Proton Structure

and Prediction of pp Elastic Scattering

at LHC at Center-of-Mass Energy 7 TeV

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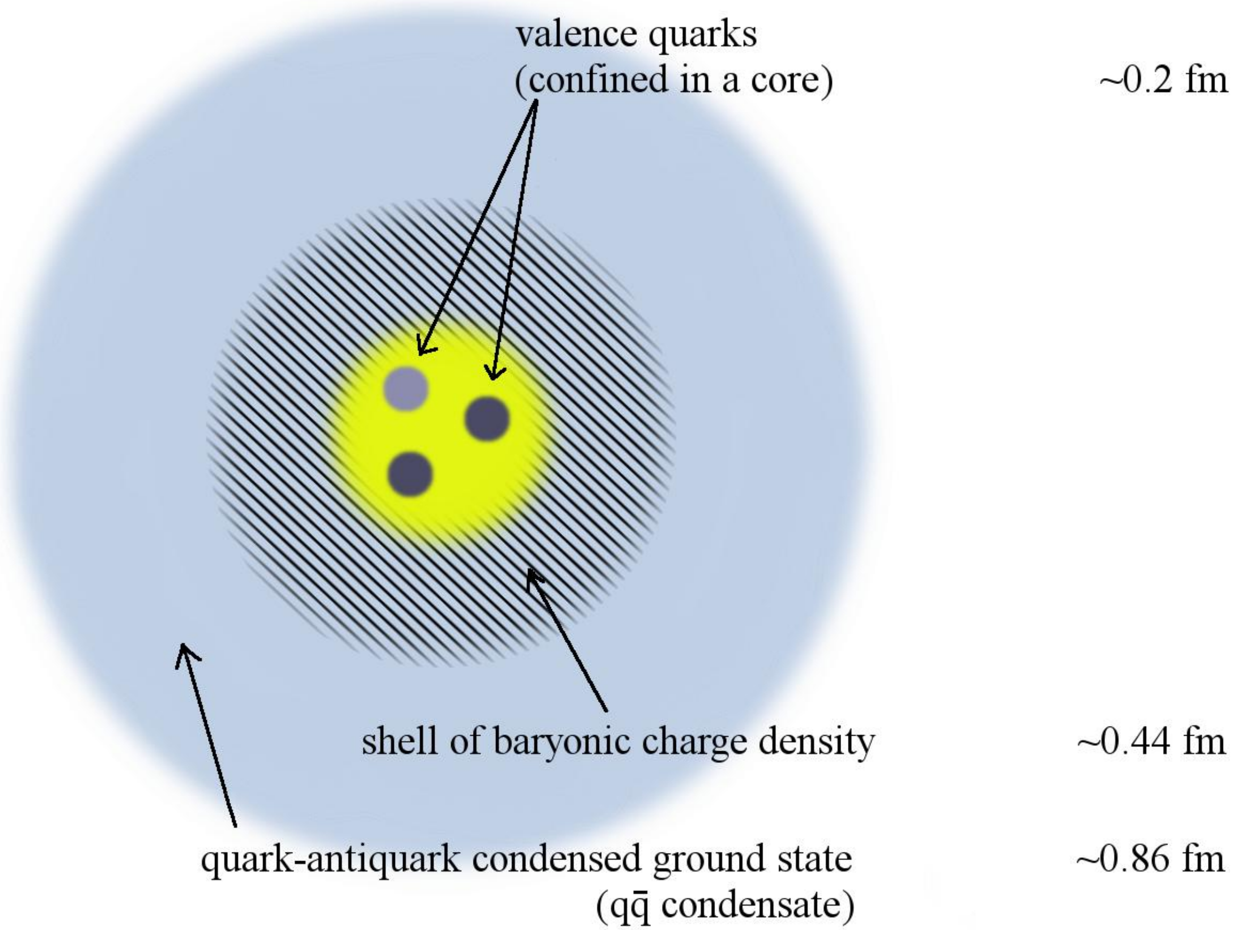
A physical picture of the proton has emerged from:

1) our phenomenological investigation of high energy proton-proton and proton-antiproton elastic scattering measurements;

2) our study of nonperturbative models of nucleon structure based on low energy properties of the nucleon.


valence quarks
(confined in a core)
~0.2 fm
shell of baryonic charge density
~0.44 fm
quark-antiquark condensed ground state
($q\bar{q}$ condensate)
~0.86 fm

High energy pp and $\bar{p}p$ elastic scattering measurements were done:

a) beginning with the CERN pp Collider in early seventies,
b) followed by Fermi lab fixed target pp scattering,
c) continued with the CERN SPS $\bar{p}p$ Collider in mid eighties,
d) finally, ending with the Tevatron measurements of $\bar{p}p$ elastic scattering at 1.8 TeV c.m. energy in mid nineties.

During the same period, many theoretical groups devoted enormous effort to develop nonperturbative models of nucleon structure. Here are some examples: i) Skyrmion model, ii) MIT Bag model, iii) Little Bag model, iv) Topological Soliton model, v) Chiral Bag model, etc.

Currently, pp elastic scattering at c.m. energy 7 TeV is being measured at LHC by the TOTEM (TOTal, Elastic, and diffractive scattering Measurement) Collaboration over a large momentum transfer range: $|t| = 0$ to $|t| > 10\ GeV^2$. And we just heard Karsten Eggert presenting preliminary results of the TOTEM Collaboration.

We present our prediction of pp elastic differential cross sections at c.m. energy 7 TeV in the range $|t| = 0 - 10\ GeV^2$ and compare our results with those of Block et al. model and Bourrely, Soffer, Wu (BSW) model.

Precise measurement of $d\sigma/dt$ by the TOTEM Collaboration will determine how well our predictions hold up and provide a major test of our model.

Gell-Mann-Levy linear σ-model with $SU(2)_L \times SU(2)_R$ global symmetry

$$\mathcal{L} = \bar{\psi}\, i\, \gamma^\mu \partial_\mu \psi + \frac{1}{2}\left(\partial_\mu \sigma\, \partial^\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}\right) - G\, \bar{\psi}[\sigma + i\vec{\tau} \cdot \vec{\pi}\, \gamma^5]\psi - \lambda(\sigma^2 + \vec{\pi}^2 - f_\pi^2)^2$$

Fermions: quarks (u, d)

$$\sigma(x) + i\, \vec{\tau}\, \cdot \vec{\pi}(x) = \zeta(x)\, U(x)$$

$\zeta(x)$: a scalar field $\qquad$ $U(x)$: a unitary field

$$\zeta(x) = \sqrt{\sigma^2(x) + +\vec{\pi}^2(x)}$$

Using right and left fermion fields:

$$\psi_R(x) = \tfrac{1}{2}(1+\gamma^5)\psi(x), \qquad \psi_L(x) = \tfrac{1}{2}(1-\gamma^5)\psi(x)$$

the Lagrangian density becomes

$$\mathcal{L} = \overline{\psi}_R\;\, i\gamma_\mu \partial_\mu \psi_R + \overline{\psi}_L\;\, i\gamma^\mu \partial_\mu \psi_L + \tfrac{1}{2}\partial_\mu \zeta\;\, \partial^\mu \zeta + \tfrac{1}{4}\zeta^2\, tr[\partial_\mu U\;\, \partial^\mu U^\dagger]$$

$$-G\zeta[\overline{\psi}_L\, U\, \psi_R + \overline{\psi}_R\, U^\dagger\, \psi_L] - \lambda(\zeta^2 - f_\pi^2)^2$$

Under global right and left transformations

$$\psi_R(x) \to R\,\psi_R(x), \quad \psi_L(x) \to L\,\psi_L(x)$$

$$U(x) \to LU(x)R^\dagger, \quad \zeta(x) \to \zeta(x)$$

If we now consider chiral symmetry ($\psi \to e^{\theta\gamma^5}\psi$), then

$$\psi_R(x) \to e^{\theta}\psi_R(x), \quad \psi_L(x) \to e^{-\theta}\psi_L(x),$$

$$U(x) \to e^{-\theta}\,U(x)e^{-\theta}, \quad \zeta(x) \to \zeta(x)$$

where $\theta = -i\,T^a\,\theta^a$ $\quad (T^a = \frac{1}{2}\,\tau^a)$ and θ^a's are global.

Potential energy density: $v(\zeta) = \lambda\,(\zeta^2 - f_\pi^2)^2$

$$\zeta(x) = f_\pi$$

(corresponds to the ground state)

Under a chiral transformation,

$$\zeta_0\, U(x) \rightarrow \zeta_0\, U'(x) = \zeta_0\, e^{-\theta}\, U(x)\, e^{-\theta} \quad (\zeta_0 = f_\pi)$$

(spontaneous breakdown of chiral symmetry)

$U(x) = \exp[i\, \vec{\tau} \cdot \frac{\vec{\varphi}(x)}{f_\pi}]$ ($\vec{\varphi}(x)$: pion field,

f_π: pion decay coupling constant)

$$U(x) = \xi_L^\dagger(x)\,\xi_R(x)$$

$$\xi_R(x) \to h(x)\xi_R(x)R^\dagger \qquad\qquad \xi_L(x) \to h(x)\xi_L(x)L^\dagger$$

$$U(x) = \xi_L^\dagger(x)\,\xi_R(x) \to L\,U(x)\,R^\dagger$$

$$\mathcal{L}_U = \tfrac{1}{4}\zeta^2\, tr\left[\partial_\mu U\,\partial^\mu U^\dagger\right]$$

$$= -\tfrac{1}{4}\zeta^2\, tr\left[\partial_\mu\xi_L\,\xi_L^\dagger - \partial_\mu\xi_R\,\xi_R^\dagger\right]^2$$

$$\mathcal{V}_\mu = -\tfrac{i}{2}\, g[\vec{\tau}\cdot\vec{\rho}_\mu + \omega_\mu]$$

$$\partial_\mu \to D_\mu = \partial_\mu + \mathcal{V}_\mu$$

$$\mathcal{L}_U = -\tfrac{1}{4}\zeta^2 tr[(D_\mu\,\xi_L)\xi_L^\dagger - (D_\mu\xi_R)\xi_R^\dagger]^2$$

$$-\tfrac{1}{2}\zeta^2 tr\left[(D_\mu\,\xi_L)\xi_L^\dagger + (D_\mu\xi_R)\xi_R^\dagger\right]^2 + \frac{1}{2g^2}\, tr[F_{\mu\nu}F^{\mu\nu}]$$

$$\mathcal{L}_q = \bar{\psi}_L \, i \, \gamma^\mu \left(\partial_\mu + A_\mu^L\right)\psi_L + \bar{\psi}_R \, i \, \gamma^\mu \left(\partial_\mu + A_\mu^R\right)\psi_R$$
$$- G \, \zeta[\bar{\psi}_L \, \xi_L^\dagger \xi_R \, \psi_R + \bar{\psi}_R \, \xi_R^\dagger \xi_L \, \psi_L]$$

$$\int d\psi_L \, d\psi_L^\dagger \, d\psi_R \, d\psi_R^\dagger = e^{i \, \Gamma[A^L, A^R, U]} \int d\psi_L^0 \, d\psi_L^{0\dagger} \, d\psi_R^0 \, d\psi_R^{0\dagger}$$

$$\Gamma[A^L, A^R, U] = \Gamma_{WZW}[A^L, A^R, U]$$

Wess-Zumino-Witten (WZW) action

$$\psi_L^0(x) = \xi_L(x)\psi_L(x) \to \xi_L(x)L^\dagger(x)L(x)\psi_L(x) = \psi_L^0(x)$$

$$\psi_R^0(x) = \xi_R(x)\psi_R(x) \to \xi_R(x)R^\dagger(x)R(x)\psi_R(x) = \psi_R^0(x)$$

$$e^{i\,W\left[A^L,A^R,U,\zeta,\psi_L,\psi_R\right]}$$

$$= \frac{1}{\mathcal{N}} \int dU\, d\zeta\; e^{i\,\Gamma_{WZW}\left[A^L,A^R,U\right]} e^{iS_{U,\zeta}\left[A^L,A^R,U,\zeta\right]} \int d\psi^0\; d\psi^{0\dagger} e^{iS_q\left[A^{L0},A^{R0},\psi^0,\psi^{0\dagger}\right]}$$

$$\psi_0 = \psi_0^L + \psi_0^R$$

$$A_\mu^{L0} = \xi_L\, A_\mu^L\, \xi_L^\dagger + \xi_L\, \partial_\mu\, \xi_L^\dagger$$

$$A_\mu^{R0} = \xi_R\, A_\mu^R\, \xi_R^\dagger + \xi_R\, \partial_\mu\, \xi_R^\dagger$$

$$e^{iW\left[A^L,A^R,U,\zeta,\psi_L,\psi_R\right]}$$

$$\simeq \frac{1}{\mathcal{N}} \int dU\; e^{i\,\Gamma_{WZW}\left[A^L,A^R,U\right]+iS_U\left[A^L,A^R,U,f_\pi\right]} \int d\zeta\; d\psi^0\; d\psi^{0\dagger} e^{iS_{q,\zeta}\left[A^L,A^R,\zeta,\psi^0,\psi^{0\dagger}\right]}$$

$$\Gamma_{WZW}\left[A^L, A^R, U\right] + S_U\left[A^L, A^R, U, f_\pi\right]$$

describes the Topological Soliton Model of the nucleon.

$$S_{q,\zeta}[A^{L0}, A^{R0}, \zeta, \psi^0, {\psi^0}^\dagger]$$

describes the quark-scalar sector:

$$S_{q,\zeta}\left[A^{L0}, A^{R0}, \zeta, \psi^0, {\psi^0}^\dagger\right]$$
$$= \int d^4x \,\{\bar{\psi}_L^0 \;\, i\,\gamma^\mu\left(\partial_\mu + A_\mu^{L0}\right)\psi_L^0 + \bar{\psi}_R^0 \;\, i\,\gamma^\mu\left(\partial_\mu + A_\mu^{R0}\right)\psi_R^0$$
$$+ \frac{1}{2}\,\partial_\mu\zeta\,\partial^\mu\zeta - \lambda(\zeta^2 - f_\pi^2)^2 - G\,\zeta\,\bar{\psi}^0\psi^0\}$$

$$(\psi^0 = \psi_L^0 + \psi_R^0)$$

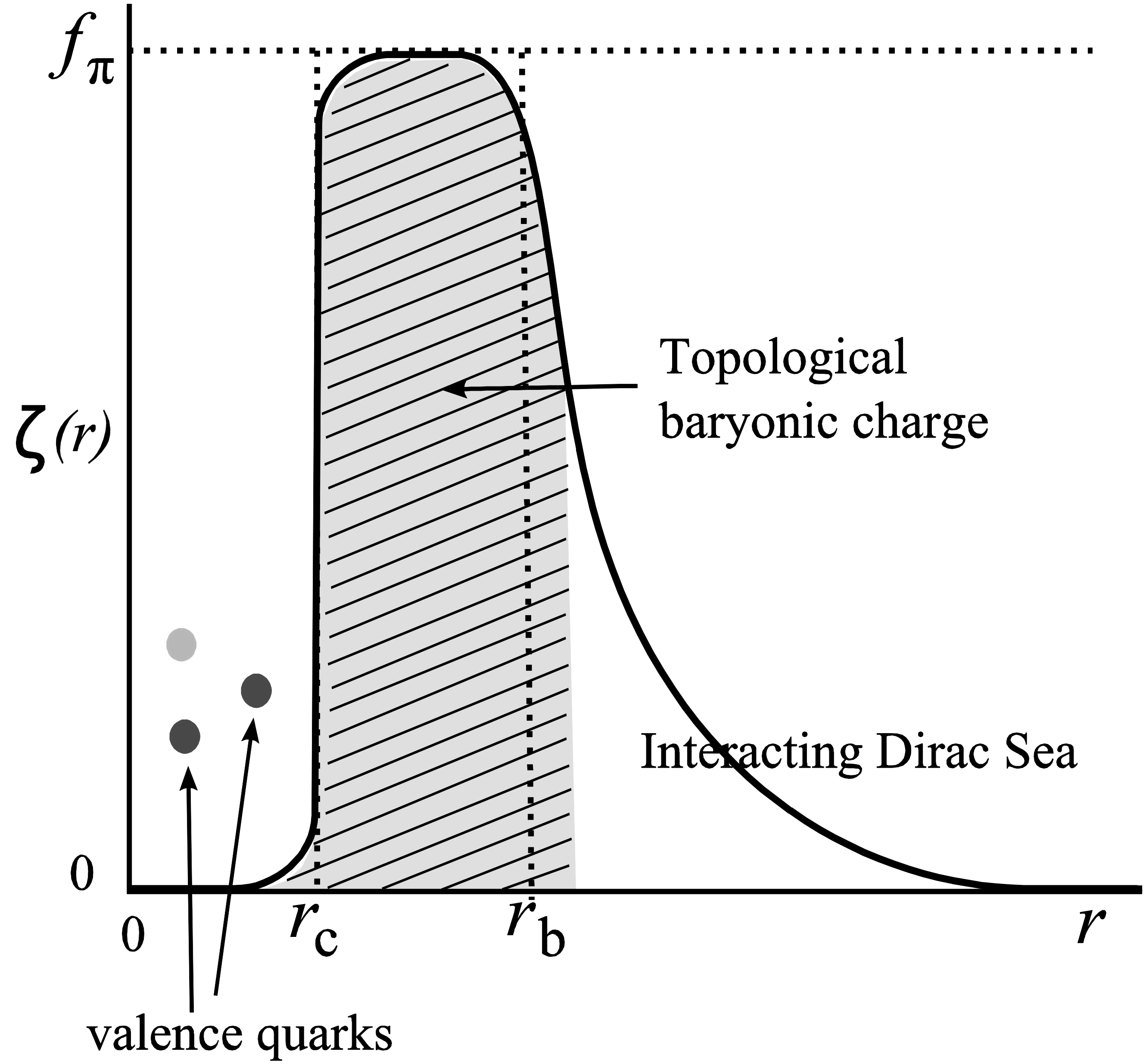
If we take a ζ-field which is

$$\zeta(r) = f_\pi\, \theta(r - r_c),$$

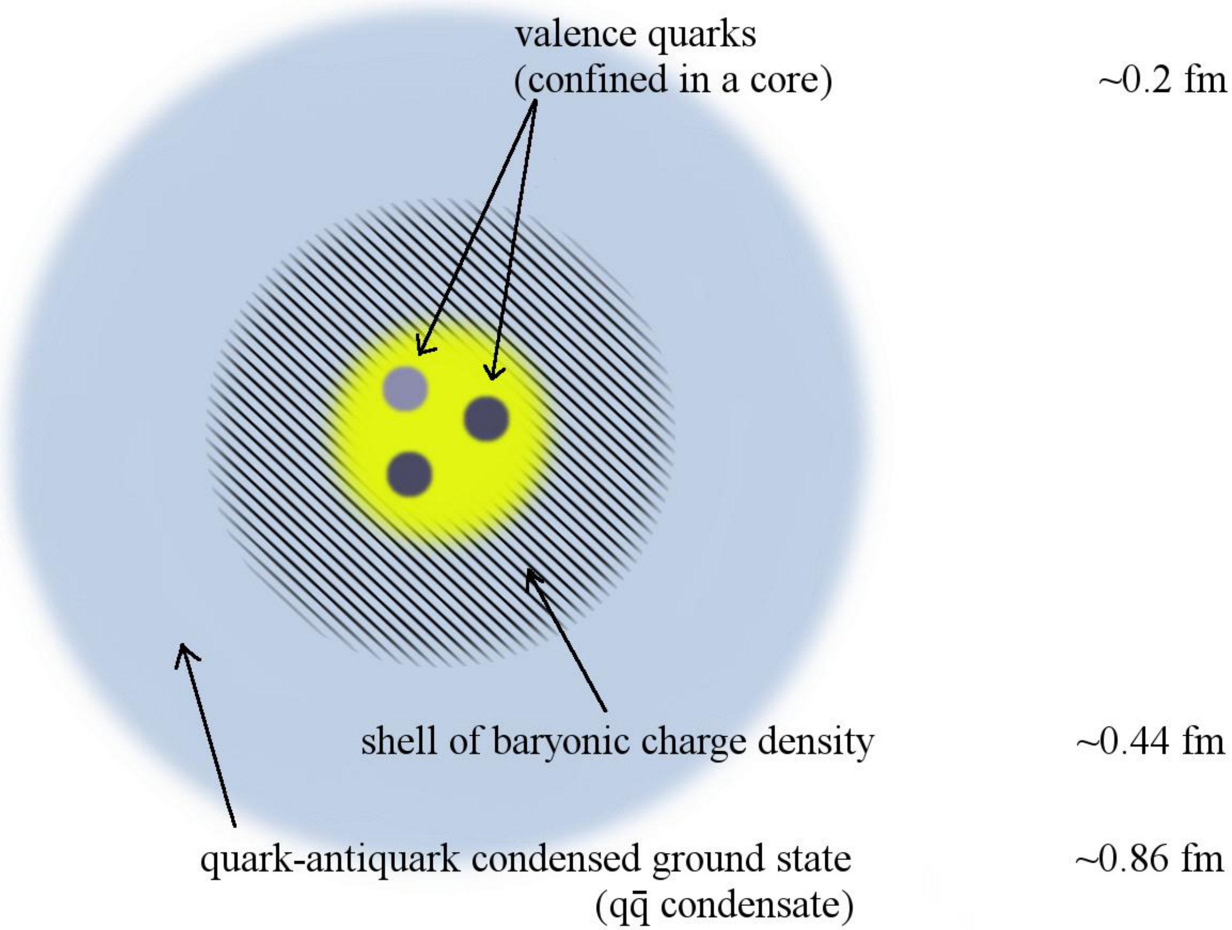
we obtain an infinitely negative surface energy:

$$-\frac{1}{2}\, f_\pi^2\, 4\,\pi\, r_c^2\, \delta(0)$$

What this means is that, if $\zeta(r)$ falls off steeply enough, the mass of the soliton will be reduced by a large amount. This resolves a major problem for the topological soliton model, which has consistently led to a large mass for the soliton.



f_π
$\zeta(r)$
Topological
baryonic charge
Interacting Dirac Sea
0
0
r_c
r_b
r
valence quarks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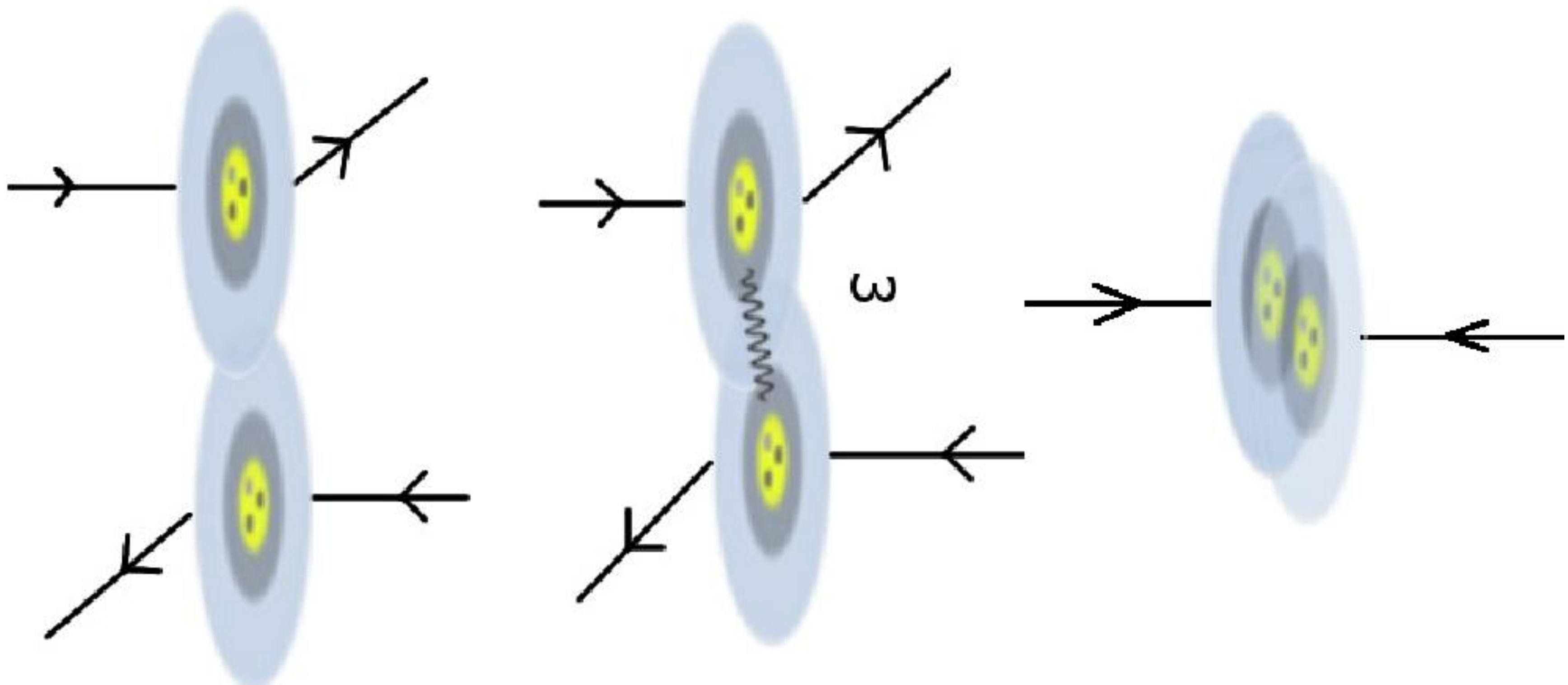
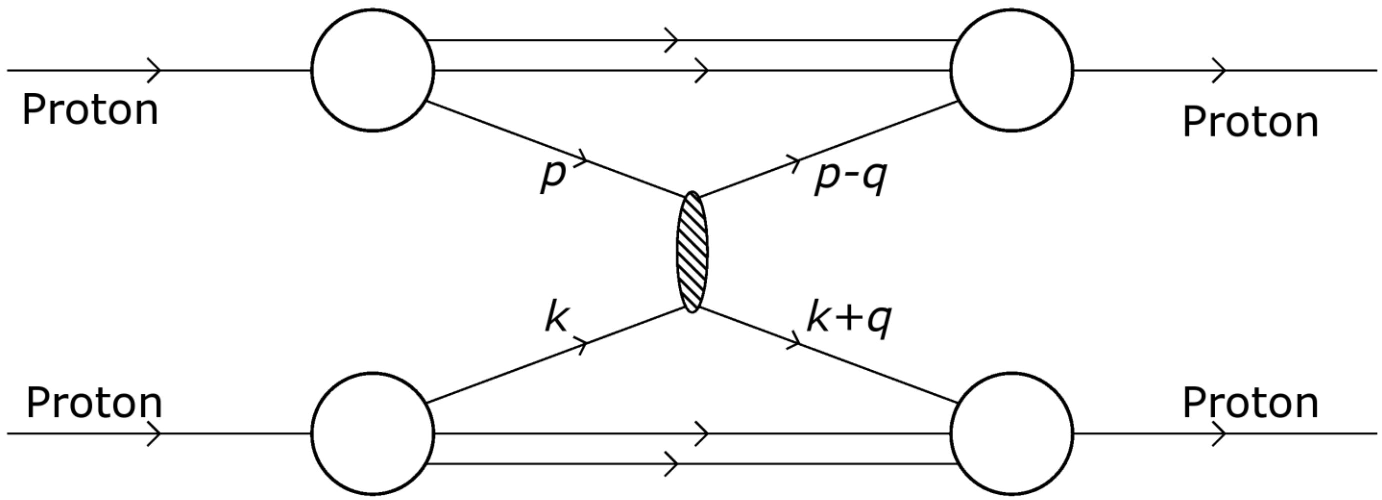
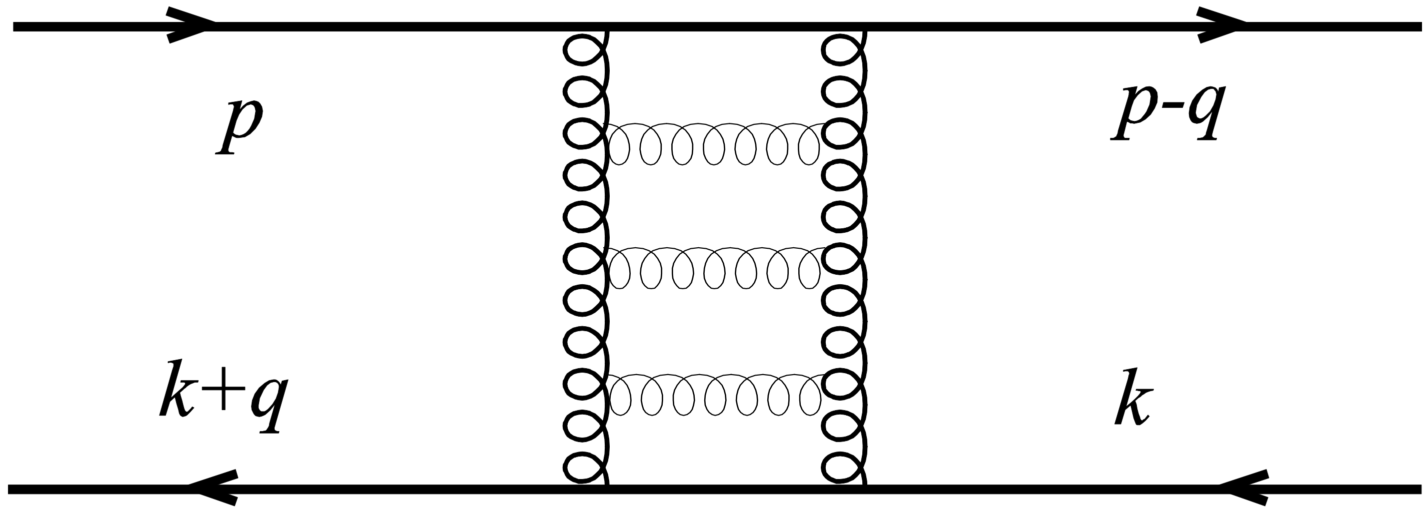
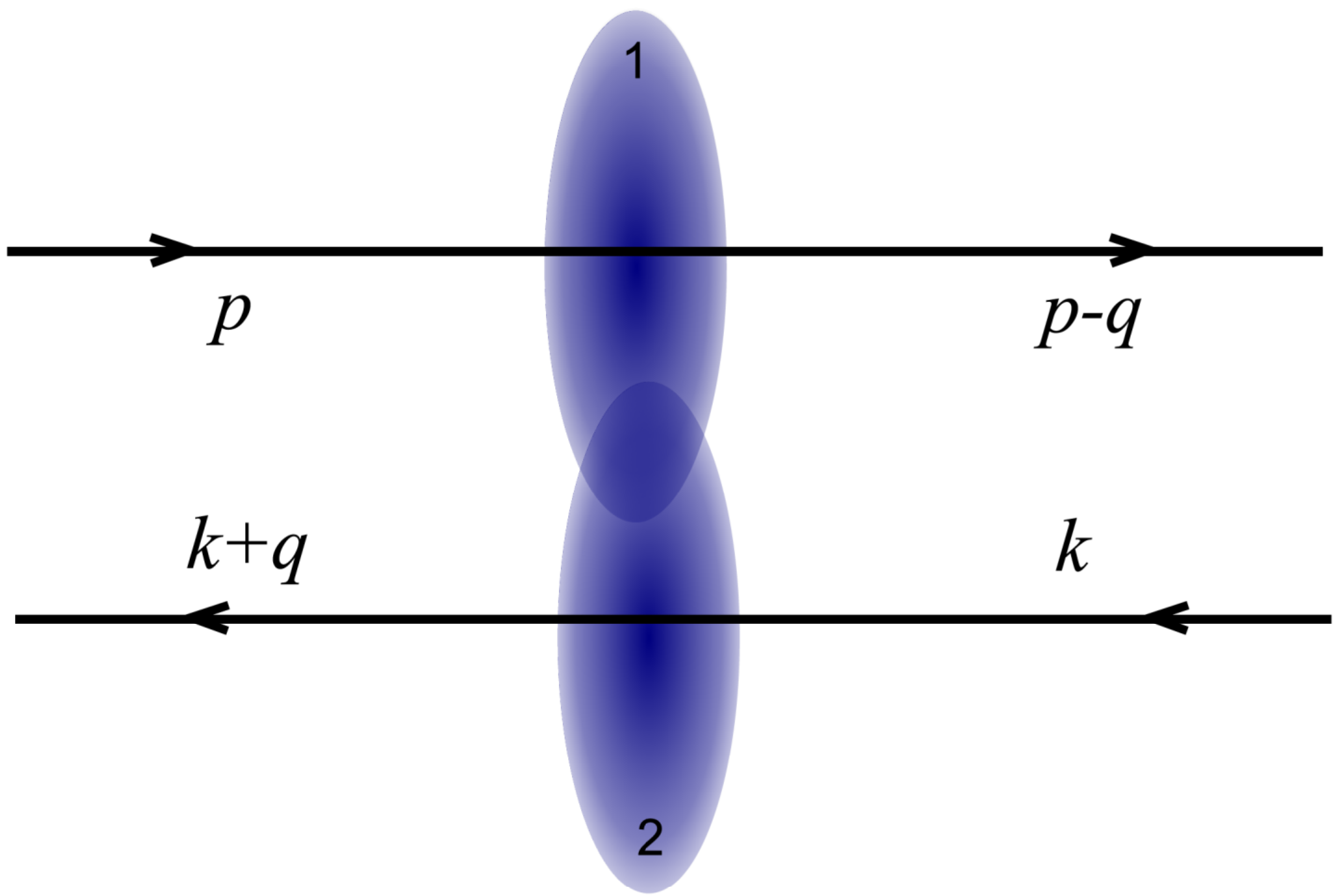
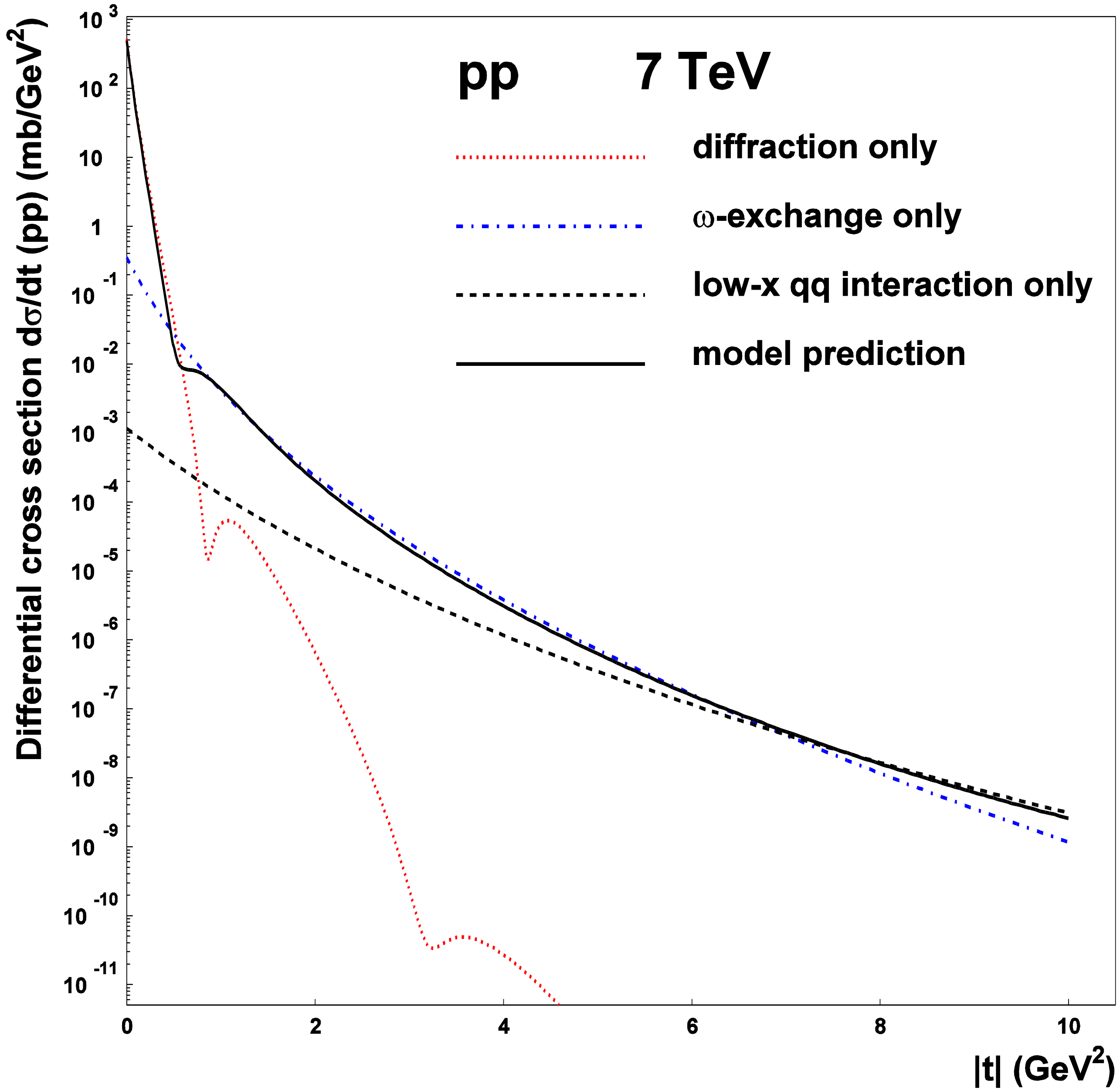

ω


Proton
Proton
Proton
Proton
p
p-q
k
k+q

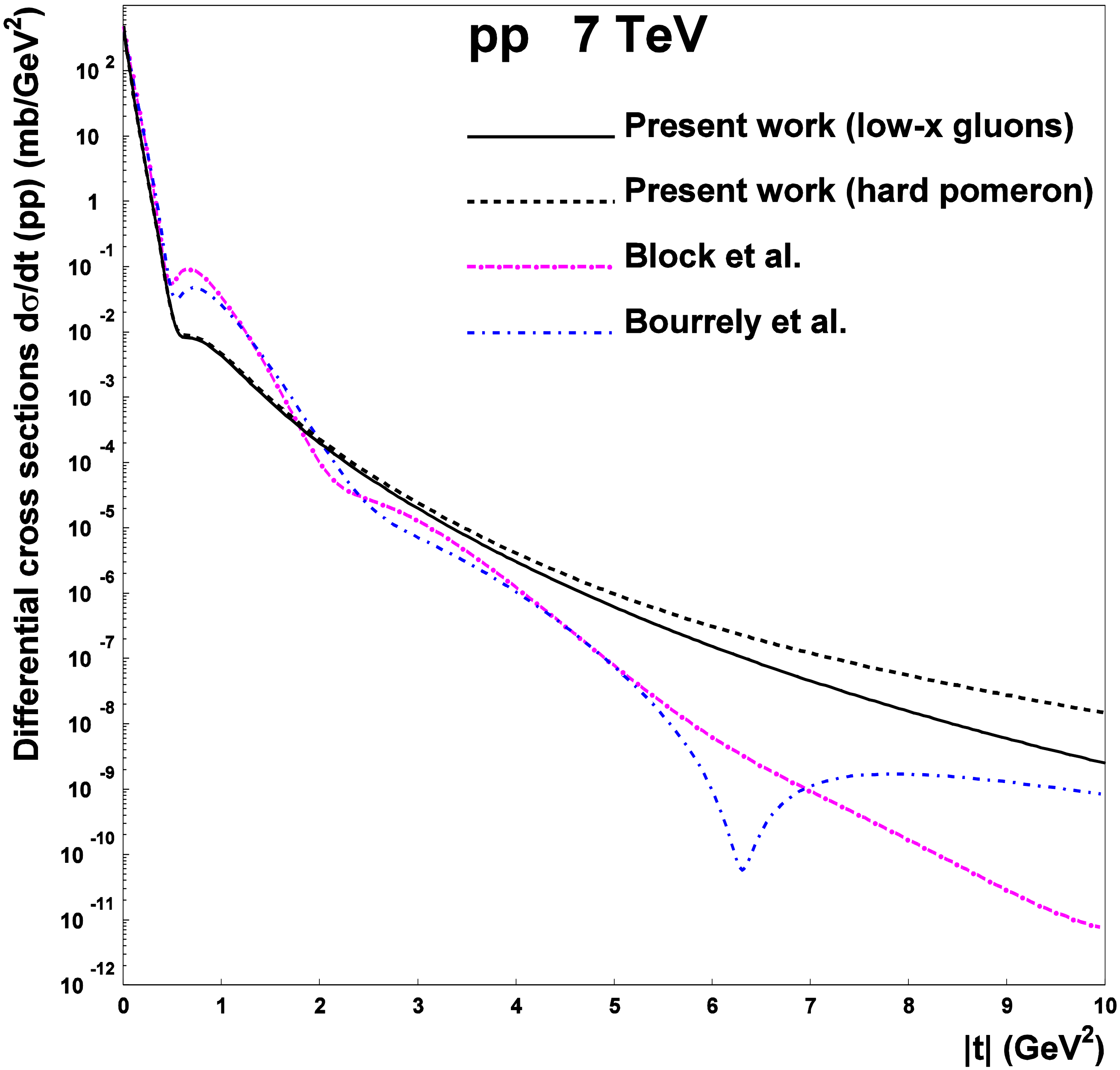


p
p-q
$k+q$
k


1
p
p-q
k+q
k
2



pp 7 TeV
diffraction only
ω-exchange only
low-x qq interaction only
model prediction
Differential cross section dσ/dt (pp) (mb/GeV²)
|t| (GeV²)


pp 7 TeV
Present work (low-x gluons)
Present work (hard pomeron)
Block et al.
Bourrely et al.
Differential cross sections dσ/dt (pp) (mb/GeV²)
|t| (GeV²)

Concluding Remarks

1. We find that the topological soliton model provides an appropriate backdrop for the proton structure that we have arrived at.

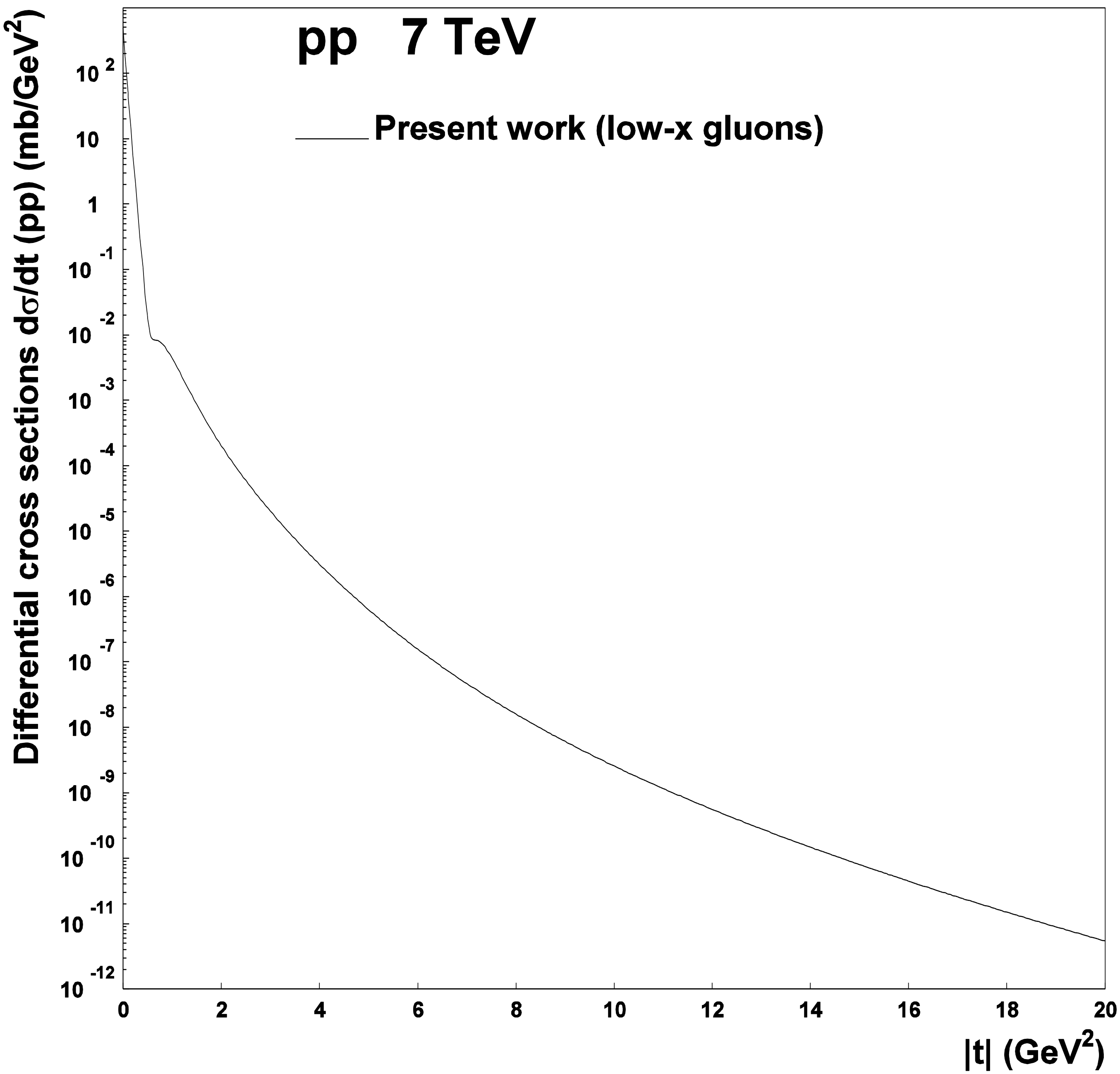
2. The large mass problem of the topological soliton model is resolved by realizing that, if the scalar field falls steeply from $\zeta(r) = f_\pi$ to $\zeta(r) = 0$ – somewhat like $\zeta(r) = f_\pi\, \theta(r - r_c)$, then the mass of the soliton decreases by a significant amount. This also means the model becomes essentially a Chiral Bag model.

3. In the region beyond the topological baryonic charge density ($r > r_b$), the scalar field decreases smoothly (p. 14), its nonvanishing value makes the quarks and antiquarks massive, lowers the energy of the Dirac sea,

and we have a $q\bar{q}$ condensed ground state forming an outer cloud of the proton.

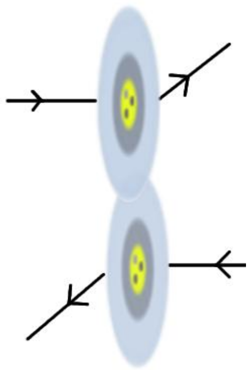
4. Our quantitative $d\sigma/dt$ calculations at 7 TeV show that there are three distinct processes which come to play. For small $|t|$ - it is diffraction, for intermediate $|t|$ $(1\ GeV^2 < |t| < 7\ GeV^2)$ - it is ω-exchange probing charge densities of the colliding protons, and for large $|t|$ $(|t| > 7\ GeV^2)$ - it is valence quark-quark scattering via gluon ladders or low-x gluon-gluon cloud interaction.

5. Our predicted $d\sigma/dt$ decreases smoothly from $|t| \approx 1\ GeV^2$ to $|t| = 10\ GeV^2$, while $d\sigma/dt$ predicted by Block et al. model and Bourrely et al. model shows oscillations and at large $|t|$ $(> 5\ GeV^2)$ leads to differential cross sections much smaller than those obtained by us.



pp 7 TeV
Present work (low-x gluons)
Differential cross sections dσ/dt (pp) (mb/GeV^2)
|t| (GeV^2)
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
0
2
4
6
8
10
12
14
16
18
20

Diffraction Amplitude:



$$T_D(s,t) = i\,pW \int_0^\infty b\,db\, J_0(bq)\,\Gamma_D(s,b)$$

$q = \sqrt{|t|}$, Profile function: $\Gamma_D(s,b) = 1 - e^{i\chi_D(s,b)}$

$$\Gamma_D(s,b) = g(s)\left[\frac{1}{1+\exp[(b-R)/a]} + \frac{1}{1+\exp[-(b+R)/a]} - 1\right]$$

$$R = R(s) = R_0 + R_1\left(\ln s - i\pi/2\right), \quad a = a(s) = a_0 + a_1\left(\ln s - i\pi/2\right)$$

General properties associated with diffraction:

1. $\sigma_{tot}(s) \sim (a_0 + a_1 \ln s)^2$ Froissart – Martin bound

2. $\rho(s) \simeq \dfrac{\pi\, a_1}{(a_0 + a_1 \ln s)}$ Derivative dispersion relation

$$\rho(s) = \frac{Re\, T(s,0)}{Im\, T(s,0)}$$

3. $T_D(s,t) \sim i\, s \ln^2 s\;\, f(|t| \ln^2 s)$ Auberson – Kinoshita – Martin scaling

4. $T_D^{\bar{p}p}(s,t) = T_D^{pp}(s,t)$ Crossing even

$$\Gamma_{WZW}[A^L, A^R, U] = \int d^4x\, \mathcal{L}_{WZW}$$

$$\mathcal{L}_{WZW} = g_\omega \omega_\mu B^\mu$$

B^μ is the conserved topological baryonic current

$$B^\mu(x) = \frac{1}{24\,\pi^2} \epsilon^{\mu\nu\rho\sigma}\, tr[U^\dagger \partial_\nu U\, U^\dagger\, \partial_\rho U\, U^\dagger\, \partial_\sigma U]$$