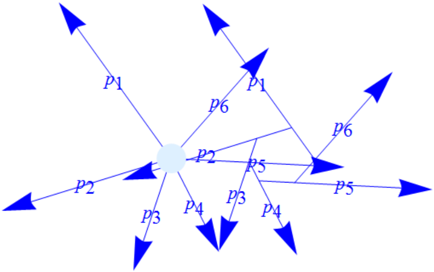

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'$=$'

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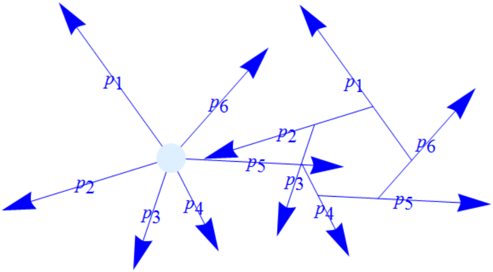

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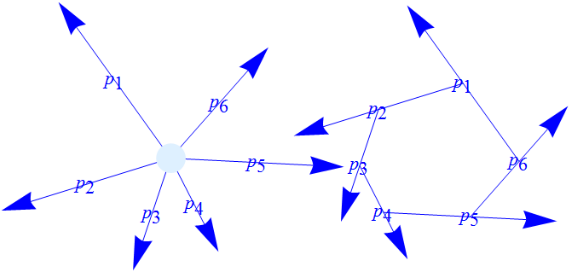

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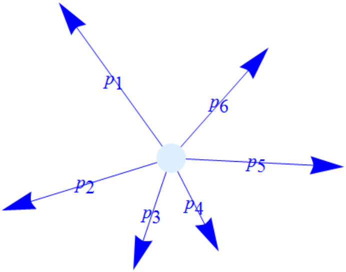

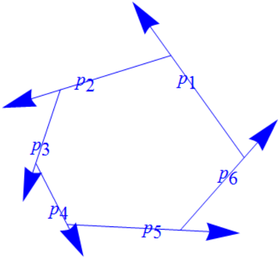

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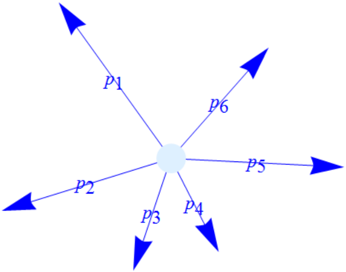

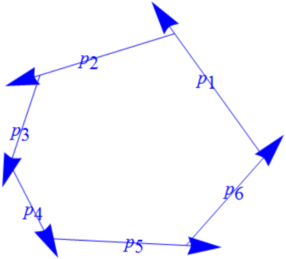

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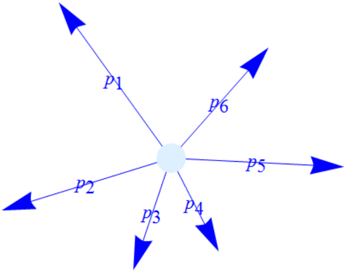

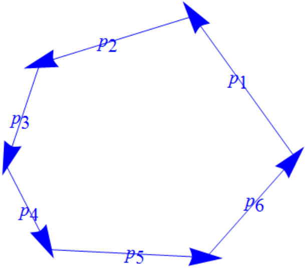

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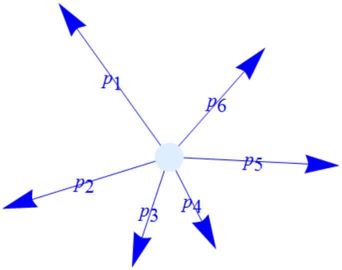

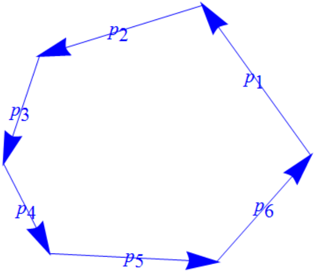

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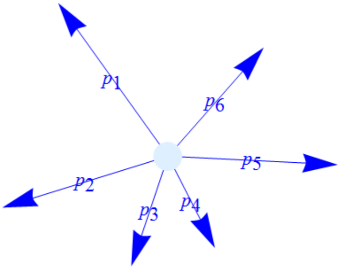

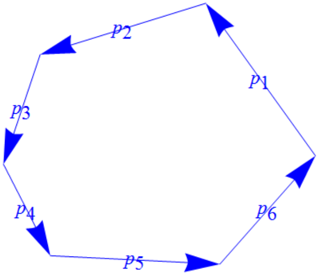

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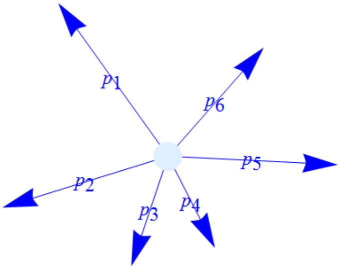

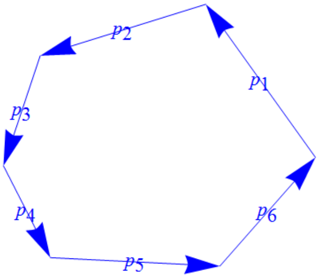

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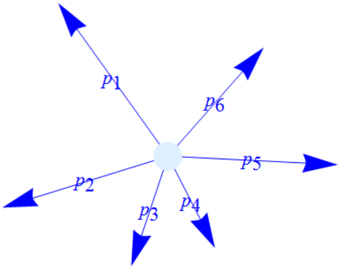

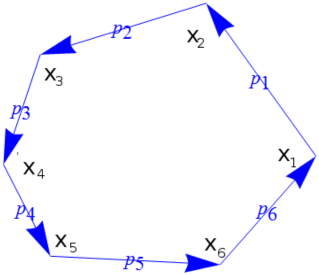

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Evidence for the WL/ MHV amplitude duality

- AdS/CFT: amplitudes = Wilson loops at strong coupling
  [Alday Maldacena 2007]
- 1 loop amplitudes = Wilson loops
  [Drummond Henn Korchemsky Sokatchev, Brandhuber Travaglini PH]
- 2 loop amplitudes = Wilson loops 4,5,6 points
  [Drummond Henn Korchemsky Sokatchev, Bern Dixon Kosower Roiban Spradlin Vergu Volovich]
- duality derived and generalised in twistor space [Adamo Bullimore Mason Skinner] and Minkowski space [Caron-Huot] at the integrand level.
  (Somewhat formal however, leaves several questions unanswered (regularisation issues))

Consequences/uses of the duality

New hidden symmetry of amplitudes

[Drummond Henn Korchemsky Sokatchev, Brandhuber Travaglini PH]

- Wilson loops conformally invariant $\longrightarrow$ new hidden symmetry of planar amplitudes dual conformal symmetry
- Lead to many new developments (momentum twistors, new formulae for amplitude integrands, correlation function duality etc.)

New results for perturbative amplitudes

- dual conformal symmetry $\Rightarrow$ 4,5 point MHV amplitude “trivial” (given by BDS [Bern Dixon Smirnov 2005], to all orders)
- non-trivial from two-loop six-points, but
- Wilson loop at two loops yields n-point integrands (completely different - easier - than the amplitude integrands)
- Integrals written down and done numerically: Numerical algorithm gives two loop MHV amplitudes for any number of points n [Anastasiou Brandhuber Khoze Spence Travaglini P. H.]

Analytic results: Six points 2 loops Wilson loop/MHV amplitude

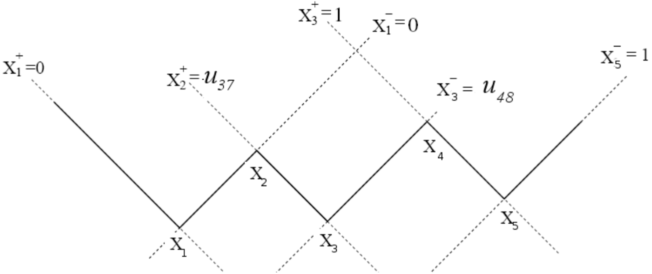
[Del Duca Duhr Smirnov, Goncharov Spradlin Vergu Volovich]

$$R_6^{(2)}(u_1,u_2,u_3) = \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}\mathrm{Li}_4(1-1/u_i)\right) - \frac{1}{8}\left(\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}$$

2 line result simplified down from 15 pages of Goncharov polylogs using the "Symbol"
(see talks by Vittorio del Duca and Claude Duhr)

Further analytic results: 8 point special kinematics

- The first non-trivial result at strong coupling: Alday and Maldacena considered the 8-point amplitude at strong coupling via string theory
- special kinematics lying in $1+1$ dimensions

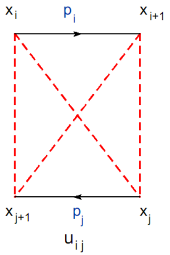


- depends on only two parameters (up to conformal transformations)
- $u_{15},\ u_{26}$ with $u_{37} = 1 - u_{15}, u_{48} = 1 - u_{26}$

Weak coupling

- The corresponding weak coupling result was found by [Del Duca Duhr Smirnov]
- It has the amazingly simple form:

$$R_8^{\text{DDS}} = -\frac{1}{2}\log(u_{15})\log(u_{26})\log(u_{37})\log(u_{48}) - \frac{\pi^4}{18}\,.$$

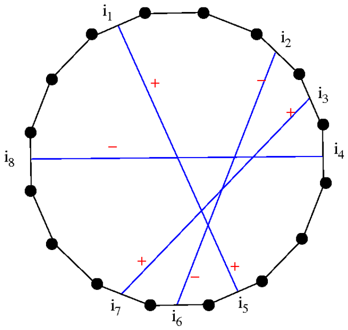


$$u_{ij} = \frac{x_{ij+1}^2 x_{ji+1}^2}{x_{ij}^2 x_{i+1j+1}^2}$$

- Can we generalise this result for higher numbers of points?

2 loops *n*-point MHV amplitudes in 1+1 kinematics

[Khoze PH]



$$R_n = -\frac{1}{2}\Big(\sum_{\mathcal{S}} \log(u_{i_1 i_5})\log(u_{i_2 i_6})\log(u_{i_3 i_7})\log(u_{i_4 i_8})\Big) - \frac{\pi^4}{72}(n-4)\,,$$

$$\mathcal{S} = \Big\{ i_1, \dots i_8 : 1 \le i_1 < i_2 < \dots < i_8 \le n, \qquad i_k - i_{k-1} = \text{odd} \Big\}$$

How the result is obtained

Major assumption:

We assume the result is written in terms of $\log(u_{ij})$

- $\mathcal{R}_n(u)$ should satisfy all collinear limits
- For k lines becoming collinear

$$\mathcal{R}_n \to \mathcal{R}_{n-k+1} + \mathcal{R}_{k+3}$$

Higher loops

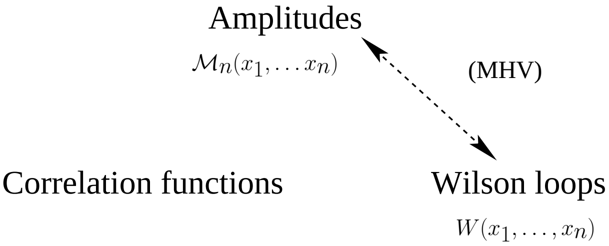
- Satisfies all collinear limits - highly non-trivial (only the simplest was used as input, the other limits remarkably came for free.)
- natural higher loop generalisation
- But it is not present at 8 points
- Thus disagrees with strong coupling (although not by much!) [Brandhuber Khoze Travaglini PH])
- Confirmed by OPE method [Gaiotto Maldacena Sever Vieira]
- also confirmed by recent *n*-point 2 loop symbol [Caron-Huot]

amplitudes versus integrands

- so far focussed on the amplitude itself
- analytic results up to two-loop, MHV amplitudes
- Wilson loop gave completely different integrals - easier to evaluate

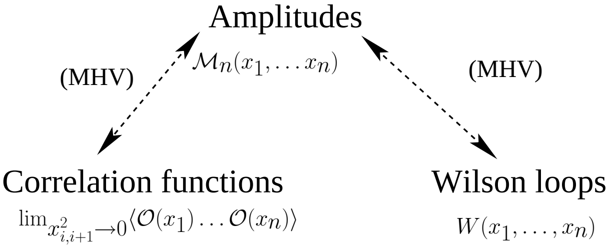
- Other big recent developments involve new expressions for the integrand of loop level amplitudes
  [Arkani-Hamed Bourjaily Cachazo Trnka + Caron-Huot]
- Goes much further (arbitrary helicity, higher loop) but the integrals themselves are hard to evaluate
- Duality/triality can be expressed in terms of integrands ...

Triality



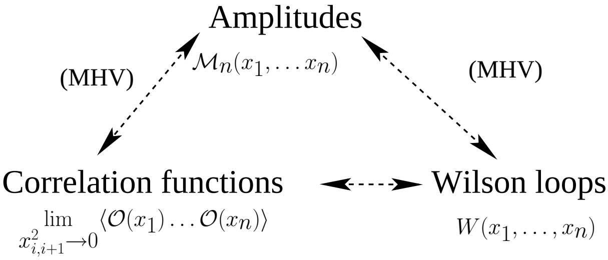
The story so far...

Triality



Last summer, [Eden Korchemsky Sokatchev]. Tests, later integrand identity. Dual conformal symmetry $\to$ conformal symmetry of correlators

Triality



The same day, [Alday Eden Korchemsky Maldacena Sokatchev]. Regularisation: correlators as regularised Wilson loops/amplitudes?

Correlation function / MHV amplitude duality

- The easiest correlator to test is

$$G_{n;0} := \langle \mathcal{O}(x_1)\tilde{\mathcal{O}}(x_2)\ldots\mathcal{O}(x_{n-1})\tilde{\mathcal{O}}(x_n)\rangle$$

- half BPS, energy momentum supermultiplet
  $\mathcal{O} = \mathrm{Tr}(\phi_{12}\phi_{12})\,, \qquad \tilde{\mathcal{O}} = \mathrm{Tr}(\bar{\phi}^{12}\bar{\phi}^{12})$

conjectured duality

$$\lim_{x^2_{i,i+1}\to 0} \left(G_{n;0}/G^{(0)}_{n;0}\right) = \left(A_{n;0}/A^{(0)}_{n;0}\right)^2$$

- Viewed as an integrand identity (everything is then finite, well-defined and rational even in the light-like limit

Integrands = correlators with Lagrangian insertions

- Loop corrections $\Rightarrow$ Lagrangian insertions.

1 loop correlator

$$G^{(1)}_{n;0} = \int d^4x_0 \, \langle \mathcal{L}(x_0)\mathcal{O}(x_1)\dots\mathcal{O}(x_n)\rangle^{(0)}$$

- so the *Born-level* $(n+1)$-point correlator with Lagrangian inserted at new point x_0 defines the 1 loop integrand
- l-loops $\Rightarrow$ l Lagrangian insertions

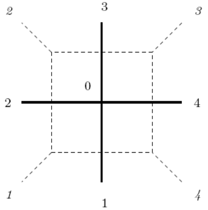
Example 1: 4-points 1 loop

$$G^{(1)}_{4;0}(x_1, x_2, x_3, x_4) \sim \frac{x^2_{13}x^2_{24}}{x^2_{12}x^2_{23}x^2_{34}x^2_{41}}(1-u-v)g(1,2,3,4)$$

$$\left(u = \frac{x^2_{12}x^2_{34}}{x^2_{13}x^2_{24}}, \qquad v = \frac{x^2_{14}x^2_{23}}{x^2_{13}x^2_{24}}\right)$$

- light-like limit: $\lim_{x^2_{i,i+1}\to 0} \frac{G^{(1)}_{4;0}}{G^{\text{tree}}_{4;0}} = 2x^2_{13}x^2_{24}g(1,2,3,4)$
- amplitude: $\frac{A^{(1)}_4}{A^{(0)}_4} = x^2_{13}x^2_{24}g(1,2,3,4)$

$$g(1,2,3,4) = 1 \text{ loop box} = \int d^Dx_0 \, \frac{1}{x^2_{01}x^2_{02}x^2_{03}x^2_{04}} =$$



(Divergent in limit $\Rightarrow$ consider integrand)

- The MHV duality has been further checked at the integrand level:[Eden Korchemsky Sokatchev]
- All n-point 1 loop (reproduce 2me box functions, but also parity odd part - momentum twistor pentagons found recently by Arkani-Hamed et al)
- 4,5,6 point 2 loop
- See Burkhard Eden's talk for more details.

beyond MHV: superduality

[Eden Korchemsky Sokatchev PH]

- So far, specific amplitude (MHV) $\leftrightarrow$ specific correlation function
- How general is the duality?
- amplitude $\rightarrow$ superamplitude
- MHV $\rightarrow$ N^{k}MHV
- Correlation functions $\rightarrow$ supercorrelation functions

Superspaces: superamplitudes

- Use Nair's $\mathcal{N}$=4 on-shell superspace, all particles → superparticle

super-particle

$$\Phi(p,\theta) = G^+(p) + \theta\psi + \theta^2\phi(p) + \theta^3\bar{\psi}(p) + \theta^4 G^-(p)$$

- All amplitudes → superamplitudes

$$A(x_i) \to \mathcal{A}(x_i,\theta_i)$$

super-amplitude structure

$$\mathcal{A}(x,\theta) = A^{MHV} + \theta^4 A^{NMHV} + \theta^8 A^{NNMHV} + \ldots$$

Superspace: correlation functions

Similarly for correlation functions:

energy momentum supermuliplet

$$\mathcal{T}(x,\theta,\bar\theta) = \mathcal{O} + \ldots + \theta^4 \mathcal{L} + \ldots + \bar\theta^4 \bar{\mathcal{L}} + \ldots + (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)\, T_{\mu\nu} + \ldots\,,$$

- correlation function of $\mathcal{T}$s: θ-expansion organised in powers of $\theta^m\bar\theta^n$ with $m - n = 4k$ (non-chiral)
- How can we compare with the amplitude?

- Simply set $\bar{\theta} = 0$. Somewhat unnatural, but...
- then

$$\begin{aligned} G_n &:= \langle \mathcal{T}(1)\mathcal{T}(2)\ldots\mathcal{T}(n)\rangle \\ &= G_{n;0} + \theta^4 G_{n;1} + \theta^8 G_{n;2} + \ldots \end{aligned}$$

very similar to the amplitude

- Correlation functions have full superconformal symmetry, but $\bar{Q}$, S is explicitly broken by setting $\bar{\theta} = 0$

Superamplitude/ supercorrelation function duality

We conjecture that (a='tHooft coupling)

Superduality

$$\lim_{x^2_{i\,i+1}\to 0} \frac{\langle \mathcal{T}(1)\dots\mathcal{T}(n)\rangle}{\langle \mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{\text{tree}}_{n;0}}(x, a^{-1/4}\theta, \bar\theta = 0) = \left(\frac{\mathcal{A}_n}{\mathcal{A}^{\text{tree}}_{n;\text{MHV}}}(x,\theta)\right)^2$$

- duality considered at the level of the integrand...
- technical difficulty in identifying the superspaces on the two sides solved

Lagrangian insertions are also T-correlators!

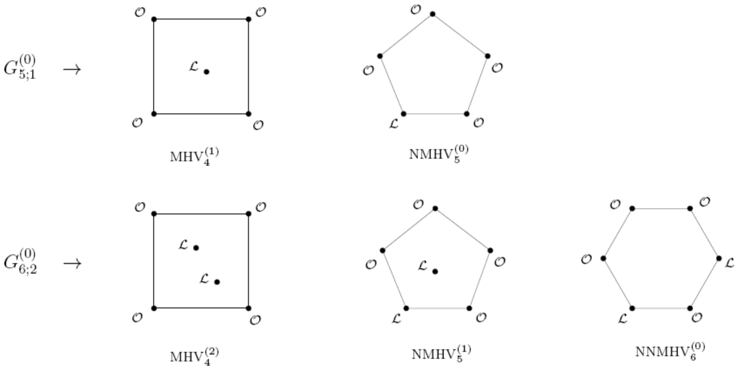
- Defined integrands of loop level correlators via insertion of $\mathcal{L}$
- But we saw that $\mathcal{L}$ is part of $\mathcal{T}$ (at $O(\theta^4)$)!
- Therefore integrands of loop level correlators of $\mathcal{T}$'s are in fact Born level correlators of $\mathcal{T}$'s

$$
\begin{aligned}
\langle \mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(l)} &= \int d^4x_{0_1}\dots d^4x_{0_l}\langle \mathcal{L}(0_1)\dots\mathcal{L}(0_l)\mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(0)} \\
&= \int d\mu_{0_1}\dots d\mu_{0_l}\langle \mathcal{T}(0_1)\dots\mathcal{T}(0_l)\mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(0)}\,,
\end{aligned}
$$

- $d\mu := d^4x d^4\theta$

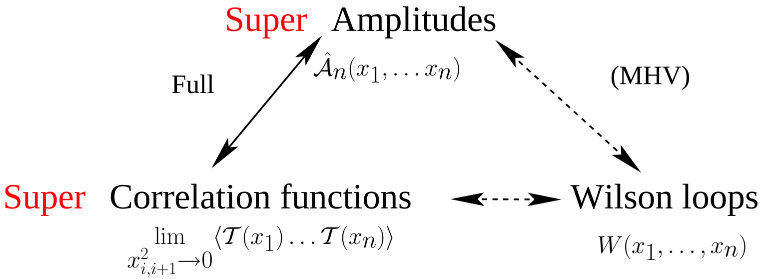
All loop amplitude integrands from Born level correlators

- Putting all these facts together we predict that *all* loop amplitude integrands can be obtained from Born level correlators
- Same correlator gives rise to different amplitudes

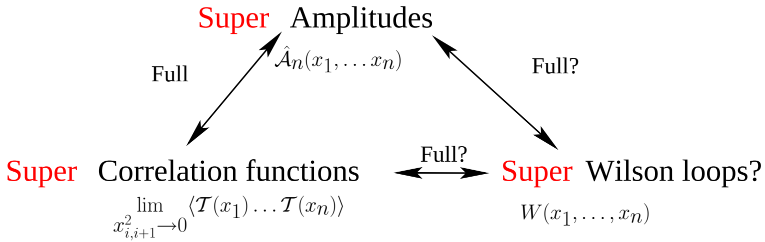


Further checks beyond MHV

- tree level: all *n*-point NMHV and 6 point NNMHV
- 1 loop: 5,6 points NMHV
- 6 point 1-loop NMHV integrand in particular has a highly non-trivial parity odd sector [Arkani-Hamed Bourjaily Cachazo Trnka + Caron-Huot] which is reproduced exactly (shown numerically)



- Super Wilson loop on twistor space: [Adamo Bullimore Mason Skinner]
- Super Wilson loop on Minkowski space: [Caron-Huot]
- Correlation functions on twistor space $\leftrightarrow$ super Wilson loop on twistor space (finite N_c adjoint rep.) [Adamo Bullimore Mason Skinner]
- However at the moment the super Wilson loop is formal, not regularised (even integrands are divergent).
- Only attempt so far to regularise (dim reg) lead to an anomaly [Belitsky Korchemsky Sokatchev]



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Questions

- Correlation functions of other operators?
- Reconstruction of correlators from amplitudes?
- Traces of x's versus momentum twistors?
- Implications for AdS/CFT?
- Off-light cone regularisation?
- Relation to OPE
- Form factors/ operators between states (see Gang Yang's talk) [Alday Maldacena Zhiboedov, Brandhuber Spence Travaglini Yang, Raju]
- Turning on $\bar{\theta}$? [Caron-Huot]

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Bern Dixon Kosower 2004

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