Real-Time AdS/CFT and applications: jet quenching and hydrodynamic coefficients

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arXiv: 1004.1179 w P. Arnold, E. Barnes and C. Wu (Phys.Rev.D)
arxiv: 1008.4023 w P. Arnold (JHEP)
arxiv: 1101.2689 w P. Arnold (JHEP)
arxiv: 1105.4645 w P. Arnold, C. Wu, W. Xiao
work in progress with C. Wu, E. Barnes

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Real-time finite temperature AdS/CFT

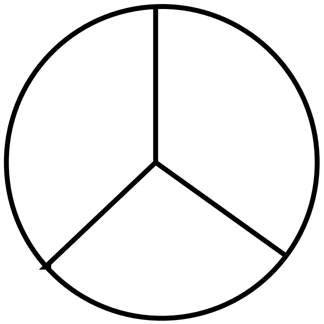
AdS/CFT at $T = 0$: Strong-weak duality between a conformal field theory and string theory in a curved (Anti-de Sitter) background.
Trademark example: $\mathcal{N} = 4$ super Yang-Mills (sYM) and IIB string theory on $AdS_5 \times S^5$.

Identification of partition functions: $Z_{sYM} = Z_{IIBstring}$
In a simplified set-up, consider the supergravity (sugra) modes on $AdS_5 \times S^5$.

Witten:

$$Z_{sYM}[J = \text{source for a BPS opt. } O] = Z_{sugra}[\phi_{sugra}(x^\mu, z) \longrightarrow J(x^\mu) \text{ at the AdS bdy}]$$

$\Longrightarrow$ Correlators of BPS sYM operators can be computed at strong coupling by doing a weakly coupled gravity computation.

Witten diagram: 

Witten's prescription was for Euclidean AdS.

The correlators are then computed in imaginary time. What about real-time correlators?

The problem:

AdS_5 can be described using different coordinates: global coordinates, which cover the whole space (two time-like boundaries!), or coordinates which make manifest the Poincare symmetry of the field theory (plus a radial coordinate)

$$ds^2_{AdS} = \frac{1}{z^2}(dx^\mu dx_\mu + dz^2)$$

and which cover only half of the space.

Which region of AdS must one integrate to get real-time correlators?

What about the different types of real-time correlators? How are they computed from sugra?

To find the answer, we used reverse engineering:

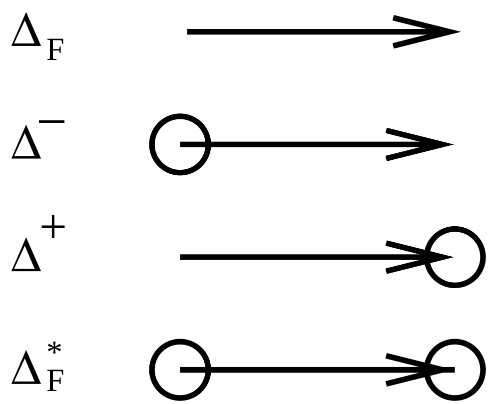
-start from the known expressions of the real-time 3-point correlators

$$
\begin{aligned}
G_F(x_1,x_2,x_3) &= (-i)^2\langle 0|\mathcal{T}O(x_1)O(x_2)O(x_3)|0\rangle \\
&= (-i)^2\left(\frac{1}{-t_{12}^2+\vec{x}_{12}^2+i\epsilon}\,\frac{1}{-t_{23}^2+\vec{x}_{23}^2+i\epsilon}\,\frac{1}{-t_{31}^2+\vec{x}_{31}^2+i\epsilon}\right)^{\Delta/2} \\
G_{123}(x_1,x_2,x_3) &= (-i)^2\langle 0|O(x_1)O(x_2)O(x_3)|0\rangle \\
&= (-i)^2\left(\frac{1}{-t_{12}^2+\vec{x}_{12}^2+i\epsilon t_{12}}\,\frac{1}{-t_{23}^2+\vec{x}_{23}^2+i\epsilon t_{23}}\,\frac{1}{-t_{31}^2+\vec{x}_{31}^2-i\epsilon t_{31}}\right)^{\Delta/2} \\
G_R(x_1,x_2,x_3) &= \theta(t_{31})\theta(t_{12})\Big(G_{312}-G_{132}+G_{213}-G_{231}\Big) \\
&+ \theta(t_{32})\theta(t_{21})\Big(G_{321}-G_{231}+G_{123}-G_{132}\Big)
\end{aligned}
$$

and manipulate until the structure of a Witten diagram with 3 bulk-to-bdy propagators emerges: the integration needs to be done only over the Poincare patch.

What about the different types of real-time correlators? How are they computed from sugra?

Our answer: for Feynman (time-ordered) correlators use Feynman sugra propagators; for retarded/ (causal) correlators use causal sugra propagators. Tantamount to using Veltman's circling rules to build real-time diagrams at $T = 0$ in supergravity [1004.1179].



This is the jump-off point for real-time $T \neq 0$ computations from AdS/CFT.

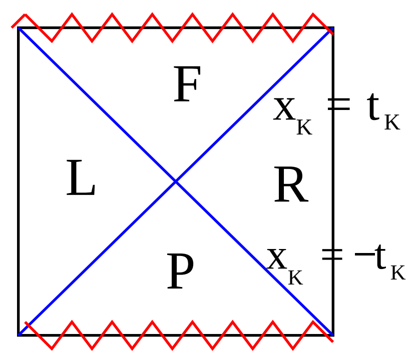
Finite temperature AdS/CFT: the finite temperature CFT is holographically dual to AdS with a black hole in it: AdS-Schwarzschild (AdS-S)

$$ds^2_{10} = \frac{r^2}{R^2}(-f(r)dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f(r)}dr^2 + R^2 d\Omega_5$$

$$= R^2\left(\frac{-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{z^2 f(z)}}{z^2} + d\Omega_5^2\right), \qquad z = \frac{R^2}{r}$$

$$f(r) = 1 - \frac{r_0^4}{r^4}$$

The Hawking temperature is $T_H = \frac{r_0}{\pi R^2}$.



AdS-S Penrose diagram

How to integrate over the black hole bulk (given the presence of singularities, horizons)?

Tools of the trade: supergravity bulk-to-boundary propagators, and supergravity vertices.

Consider a massless scalar field $\phi(p^\mu, u) = F(\omega, \vec{p}, u)\phi_0(p^\mu)$ in the AdS-S background obeys $\Box F = 0$:

$$F'' - \frac{1+u^2}{u(1-u^2)}F' + \left(\frac{\omega^2}{u(1-u^2)^2} - \frac{\vec{p}^2}{u(1-u^2)}\right)F = 0$$

where $\omega, \vec{p}$ and u are dimensionless quantities

$$\omega = \frac{E}{2\pi T_H}, \vec{p} = \frac{\vec{P}}{2\pi T_H}, \qquad u = \frac{r_0^2}{R^2}z^2.$$

Furthermore, impose the condition that F is an incoming wave at the horizon.

- Retarded propagator: $G_R \equiv F(\omega, \vec{k}, u)$.
- Gibbons & Perry The "Kruskal" vacuum Feynman propagator is the one which exhibits periodicity in imaginary "Schwarzschild" time. Hallmark characteristic of thermal propagators.

  Feynman propagator: $G_F = ReG_R + i\coth(\omega\pi)ImG_R$.

  $G_R = G_F - G^-$ etc. $\Rightarrow$ get the other (Wightman) Green's functions.

What was known since 2002 or how to compute 2-point functions:

Son & Starinets conjectured that the retarded 2-point CFT correlator at finite temperature is given by

$$\langle O(\omega,\vec{k})O(0)\rangle_\beta \propto \sqrt{g}g^{uu}\partial_u F(\omega,\vec{k},u)\Big|_{u=0}$$

based on the zero-temperature limit.

Son & Herzog gave a geometric interpretation of the finite temperature Schwinger-Keldysh matrix 2-point correlator by adding sources for the physical and ghost/doubler fields on the two boundaries of the AdS-S Penrose diagram.

Comments: Peculiar feature of 2-point functions which arise from $\int\sqrt{g}\partial\phi\cdot\partial\phi$: when evaluating the quadratic action on-shell, a 2-point function reduces to a boundary term. A genuine integration over the black hole bulk is not needed. Son and Herzog's prescription was not precise in how the integration over the black hole needs to be carried out.

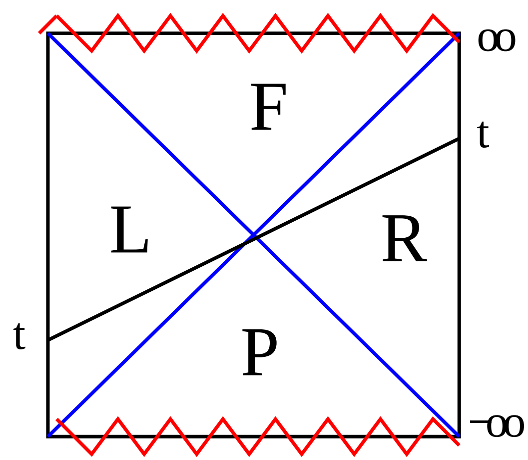
Our answer: $R - L$ prescription

If one follows Son and Herzog, then

$$\mathcal{S} \sim \left(\int_R - \int_L \right) \sqrt{g} \partial\phi\partial\phi,$$

use EOM and keep the R and L boundary terms, with a relative sign contribution [1004.1179].

This gives the 2-point finite temperature Schwinger-Keldysh Green's function.



Frolov-Martinez

Simpler interpretation: the $R - L$ prescription is merely enforcing Veltman's circling rules at finite temperature [1004.1179].

Kobes, Kobes& Semenoff: Trade off the matrix Schwinger-Keldysh propagator for circling rules diagrams.

Any finite temperature "Feynman" diagram is given by the sum of all diagrams with vertices either circled or uncircled, with the exception of vertices connected to external lines which remain uncircled.

Veltman's Largest Time Equation (identity): The sum of all diagrams with all vertices either circled or uncircled is 0.

A retarded n-point function, with one external vertex having the largest time is given by the sum of all diagrams, with all other vertices being either circled or uncircled.

It is this Green's function computed in real-time formalism that coincides with the analytic continuation of the imaginary time formalism.

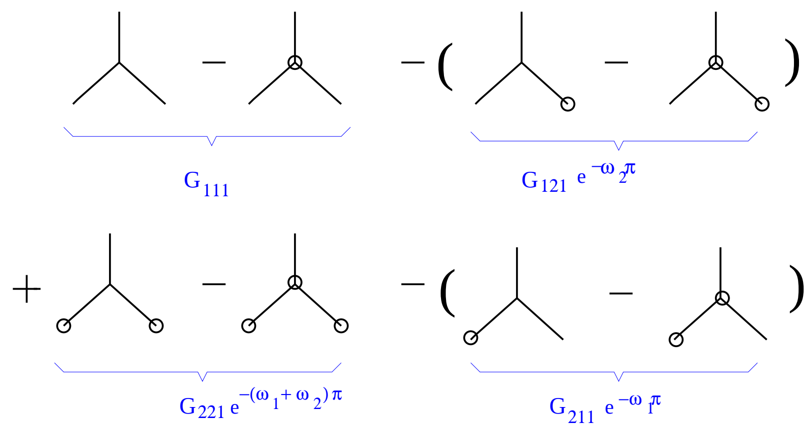
The same rules apply to gravity, with the integration over the bulk region only up to horizon. (We need not be concerned with the global black hole space-time.)

The Poincare coordinates are singled out since they are the preferred coordinates in the dual field theory.

Upshot: we give the first concrete formulas for real-time finite-temperature 3-point correlators computed from AdS/CFT [1004.1179]

At finite temperature,

- Retarded 3-point definition



We define the retarded 3-point correlator to be given by the sum of all diagrams above. After substituting the various G_{abc}, the final expression is

$$G_R(r; p, q) \propto \int_0^1 du \sqrt{g} F^*(q) F^*(p) F(r)$$

which is consistent with the zero temperature limit result, it is consistent with analytic continuation of the imaginary time Green's function, and it is manifestly causal.

Applications

Jet quenching revisited

How far does a localized high-energy excitation travel through the quark-gluon plasma before stopping and thermalizing?

Weakly-coupled plasma: $E^{1/2}$

Strongly-coupled $\mathcal{N}=4$ super Yang-Mills: $E^{1/3}$
(Maximal distance traveled $\sim (E/\sqrt{\lambda})^{1/3}$ for excitations dual to semi-classical strings. No $\sqrt{\lambda}$-dependence for excitations dual to sugra modes.)

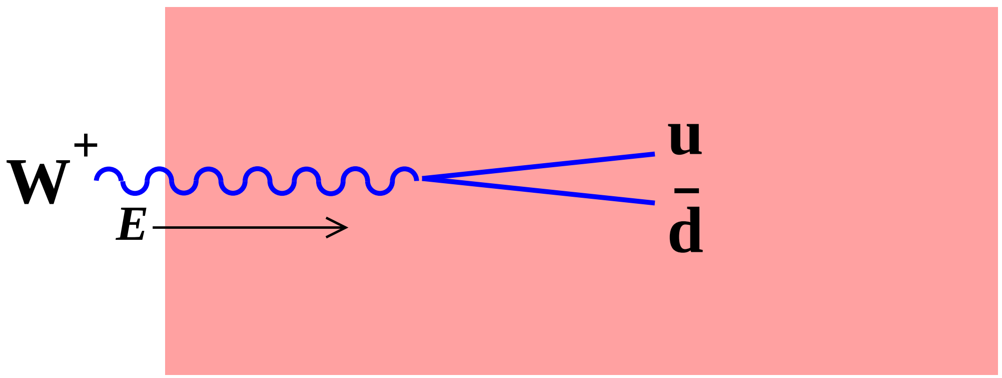
In [1008.4023] we re-opened the problem by posing the question on the field theory side: namely we specify the excitation created on the gauge theory side, and the response (in terms of conserved charge densities) is later measured in the field theory as well. We work at strong coupling and use AdS/CFT duality.

$$\mathcal{L} \to \mathcal{L} + j^a_\mu A^{a\,\mu}_{\rm cl},$$

$$A^{a\,\mu}_{\rm cl}(x) = \bar{\varepsilon}^\mu \mathcal{N}_A \left[\frac{\tau^+}{2}\, e^{i\bar{\boldsymbol{k}}\cdot\boldsymbol{x}} + \text{h.c.}\right] e^{-\frac{1}{2}(x_0/L)^2} e^{-\frac{1}{2}(x_3/L)^2},$$

$$\bar{\varepsilon}^\mu = (0,1,0,0), \qquad \bar{k}^\mu = (E,0,0,E), \qquad E \gg T, EL \gg 1.$$

Analogy: A very high energy W^+ boson decaying inside a standard-model quark-gluon plasma and producing high-energy partons moving to the right with net 3rd component of isospin, $\tau^3/2$:



The problem:

The source $A_{\rm cl}^{a\mu}$ creates an excitation that carries energy, momentum, and R charge. We track the R charge density, specifically the large-time behavior ($t \gg$ both T^{-1} and L) of

$$\left\langle j^{(3)0}(x) \right\rangle_{A_{\rm cl}}$$

if the system starts in thermal equilibrium at $t = -\infty$.

This reduces to a retarded 3-point function!

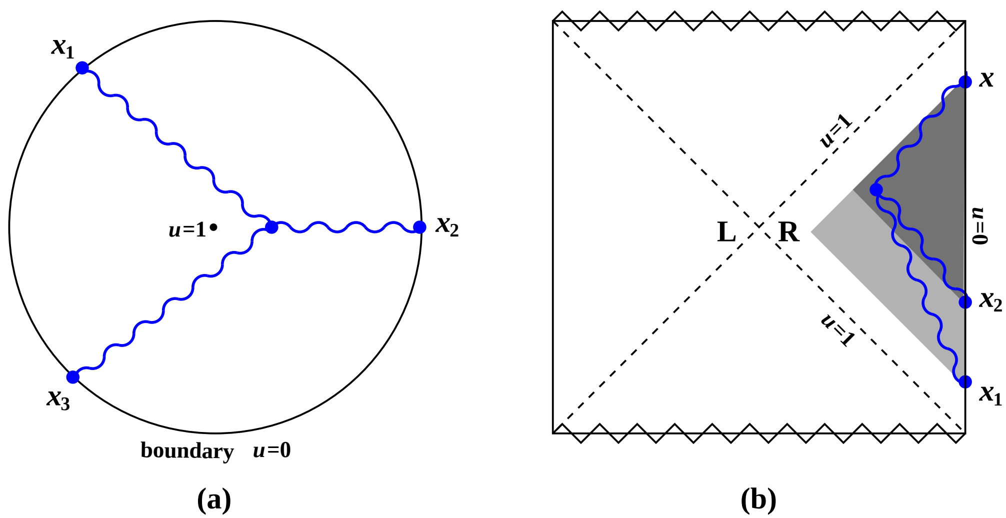
$$\left\langle j^{(3)\mu}(x)\right\rangle_{A_{\rm cl}} = \frac{1}{2}\int d^4x_1\; d^4x_2\; G_{\rm R}^{(ab3)\alpha\beta\mu}(x_1,x_2;x)\; A^a_{\alpha,{\rm cl}}(x_1)\; A^b_{\beta,{\rm cl}}(x_2)$$

where

$$\begin{aligned} G_{\rm R}^{(ab3)\alpha\beta\mu}(x_1,x_2;x) &= \theta(t-t_1)\theta(t_1-t_2)\langle[[j^{(3)}(x),j^a(x_1)],j^b(x_2)]\rangle \\ &+ \theta(t-t_2)\theta(t_2-t_1)\langle[[j^{(3)}(x),j^b(x_2)],j^a(x_1)]\rangle \end{aligned}$$

The physical problem of tracking the jet evolution reduces to a technical problem: how to actually compute the 3-point correlators.

Witten diagram for (a) 3-point boundary correlator in imaginary-time AdS$_5$-Schwarzschild and (b) retarded 3-point boundary correlator $G_{\rm R}(x_1, x_2; x)$ in real-time AdS$_5$-Schwarzschild.



Technical comments (helpful approximations):

-the jet has large energy (WKB approximation useful).

-the R-charge density is measured at scales which are large comparative to $1/E$ or even $1/T$ (one measures a “smeared response”)

Then, the Fourier-transform 3-point correlator factorizes (almost).

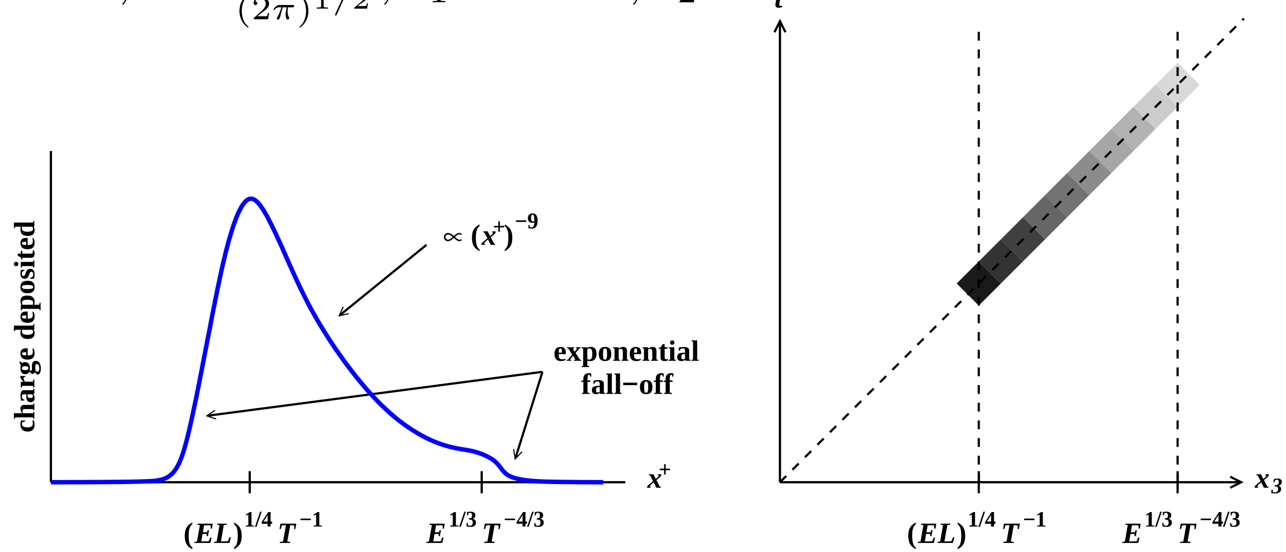
Final result:

$$\left(\partial_t - \frac{1}{2\pi T}\nabla^2\right)\left\langle j^{(3)0}(x)\right\rangle_{A_{\rm cl}} \simeq \bar{\mathcal{Q}}^{(3)}\,\Theta(x),$$

where the charge deposition function is

$$\Theta(x) \simeq 2\,\delta_L(x^-)\,\theta(x^+)\begin{cases} \frac{(4c^4EL)^2}{(2\pi T)^8(x^+)^9}\,\Psi\left(-\frac{c^4EL}{(2\pi Tx^+)^4}\right), & x^+ \ll E^{1/3}/(2\pi T)^{4/3}; \\ \frac{(2\pi T)^4 2(c_2L)^2}{E}\,\Psi(0)\exp\left(-\frac{c_1(2\pi T)^{4/3}x^+}{E^{1/3}}\right), & x^+ \gg E^{1/3}/(2\pi T)^{4/3}. \end{cases}$$

with $\Psi(y) = e^{-2y^2}$, $c \equiv \frac{\Gamma^2(\frac{1}{4})}{(2\pi)^{1/2}}$, $c_1 \simeq 0.927$, $c_2 \simeq 3.2$..



(a) (b)

Jet quenching revisited, simplified

Consider a source

$$\text{source}(x) \sim e^{i\bar{\boldsymbol{k}}\cdot\boldsymbol{x}}\Lambda_L(x)$$

which creates a localized perturbation at the boundary, which then propagates in the 5th dimension, eventually falling into the horizon.

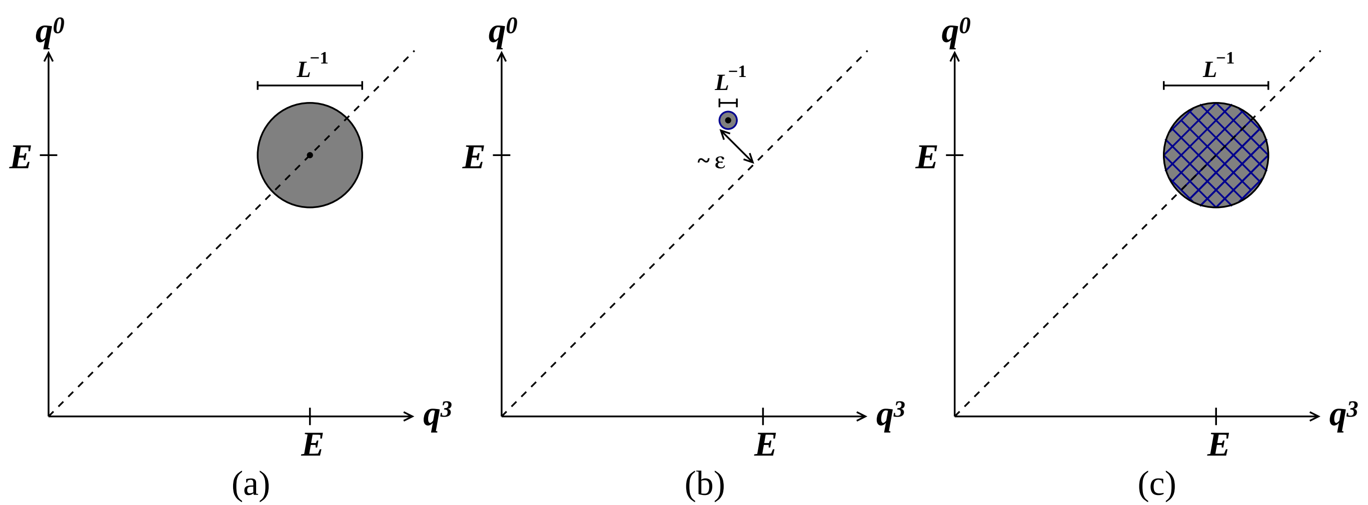
Previously, $\bar{\boldsymbol{k}}$ was light-like:

$$\text{(a)} \qquad \bar{\boldsymbol{k}}^\mu = (E, 0, 0, E)$$

Now, let us choose $\bar{\boldsymbol{k}}$ off the light-cone

$$\text{(b)} \qquad \bar{\boldsymbol{k}}^\mu = (E+\epsilon, 0, 0, E-\epsilon)$$

Qualitative picture of momenta used to generate jets.

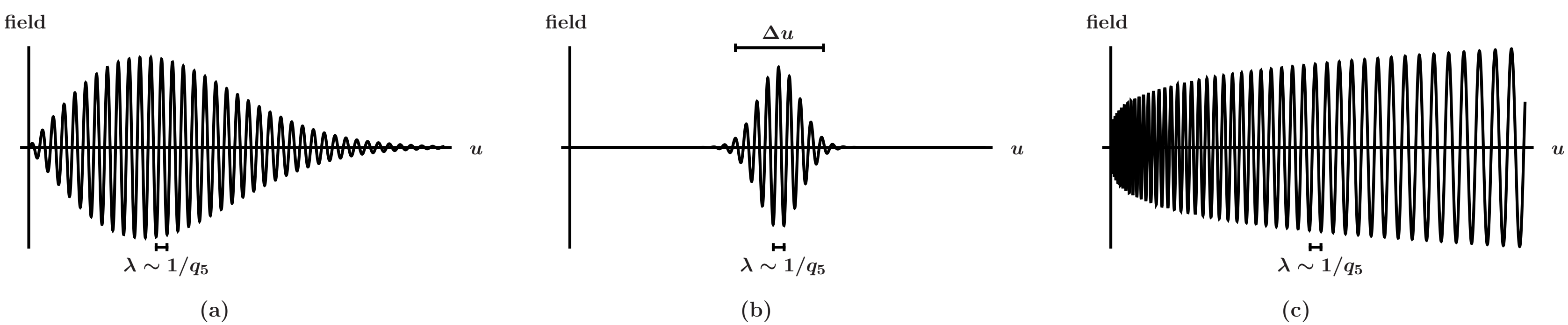


(a,b) A snapshot in time of waves in the fifth dimension u for times after the boundary source has turned off but relatively early (before the wave gets very close to the horizon).

(a) shows the type of wave generated by a localized source that superposes a range of q^2 values.

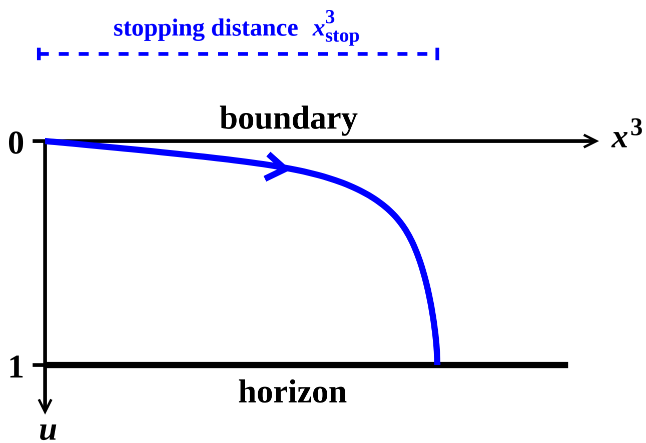
(b) shows the wave packet generated by a source with approximately well-defined q^2

(c) shows a single 4-momentum component, corresponding to a single, definite value of 4-momentum q_μ.

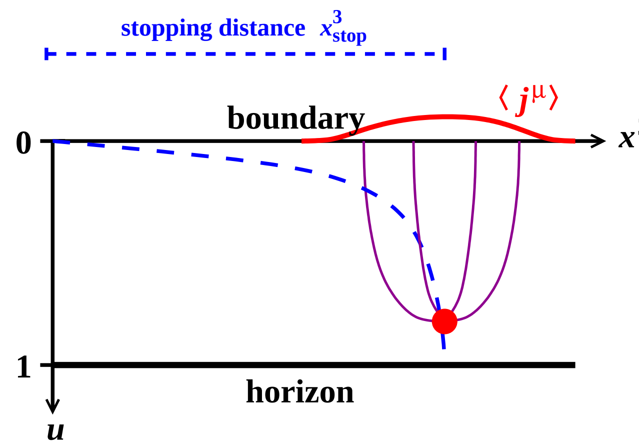


(a) A classical particle in the AdS$_5$-Schwarzschild space-time, moving in the $\boldsymbol{x^3}$ direction as it falls from the boundary to the black brane in the fifth dimension u.

(b) The presence of the particle perturbs the boundary theory in a manner that spreads out diffusively as the particle approaches the horizon for $\boldsymbol{x^0} \to \infty$.



(a)



(b)

The $x^{\mathbf{3}}$ distance traveled is estimated from the geodesic equation:

$$x^{\mathbf{3}}_{\text{stop}} \simeq \frac{c}{\sqrt{2}} \left(\frac{\vec{q}^2}{-\boldsymbol{q}^2} \right)^{1/4} \simeq \frac{c}{2} \left(\frac{E}{\epsilon} \right)^{1/4}$$

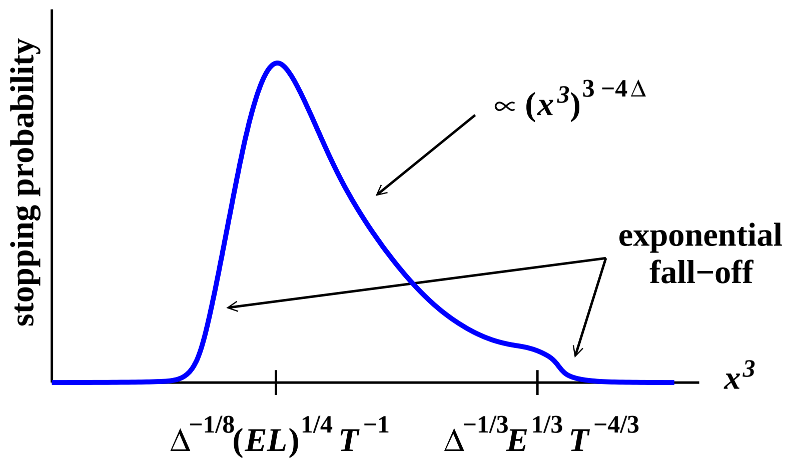
L enters $x^{\mathbf{3}}_{\text{stop}}$ through the virtuality $\boldsymbol{q}^2 \sim E\epsilon \sim E/L$ [1101.2689].

Massive particles

For supergravity modes on $AdS_5 \times S^5$ mass is related to the conformal dimension of the CFT BPS operator

$$(Rm)^2 = \Delta(\Delta - d)$$

The probability distribution of jet stopping distances for scalar or transverse BPS sources with conformal dimension Δ [1101.2689]. (R-charge current case corresponds to $\Delta{=}3$.) Here we assume that Δ is held fixed when taking the limit of large energy E (as well as large coupling $g^2 N_\mathrm{c}$ and large N_c).



The typical scale, with the same $(EL)^{1/4}$ dependence, is again where most of the charge is being deposited; however, the heavier a KK mode, the sooner it stops; this is similar to weakly coupled field theory where the more partons are available to carry the total momentum, the shorter the stopping distance.

Adding a finite chemical potential

- Start with $\mathcal{N}=4$ $d=5$ $SU(2)\times U(1)$ gauged supergravity: metric g_{mn}, dilaton ϕ, $SU(2)\times U(1)$ gauge fields A^I_m and a_m, and two antisymmetric tensor fields B^α_{mn} which are charged under the action of the $U(1)$ field. It is a consistent truncation.
- Finite chemical potential (μ) in $\mathcal{N}=4$ sYM $\leftrightarrow$ supergravity in AdS-Reissner-Nordstrom background.

$$d\bar{s}^2 = \frac{4\pi^2 R^2 T_H^2}{(2-\zeta)^2 u}(-f(u)dt^2 + d\vec{x}^2) + \frac{R^2}{4u^2 f(u)}du^2, \qquad u = \frac{r_+^2}{r^2}$$

$$f(u) = (1-u)(1+u-\zeta u^2), \qquad \bar{Z}_0 = \frac{\sqrt{3\zeta}r_+}{2R^2}(u-1)$$

$$T_H = \frac{(2-\zeta)r_+}{2\pi R^2}, \qquad \mu = \frac{\sqrt{3\zeta}}{2R^2}r_+ = \frac{\sqrt{3\zeta}}{2(2-\zeta)}2\pi T_H, \qquad 0\le\zeta\le 2.$$

- Typical stopping distance ($\mu \ll E, \quad 1/L \ll E, \quad T \ll E$):

$$x^{\mathbf{3}}_{\text{stop}} \simeq \frac{2-\zeta}{2}\frac{1}{2\pi T_H}\frac{8\Gamma^2(\frac{5}{4})}{\sqrt{\pi}(1+\zeta)^{1/4}}\left(\frac{\vec{q}^2}{-\boldsymbol{q}^2}\right)^{1/4}$$

Finite chemical potential leads to a jet quenching enhancement.

Hydrodynamic regime: second order hydro coefficients

In the hydro regime, the stress tensor of a CFT can be writen as:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + \Pi^{\mu\nu}, \qquad T^{\mu\nu}_{\text{eq}} = (\epsilon + P)U^\mu U^\nu + P g^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\tau_\Pi \left({}^{\langle}U \cdot \nabla\sigma^{\mu\nu\rangle} + \frac{1}{3}\nabla \cdot U\sigma^{\mu\nu} \right) + \kappa\left(R^{\langle\mu\nu\rangle} - 2U_\rho U_\sigma R^{\rho\langle\mu\nu\rangle\sigma} \right)$$

$$+\lambda_1 \sigma^{\langle\mu}{}_\rho \sigma^{\nu\rangle\rho} + \lambda_2 \sigma^{\langle\mu}{}_\rho \Omega^{\nu\rangle\rho} + \lambda_3 \Omega^{\langle\mu}{}_\rho \Omega^{\nu\rangle\rho} + \ldots$$

where σ and Ω are the fluid's shear and vorticity tensors:

$$\sigma^{\mu\nu} = 2\nabla^{\langle\mu}U^{\nu\rangle} \equiv \frac{1}{2}\Delta^{\mu\rho}\Delta^{\nu\sigma}(2\nabla_\rho U_\sigma + 2\nabla_\sigma U_\rho) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\rho\sigma}2\nabla_\rho u_\sigma$$

$$\Omega^{\mu\nu} = \frac{1}{2}\Delta^{\mu\rho}\Delta^{\nu\sigma}(\nabla_\rho U_\sigma - \nabla_\sigma U_\rho)$$

where $\Delta^{\mu\nu}$ are transverse (to the fluid's velocity) projectors:

$$\Delta^{\mu\nu} = g^{\mu\nu} + U^\mu U^\nu$$

Baier, Romatschke, Son, Starinets, Stephanov

How to compute the hydro coefficients: until recently η, τ_Π, κ were obtained via Kubo formulae from 2-point stress correlators. What about the others? Answer: use 3-point retarded stress correlators!

- Moore and Sohrabi 2010: compute the fluid's response to a small, slowly varying gravitational perturbation, and derive Kubo-type formulae for 2nd order hydro coefficients.

$$\begin{aligned}\langle T^{\mu\nu}(z)\rangle_h &= \langle T^{\mu\nu}\rangle_{h=0} - \tfrac{1}{2}\int d^4x\, G_{ra}^{\mu\nu|\rho\sigma}(z;x)h_{\rho\sigma}(x)\\ &+ \tfrac{1}{8}\int d^4x \int d^4y\, G_{raa}^{\mu\nu|\rho\sigma|\tau\zeta}(z;x,y)h_{\rho\sigma}(x)h_{\tau\zeta}(y) + \ldots\end{aligned}$$

- Solve the conservation law $\nabla_\mu T^{\mu\nu} = 0$, and $T^\mu_\mu = 0$ iteratively, in the fluid's velocity U^μ, and order-by-order in the metric fluctuations; compare with the previous expansion in terms of correlators $\Longrightarrow$ get Kubo-type formulae!

- Our formulae [1105.4645] for 2nd order hydro coefficients ($q \equiv (\omega, 0, 0, k)$, etc.):

$$\lim_{\substack{\omega_1\to 0\\ \omega_2\to 0}} \partial_{\omega_1}\partial_{\omega_2} \lim_{\substack{k_1\to 0\\ k_2\to 0}} G^{xy|xz|yz}(q;q_1,q_2) = -\lambda_1 + \eta\tau_\Pi$$

$$\lim_{\substack{\omega_2\to 0\\ k_1\to 0}} \partial_{k_2}\partial_{\omega_1} \lim_{\substack{\omega_2\to 0\\ k_1\to 0}} G^{xy|yz|tx}(q;q_1,q_2) = -\tfrac{1}{4}\lambda_2 + \tfrac{1}{2}\eta\tau_\Pi$$

$$\lim_{\substack{k_1\to 0\\ k_2\to 0}} \partial_{k_1}\partial_{k_2} \lim_{\substack{\omega_1\to 0\\ \omega_2\to 0}} G^{xy|0x|0y}(q;q_1,q_2) = -\tfrac{1}{4}\lambda_3$$

- Retarded supergravity bulk-to-boundary propagators:

$$
\begin{aligned}
\delta g^x_y &= C_5 (1-u)^{-i\omega/2}\Big(1 - i\frac{\omega}{2}\ln(1+u) + \omega^2(-\frac{1}{2}\mathsf{Li}(2, \frac{1-u}{2}) \\
&+ \frac{1}{8}\ln^2(1+u) + (1-\frac{\ln 2}{2})\ln(1+u)) - k^2\ln(1+u) + \dots\Big) \\
C_5 &= \Big(1 + \frac{\omega^2(\pi^2 - 6\ln^2 2)}{24} + \dots\Big) h^x_y
\end{aligned}
$$

- Supergravity quadratic action:

$$
\begin{aligned}
\delta^{(2)}\mathcal{S} &= \frac{1}{8}\int_{u=0}\frac{1}{u}\partial_5\Big(- \delta g^\mu_\mu \delta g^\nu_\nu + \delta g^\mu_\nu \delta g^\nu_\mu\Big) \\
&+ \frac{1}{4}\int_{u=0}\Big(\frac{3}{4}(h^0_0)^2 - \frac{1}{2}h^0_0 h^i_i + h^0_i h^i_0 + \frac{1}{4}h^i_i h^j_j - \frac{1}{2}h^i_j h^j_i\Big), \qquad i,j,k = 1,2,3
\end{aligned}
$$

Recover Baier, Romatschke, Son, Starinets, Stephanov:

$$\begin{aligned} G^{xy|xy}_{AdS} &= -\frac{\delta^2 \mathcal{S}}{\delta^2 h_{xy}} \\ &= \frac{N_c^2}{2^7\pi^2} - i\frac{N_c^2\omega}{2^6\pi^2} + \frac{(\omega^2(1-\ln 2)-k^2)N_c^2}{2^6\pi^2} + \ldots \end{aligned}$$

The result derived in the hydrodynamic limit from solving the conservation law of the stress tensor is

$$G^{xy|xy}_{\text{hydro}} = \frac{1}{3}\bar{\epsilon} - i\eta\omega + \eta\tau_{\Pi}\omega^2 - \frac{1}{2}\kappa(\omega^2+k^2) + \ldots$$

where the background energy density is $\bar{\epsilon} = \frac{3}{8}N_c^2\pi^2T^4$.

Determine

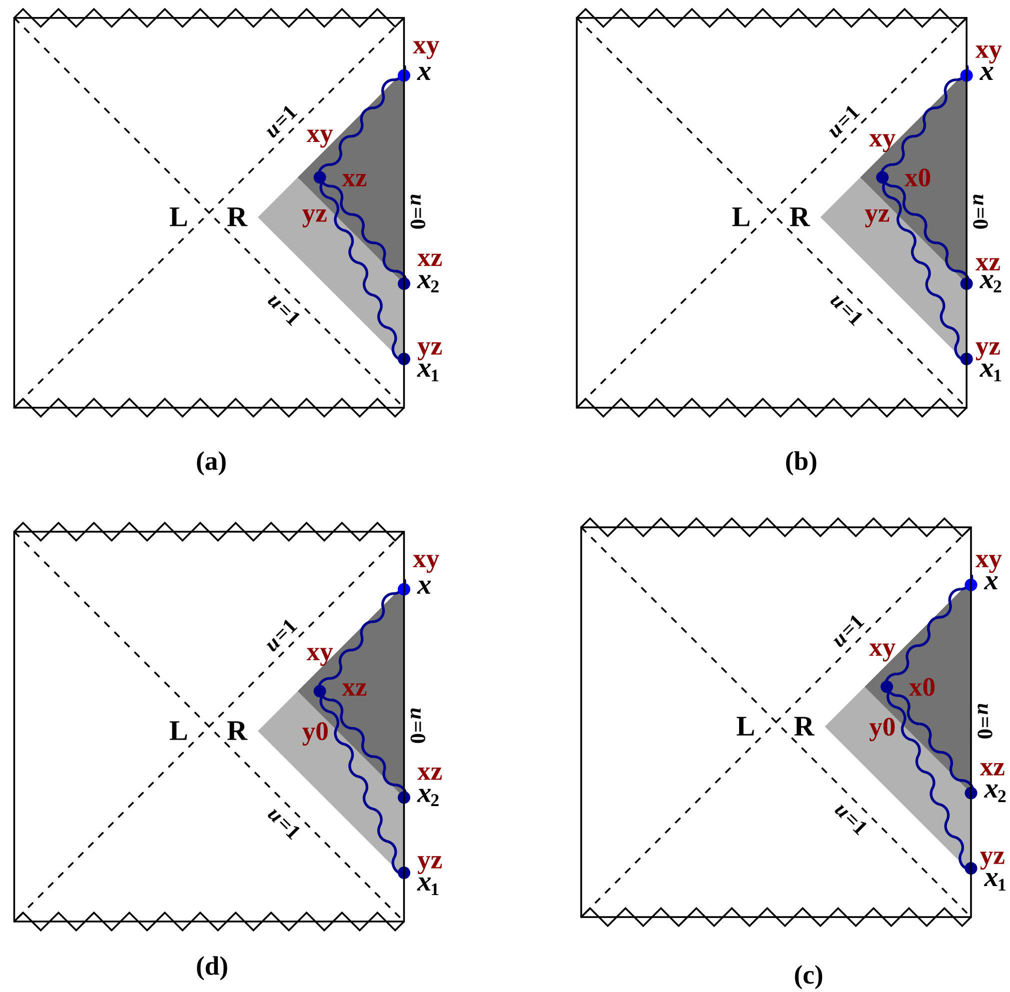
$$\eta = \frac{\pi N_c^2 T^3}{8}, \kappa = \frac{N_c^2 T^2}{8}, \eta\tau_{\Pi} = \frac{N_c^2(2-\ln 2)T^2}{16}.$$

What about λ_1, λ_2 and λ_3?

Need 3-point stress tensor correlators!

This is an independent check of the values determined by Bhattacharyya, Hubeny, Minwalla, Rangamani.

Real-time Witten diagrams for the retarded 3-point correlator $G^{xy|yz|xz}(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2)$ with the boundary point $\boldsymbol{x}$ having the largest time; $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ can have any time order.



$$\lim_{\substack{k_1\to 0\\ k_2\to 0}} G_{AdS}^{xy|yz|xz} = \frac{N_c^2}{2^4\pi^2}\left[\frac{1}{2^3} - i\frac{\omega_1+\omega_2}{2^2} - \frac{(\omega_1\omega_2+\omega_1^2+\omega_2^2)(\ln 2 - 1)}{2^2} + \ldots\right]$$

$$\lim_{\substack{k_1\to 0\\ k_2\to 0}} G_{\text{hydro}}^{xy|yz|xz} = \frac{N_c^2}{2^4\pi^2}\left[\tfrac{1}{3}\bar{\epsilon} - i\eta(\omega_1+\omega_2) + \eta\tau_\Pi(\omega_1^2+\omega_2^2+\omega_1\omega_2) - \tfrac{1}{2}\kappa(\omega_1^2+\omega_2^2) - \lambda_1\omega_1\omega_2 + \ldots\right]$$

- λ_1: $\lambda_1 = \frac{N_c^2}{2^6\pi^2}, \Rightarrow \lambda_1 = \frac{N_c^2 T^2}{16}.$

$$\lim_{\substack{\omega_1\to 0\\ \omega_2\to 0}} G_{AdS}^{xy|ty|tx} = \frac{N_c^2}{2^6\pi^2}\left[-\frac{1}{2} + (k_1^2+k_2^2) + \ldots\right]$$

$$\lim_{\substack{\omega_1\to 0\\ \omega_2\to 0}} G_{\text{hydro}}^{xy|ty|tx} = -\tfrac{1}{3}\bar{\epsilon} + \tfrac{1}{2}\kappa(k_2^2+k_1^2) - \tfrac{1}{4}\lambda_3 k_1 k_2 + \ldots$$

- λ_3: $\lambda_3 = 0.$

$$\lim_{\substack{k_1\to 0\\ \omega_2\to 0}} G_{AdS}^{xy|yz|tx} = \frac{N_c^2}{2^6\pi^2}\omega_1 k_2 + \ldots$$

$$\lim_{\substack{k_1\to 0\\ \omega_2\to 0}} G_{\text{hydro}}^{xy|yz|tx} = (-\tfrac{1}{4}\lambda_2 + \tfrac{1}{2}\eta\tau_\Pi)\omega_1 k_2 + \ldots$$

- λ_2: $\lambda_2 = -\frac{N_c^2}{2^5\pi^2}\ln 2 \Rightarrow \lambda_2 = -\frac{1}{8}N_c^2 T^2 \ln 2.$

Summary

We gave a simple prescription on how to compute higher-order correlation functions in real-time AdS/CFT at zero and finite temperature.

We computed the stopping distance of a high energy jet by measuring the R-charge in its wake at late times. This reduces to a real-time finite temperature 3-point retarded $\langle j^\dagger_{source} j_{response} j_{source} \rangle$ correlator.

While the $E^{1/3}$ scale is still present, as the maximal distance the jet travels, the typical stopping distance scale is $(EL)^{1/4}$.

Other dimensions scales: For AdS_{d+1},

- the maximal distance traveled scales as $E^{(d-2)/(d+2)}$
- the smaller scale, where most of the charge is deposited scales as $(EL)^{(d-2)/(2d)}$

We gave a simple interpretation for the new scale, as well as for the power law fall-off x^{-9} which generalizes to $x^{3-4\Delta}$. (Δ the conformal dimension of some BPS CFT operator.)

Work in progress: Correlator of two R-charge densities at late times, in the background of the source. Consider finite temperature and finite chemical potential.

We computed stress tensor 3-point correlators at finite temperature, and from Kubo-type formulae we extracted the 2nd order hydro coefficients. Note: a deformation of the AdS-Schwarzschild geometry is not necessary. BHMR's results recovered.

Back-up slides

Intermediate steps

- The supergravity vertex:

  Maximally susy 5d gauged supergavity has gauge bosons which transform in the adjoint rep of $SU(4)$. We are looking at an $SU(2)$ subgroup.

$$-\frac{1}{4g_{\rm SG}^2 R}\int d^5x\sqrt{-g}\,F^{IJa}F^a_{IJ}-\frac{k}{96\pi^2}\int d^5x\,\left[d^{abc}\varepsilon^{IJKLM}A^a_I(\partial_J A^b_K)(\partial_L A^c_M)+\cdots\right]$$

  where

$$g_{\rm SG}=\frac{4\pi}{N_c}\qquad \text{and} \qquad k=N_c^2-1$$

  The relevant contribution:

$$-\frac{f^{abc}}{2g_{\rm SG}^2 R}\int d^5x\sqrt{-g}\,g^{IM}g^{JN}(\partial_I A^a_J-\partial_J A^a_I)A^b_M A^c_N$$

Crucial approximation and zero-temperature response

The causal gauge boson $T = 0$ propagator is

$$\mathcal{G}_{\mu\nu}(q,\bar{u}) = \sqrt{4\bar{u}q^2}\,K_1\big(\sqrt{4\bar{u}q^2}\big)\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{q_\mu q_\nu}{q^2}$$

where $\bar{u} = z^2/4$, $q^2 = \eta^{\mu\nu} q_\mu q_\nu$ and $\omega \to \omega \pm i\epsilon$ for R/A propagators.

Then in the limit $\bar{u}Q^2 \ll 1$ the response bulk-to-boundary propagator simplifies to

$$\mathcal{G}^{\mathrm{R}}_{\sigma\mu}(Q,\bar{u}) \simeq \eta_{\sigma\mu}$$

and the 1-point R-charge function becomes

$$\langle j^\mu(x)\rangle \simeq -\eta^{\mu\rho}\,\frac{\mathcal{N}_A^2}{g_{\mathrm{SG}}^2}\int_0^\infty \frac{d\bar{u}}{\bar{u}}\,\mathcal{A}(x,\bar{u})^* i\overleftrightarrow{\partial_\rho}\mathcal{A}(x,\bar{u})$$

where $\mathcal{A}$ is the convolution of the source with the retarded bulk-to-boundary propagator

$$\mathcal{A}(x,\bar{u}) \equiv \int_q \mathcal{G}^{\mathrm{R}}_{\perp}(q,\bar{u})\,\tilde{\Lambda}_L(q-\bar{k})\,e^{iq\cdot x}$$

The computation is now factorized, and hinges on determining $\mathcal{A}$.

High-energy approximation

$$q^2 \simeq 4Eq_+, \qquad q_+ = \frac{1}{2}q^- = \frac{q^3 - q^0}{2}, q_- = \frac{1}{2}q^+ = \frac{q^3 + q^0}{2} \simeq E$$

Choose the envelope to be Gaussian-type

$$\Lambda_L(q_+, x^-) = 2\sqrt{\pi} L e^{-(q_+ L)^2} e^{-(x^-/2L)^2}$$

then

$$\mathcal{A}(x, \bar{u}) \simeq -i \, \frac{4\bar{u}E}{(x^+)^2} \, e^{iEx^-} \, e^{i4\bar{u}E/x^+} \Lambda_L \left(-\frac{4\bar{u}E}{(x^+)^2}; x^- \right) \theta(x^+)$$

and the response function is

$$\begin{aligned} \langle j^\mu(x) \rangle &\simeq 2\bar{k}^\mu \, \frac{\mathcal{N}_A^2}{g_{\rm SG}^2} \int_0^\infty \frac{d\bar{u}}{\bar{u}} \, |\mathcal{A}(x, \bar{u})|^2 \\ &\simeq 2\pi \bar{k}^\mu \, \frac{\mathcal{N}_A^2}{g_{\rm SG}^2} \, e^{-(x^-)^2/2L^2} \, \theta(x^+) \end{aligned}$$

Note: keeping L finite is crucial in getting a sensible (non-divergent) answer.

Schrodinger interpretation

Take the linearized gauge boson equation of motion

$$\left(\partial_{\bar{u}}^2 + i\,\frac{4E}{\bar{u}}\partial_+\right) A_\perp(x^+,\bar{u}) \simeq 0$$

redefine $A_\perp = \sqrt{z}\phi$ to cast it in a Schrodinger form

$$2i\partial_+\phi = \left(-\frac{\partial_z^2}{2E} + \frac{3}{8Ez^2}\right)\phi$$

Conserved probability

$$\int dz|\phi|^2 = \frac{1}{2}\int \frac{d\bar{u}}{\bar{u}}|\mathcal{A}|^2$$

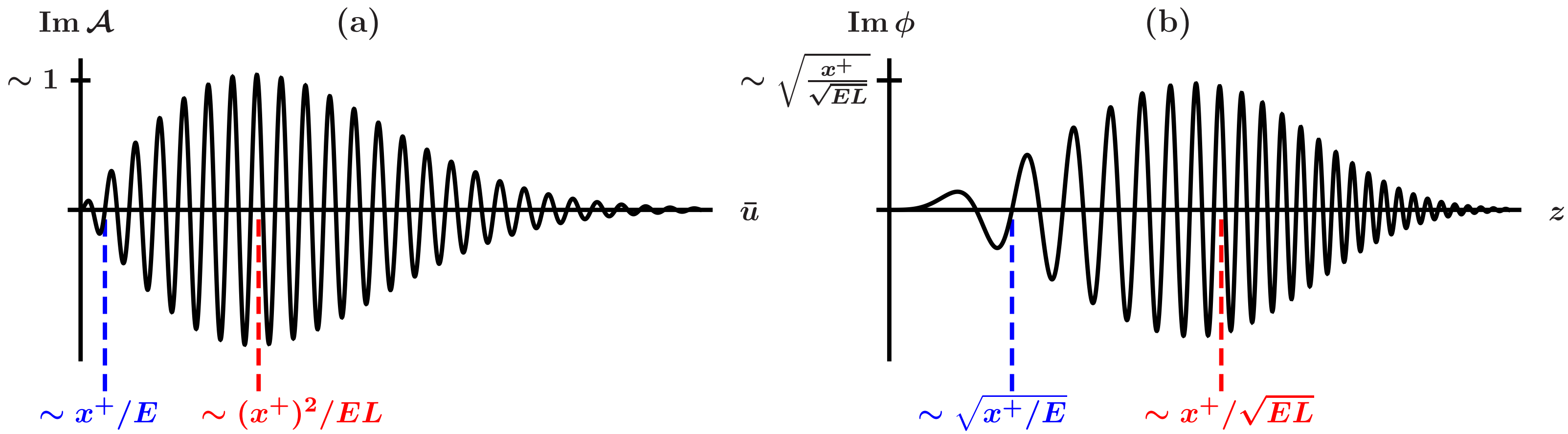
$\mathcal{A}$ solves the linearized $A_\perp$ equation.

Recall

$$\mathcal{A} \propto \frac{\bar{u}EL}{(x^+)^2}\, e^{i4\bar{u}E/x^+}\, \exp\left[-\frac{(4\bar{u}EL)^2}{(x^+)^4}\right]$$

Setting $L=0$ amounts to studying non-localized solutions, which have an infinite normalization $\int_z |\phi|^2$.

Qualitative pictures of the real or imaginary parts of (a) $\mathcal{A}$ and (b) ϕ



Two scales:

$z \sim \sqrt{x^+/E}$ called "diffusion scale" by Hatta, Iancu and Mueller, and $z \sim x^+/(EL)$ which characterizes the bulk of the prob. density.

At finite temperature the time scale for the first oscillation of $\mathcal{A}$ to fall into the horizon is $x_3 \sim E^{1/3}/T^{4/3}$, but the bulk of the probability falls on a shorter scale as $x_3 \sim (EL)^{1/4}/T$.

Consider that the original source is a superposition of wave packets with $\bar{k}$ off the light-cone by an amount ϵ, and the spread in momenta $1/L \ll \epsilon$. Each small wave packet may be approximated by a point-particle.

The particle falls into the horizon after having traveled a distance

$$x^{\mathbf{3}}_{\text{stop}} \simeq \frac{c}{\sqrt{2}} \left(\frac{|\boldsymbol{q}|^2}{-q^2} \right)^{1/4} \simeq \frac{c}{2} \left(\frac{E}{\epsilon} \right)^{1/4}$$

Each wave packet will travel a different distance, depending on its energy and ϵ. The total charge deposited by the initial source will be the weighted average of all these individual wave packets, with an weight equal to the probability that the source produces a jet of a given q^2, $\mathcal{P}$.

$$\text{Prob}(x^{\mathbf{3}}) \simeq \int d(q^2)\, \mathcal{P}(q^2)\, \delta \left(x^{\mathbf{3}} - x_{\text{stop}^{\mathbf{3}}}(q^2) \right)$$

For the original source, the typical value for q^2 is $q^2 \sim E/L$.

$x^{\mathbf{3}} \sim (EL)^{1/4}$ is then the typical distance traveled by the jet.

The distribution of stopping distances

Spectral density: $\rho(q) = 2Im(G_{\perp}^{A}(q))$ where $G_{\perp}^{A}$ is the gauge boson 2-point advanced correlator in momentum space.

For large momenta, we can approximate the 2-point function by the vacuum result

$$\begin{aligned}
\rho(q) &= Im\Big[-\frac{1}{g_{\rm SG}^2} \lim_{\bar{u}\to 0} \partial_{\bar{u}} \mathcal{G}_{\perp}^{A} \Big] \\
&= Im\Big[-\frac{q^2}{g_{\rm SG}^2} \Big(\ln(\bar{u}q^2) + 2\gamma_E \Big) \Big] \\
&= \frac{\pi}{g_{\rm SG}^2} (-q^2)\theta(-q^2)\,\text{sign}(q^0)
\end{aligned}$$

Back-of-envelope calculation:

$$x_{\text{stop}}^{\mathbf{3}} \sim \frac{E^{1/4}}{q_{+}^{1/4}}, \qquad \mathcal{P} dq_+ \sim q^2 dq_+ \sim q_+ dq_+ \sim (x_{\text{stop}}^{\mathbf{3}})^{-9} dx_{\text{stop}}^{\mathbf{3}}$$

so

$$\text{Prob}(x^{\mathbf{3}}) \simeq (x^{\mathbf{3}})^{-9}$$

The maximal $E^{1/3}$ scale from the typical $(EL)^{1/4}$ scale:

The classical particle picture must break down for a stopping distance of the order L.

Back-of-envelope calculation:

$$x_{\text{stop}}^{\mathbf{3}} \sim L \sim (EL)^{1/4} \qquad \Rightarrow \qquad L \sim E^{1/3} \sim x_{\text{stop max}}^{\mathbf{3}}$$